

# Biophysics I (BPHS 4080)

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Website: <http://www.yorku.ca/cberge/4080W2018.html>

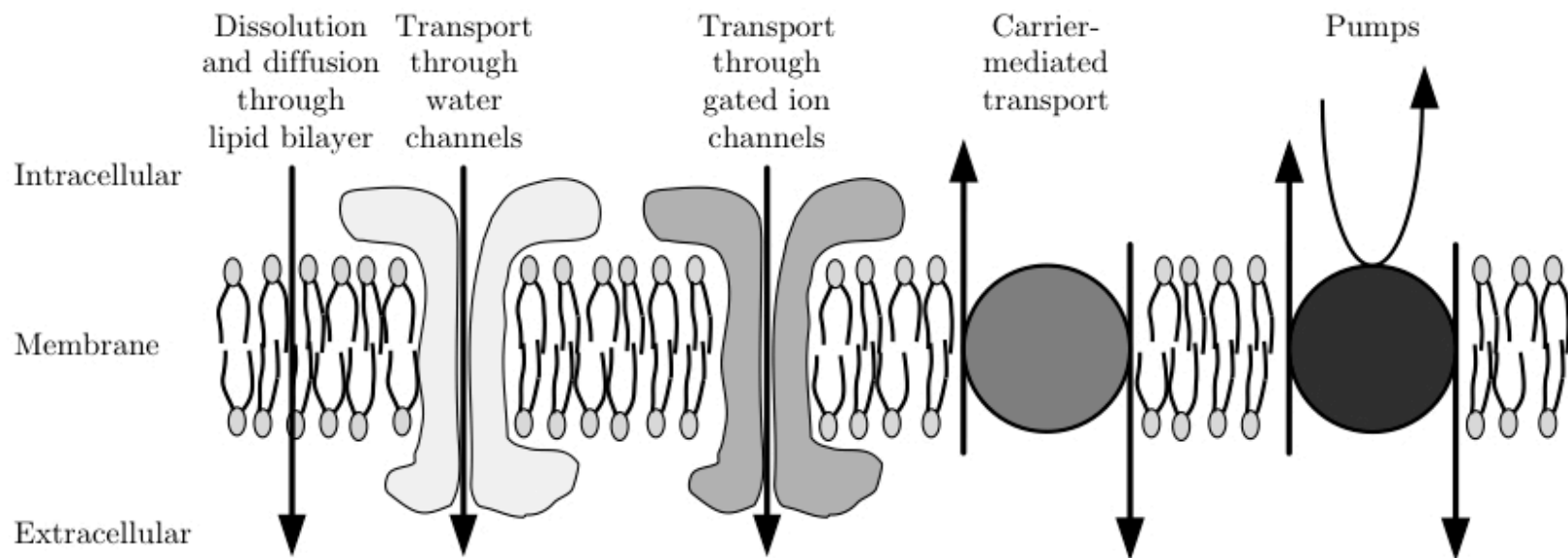


Figure 2.19

# Passive Transport: More than diffusion?

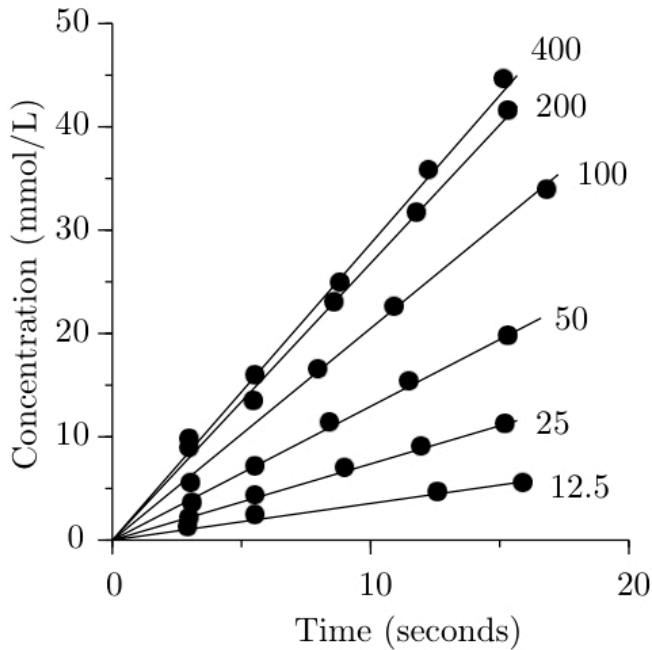


