Biophysics I (BPHS 4080)

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Website: http://www.yorku.ca/cberge/4080W2018.html
Summary: HH Equations

\[ \frac{1}{2\pi a(r_o + r_i)} \frac{\partial^2 V_m}{\partial z^2} = C_m \frac{\partial V_m}{\partial t} + G_K(V_m, t) (V_m - V_K) \]

\[ + G_{Na}(V_m, t) (V_m - V_{Na}) + G_L(V_m - V_L) \]

\[ G_K(V_m, t) = \overline{G}_K n^4(V_m, t) \]

\[ G_{Na}(V_m, t) = \overline{G}_{Na} m^3(V_m, t) h(V_m, t) \]

\[ n(V_m, t) + \tau_n(V_m) \frac{dn(V_m, t)}{dt} = n_\infty(V_m) \]

\[ m(V_m, t) + \tau_m(V_m) \frac{dm(V_m, t)}{dt} = m_\infty(V_m) \]

\[ h(V_m, t) + \tau_h(V_m) \frac{dh(V_m, t)}{dt} = h_\infty(V_m) \]

\[ \tau_x \frac{dx}{dt} + x = x_\infty \]

\[ \frac{dx}{dt} = \alpha_x (1 - x) - \beta_x x \]

\[ x_\infty = \alpha_x / (\alpha_x + \beta_x) \]

\[ \tau_x = 1 / (\alpha_x + \beta_x) \]

\[ \alpha_m = \frac{-0.1(V_m + 35)}{e^{-0.1(V_m+35)} - 1}, \]

\[ \beta_m = 4e^{-(V_m+60)/18}, \]

\[ \alpha_h = 0.07e^{-0.05(V_m+60)}, \]

\[ \beta_h = \frac{1}{1 + e^{-0.1(V_m+30)}}, \]

\[ \alpha_n = \frac{-0.01(V_m + 50)}{e^{-0.1(V_m+50)} - 1}, \]

\[ \beta_n = 0.125e^{-0.0125(V_m+60)}, \]
Four phases:

1. Local disturbance due to capacitance (behaves like cable model)

2. Onset: $V_m$ change triggers $m$ (increased $G_{Na}$ take $V_m$ with it)

3. Falloff: $h$ turns off, $n$ turns on (both work to lower $V_m$ back towards $V_k$, basis for absolute refractory period)

4. Undershoot: increased $G_k$ pushes $V_m$ beyond $V^0_m$ (basis for relative refractory period)

Note: Membrane current ($J_M$) can be parsed up into two components: a capacitive current ($J_C$) and an ionic current ($J_{ion}$)
Note: Fairly little net current across membrane (i.e., relatively few net ions transported)
Threshold

In vivo: For the same stimulus, sometimes an AP fires, sometimes it does not.

What is mechanism for a threshold?

Model exhibits ‘exceedingly narrow threshold region’

Note: Model is deterministic and does not capture stochastic behaviors manifest in vivo.
Threshold

Note lag for AP to occur (stems from capacitive build-up to threshold)

Figure 4.42

→ Note lag for AP to occur (stems from capacitive build-up to threshold)
Determine $J_{ion}-V_m$ relationship right after shock (dashed line)

- Current purely due to $C_m$
- Membrane “deciding” whether to fire AP or not

\[ J_{ion} = -J_C = -C_m \frac{dV_m}{dt} \]

Note: This picture only holds as a snapshot right after the stimulus
Equilibrium points

Stability

Threshold

Ohm’s Law: Negative resistance?
These pictures make it easy to envision a *stochastic* component too. (e.g., consider random force jittering object about)
Threshold

\[ G_K(V_m,t) = G'_K n^4(V_m,t) \]
\[ G_{Na}(V_m,t) = G_{Na} m^3(V_m,t) h(V_m,t) \]

\[ V_{Na} = \frac{RT}{F} \log \frac{c_{Na}^0}{c_{Na}} \]

\[ G_{Na}(V_m,t) = \frac{J_{Na}(V_m,t)}{V_m - V_{Na}} \]

\[ m_\infty(V_m) = m^3(V_m) h_\infty(V_m) \]

\[ J_{Na} \]
\[ (mA/cm^2) \]

\[ J_{K} + J_{L} \]
\[ (mA/cm^2) \]

\[ J_{ion} \]
\[ (mA/cm^2) \]

⇒ assume \( n \) and \( h \) are constant

⇒ Ultimately more than one ion is needed

(Na⁺ alone is insufficient)
Threshold: Phase Plane Portrait

assumes \( n \) and \( h \) are constant, but \( m \) varies dynamically

\[ V(t) = V_m(t) - V^o_m \text{ (mV)} \]
Refractory Period

Figure 1.13

Figure 4.52

Figure 4.53
Back to the question of spatial propagation...

Decremental conduction

\[ I_e(t) \]

Decrement-free conduction

\[ I_e(t) \]

Figure 1.16
Propagated APs

Space clamp
\( \frac{\partial V_m}{\partial z} = 0 \)

Step voltage clamp
\( \frac{\partial V_m}{\partial z} = \frac{\partial V_m}{\partial t} = 0 \)

Separation of ionic currents

Figure 1.22
Propagated APs

→ Solutions only stable for appropriate choice of conduction velocity
  (think back to cable model; $C_m$ matters!)

Figure 4.30

Figure 4.31
Propagated APs

Stimulus
(think cable model)

Figure 4.29
Note lag between $V_m$ and $G_m$ (stems from capacitive surge).

Similar picture as before for propagated AP.

Figure 4.32
Note lag between $V_m$ and $G_m$ (stems from capacitive surge)
Figure 5.1