Figure 6.1

➤ Saturation occurs

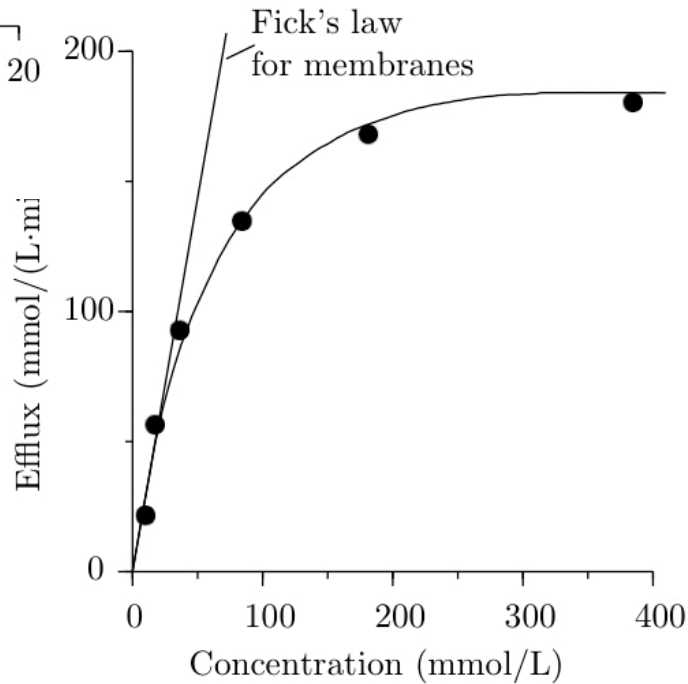


Figure 6.2

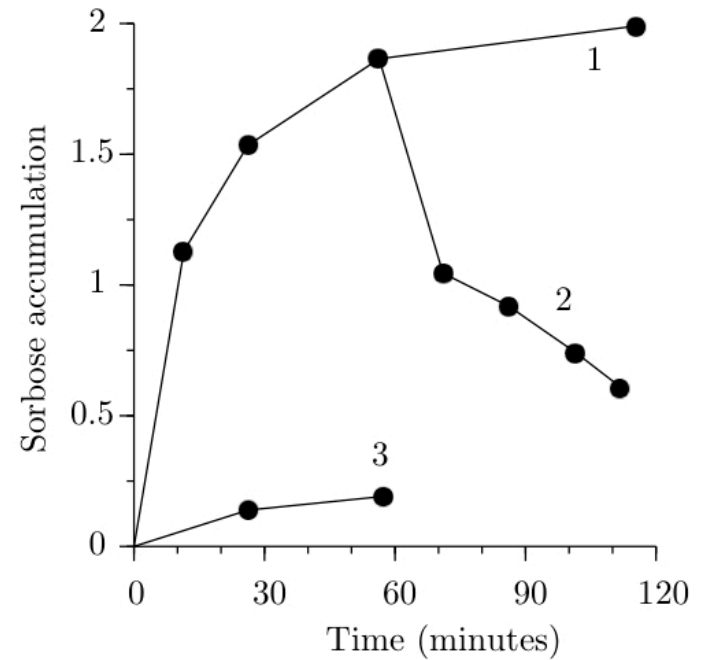


Figure 6.3

➤ Adding in other sugars affects things in a selective way

## Passive Transport: More than diffusion?

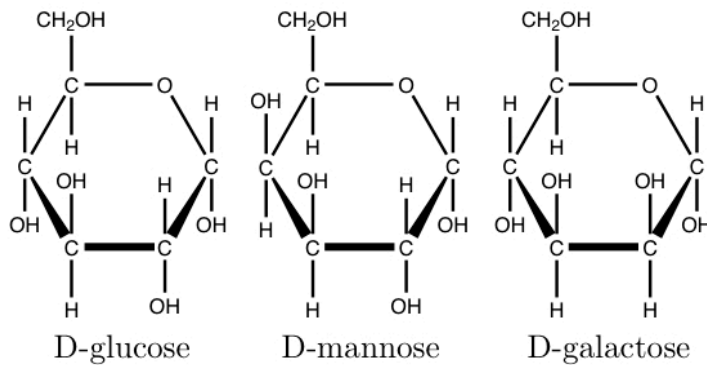


Figure 1.7

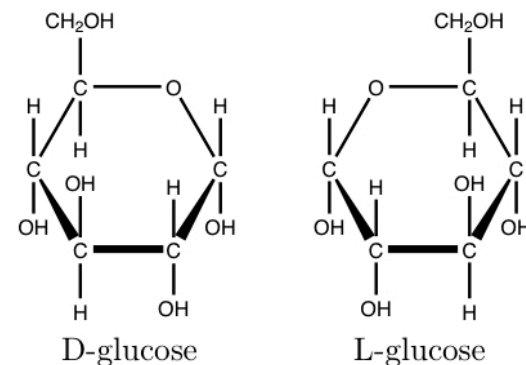


Figure 1.8

→ Structure of different solutes can have a big effect

# Notion of a “carrier”

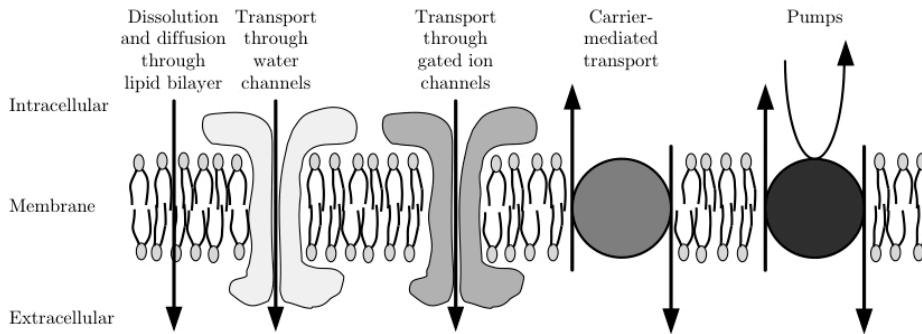
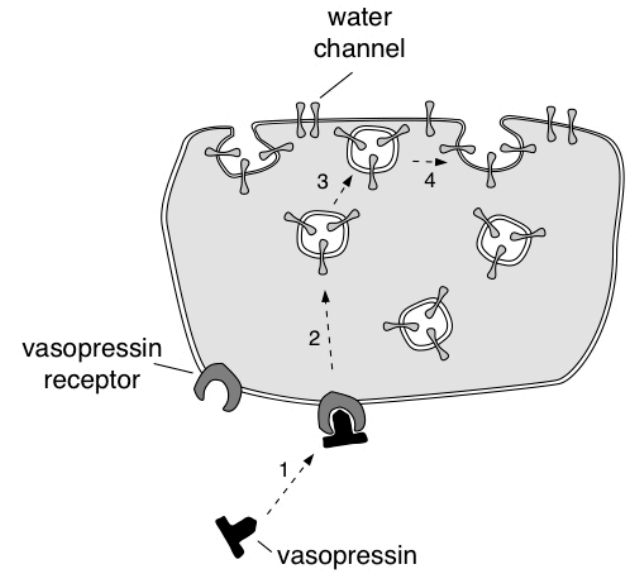


Figure 2.19



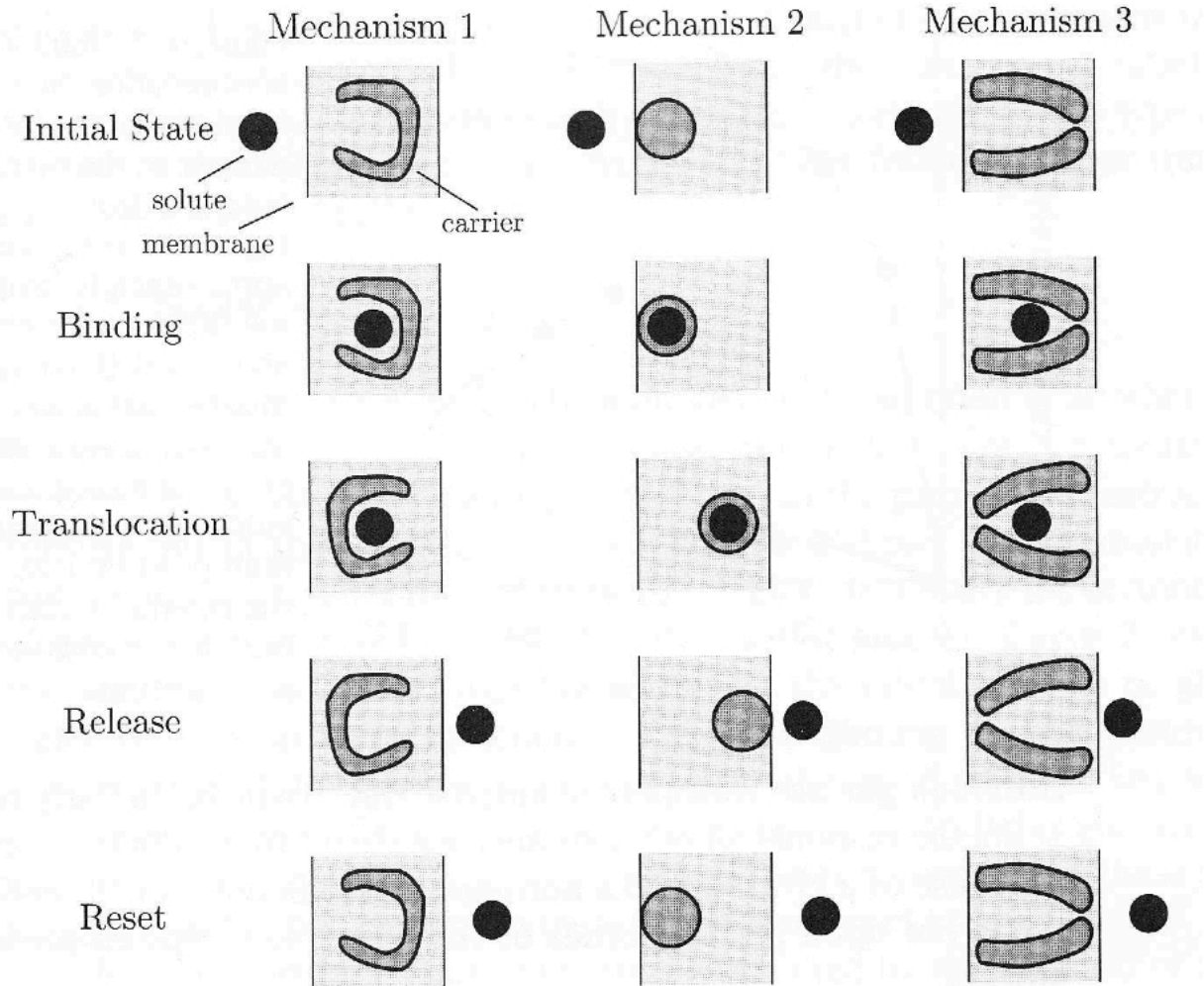
## Carrier-Mediated Transport: glucose transporter as example

### Distinguishing characteristics of glucose transport:

- facilitated -- i.e., faster than dissolve and diffuse
- structure specific -- different rates for even closely related sugars
- passive -- given a single solute, flow is down concentration gradient
- transport saturates -- solute-solute interactions
- transport can be inhibited -- solute-other interactions
- pharmacology (cytochalasin B)
- hormonal control (insulin)

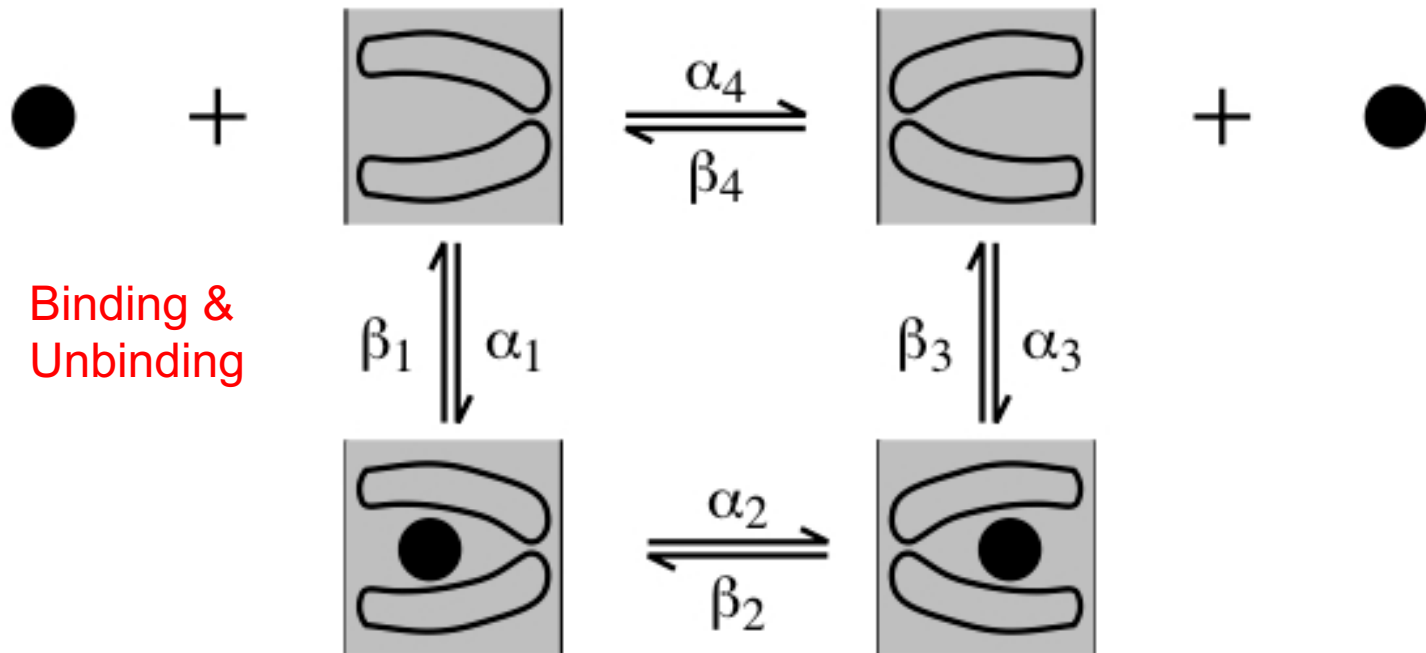
similar to water channels  
(Hg, vasopressin)

# Possible 'Carrier' Mechanisms



# General Four-State Carrier Model

## General Four-State Model

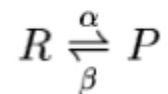


Binding &  
Unbinding

Translocation

## Chemical Kinetics (v1)

### First-order, reversible reaction



$$\frac{dc_R(t)}{dt} = \beta c_P(t) - \alpha c_R(t) \quad \text{AND} \quad \frac{dc_P(t)}{dt} = \alpha c_R(t) - \beta c_P(t)$$

**Equilibrium:**

$$\frac{dc_R(t)}{dt} = \frac{dc_P(t)}{dt} = 0 \quad \rightarrow \quad \beta c_P(\infty) = \alpha c_R(\infty)$$
$$\frac{c_P(\infty)}{c_R(\infty)} = \frac{\alpha}{\beta} = K_a \quad \left( \begin{array}{l} \text{association, equilibrium, affinity,} \\ \text{stability, binding, formation constant} \end{array} \right)$$

**Kinetics:** assume total amount of reactant and product is conserved

$$c_R(t) + c_P(t) = C$$

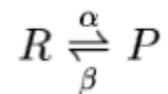
$$\frac{dc_R(t)}{dt} = \beta \left( C - c_R(t) \right) - \alpha c_R(t)$$

$$\frac{dc_R(t)}{dt} + (\alpha + \beta)c_R(t) = \beta C$$



## Chemical Kinetics (v1)

### First-order, reversible reaction

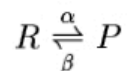


First-order linear differential equation with constant coefficients

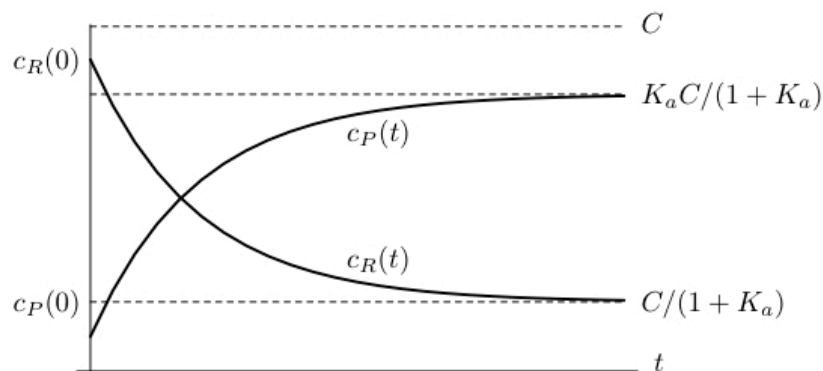
$$c_R(t) = c_R(\infty) - \left( c_R(\infty) - c_R(0) \right) e^{-t/\tau}, \text{ for } t > 0$$

$$c_R(\infty) = \frac{\beta}{\alpha + \beta} C = \frac{1}{1 + K_a} C \quad \text{AND} \quad \tau = \frac{1}{\alpha + \beta}$$

First-order, reversible reaction



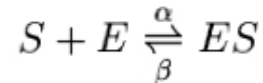
$$c_P(t) = C - c_R(t)$$



$$\tau = \frac{1}{\alpha + \beta}$$

## Chemical Kinetics (v2)

### Second-order reversible (binding) reaction



$$\begin{aligned} \frac{dc_{ES}(t)}{dt} &= \alpha c_S(t)c_E(t) - \beta c_{ES}(t), \\ \frac{dc_S(t)}{dt} &= \frac{dc_E(t)}{dt} = \beta c_{ES}(t) - \alpha c_S(t)c_E(t), \end{aligned}$$

→ Law of mass action

Equilibrium:

$$\begin{aligned} \frac{dc_{ES}(t)}{dt} &= \frac{dc_S(t)}{dt} = \frac{dc_E(t)}{dt} = 0 \\ \alpha c_S(\infty)c_E(\infty) - \beta c_{ES}(\infty) &= 0 \\ \frac{c_{ES}(\infty)}{c_S(\infty)c_E(\infty)} &= \frac{\alpha}{\beta} = K_a \quad (\text{association constant}) \\ \frac{1}{K_a} &= \frac{c_S(\infty)c_E(\infty)}{c_{ES}(\infty)} = K \quad (\text{dissociation constant}) \end{aligned}$$

Assume enzyme conserved:  $c_E(t) + c_{ES}(t) = C_{ET}$   
How does  $c_{ES}$  depend on  $c_S$ ? Eliminate  $c_E$ .

$$C_{ET} = c_E(\infty) + c_{ES}(\infty)$$

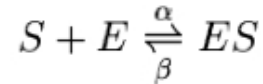
$$C_{ET} = \frac{K c_{ES}(\infty)}{c_S(\infty)} + c_{ES}(\infty) = \left( \frac{K}{c_S(\infty)} + 1 \right) c_{ES}(\infty)$$

→ Michaelis-Menten kinetics

$$c_{ES}(\infty) = \left( \frac{c_S(\infty)}{K + c_S(\infty)} \right) C_{ET}$$

# Chemical Kinetics (v2)

## Second-order reversible (binding) reaction

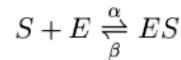


$$C_{ET} = c_E(\infty) + c_{ES}(\infty)$$

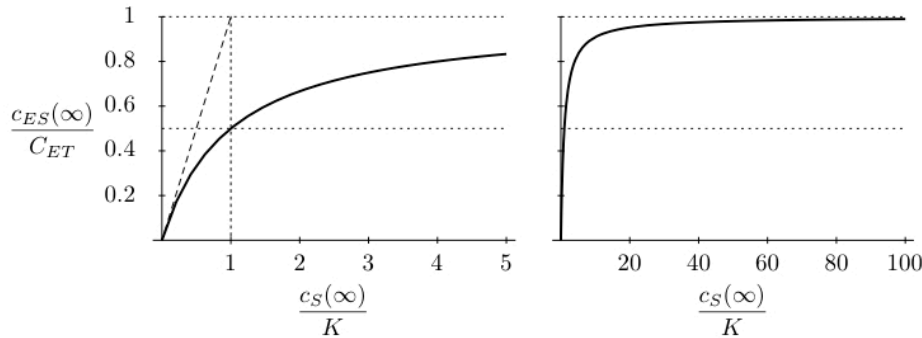
$$C_{ET} = \frac{K c_{ES}(\infty)}{c_S(\infty)} + c_{ES}(\infty) = \left( \frac{K}{c_S(\infty)} + 1 \right) c_{ES}(\infty)$$

$$c_{ES}(\infty) = \left( \frac{c_S(\infty)}{K + c_S(\infty)} \right) C_{ET}$$

### Second-order reversible (binding) reaction



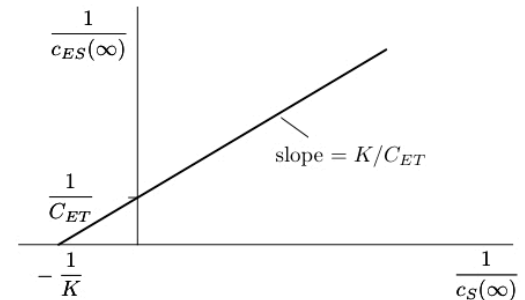
Rectangular hyperbola: Michaelis-Menten Relation



Doubly-reciprocal coordinates: Lineweaver-Burk plot

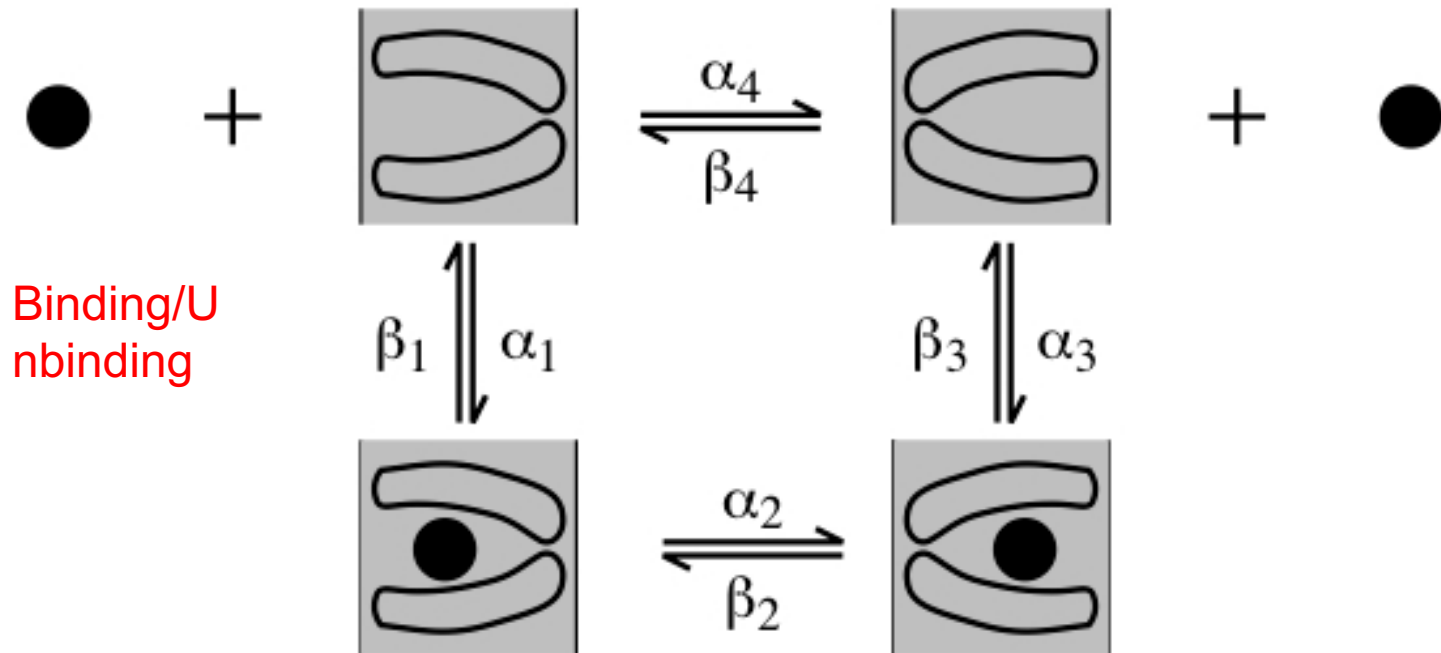
$$\frac{1}{c_{ES}(\infty)} = \left( 1 + \frac{K}{c_S(\infty)} \right) \frac{1}{C_{ET}} = \left( \frac{K}{C_{ET}} \right) \frac{1}{c_S(\infty)} + \frac{1}{C_{ET}}$$

→ Linear way to plot nonlinear relationship!



# General Four-State Carrier Model

## General Four-State Model

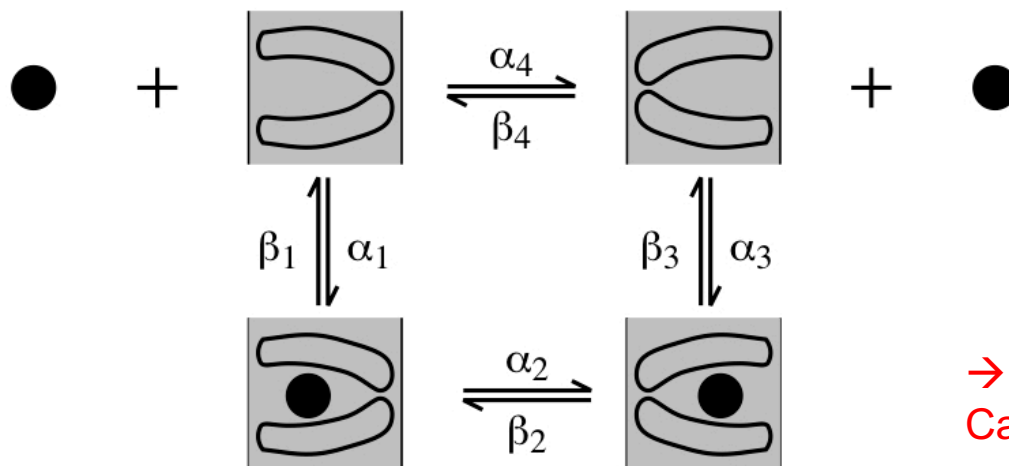


Binding/Unbinding

Translocation

# Chemical Kinetics & 'Carriers'

## General Four-State Model



→ Numerous free parameters.  
Can we simplify?

$$\frac{dC_{ES}^i}{dt} = \alpha_1 C_S^i C_E^i - \beta_1 C_{ES}^i$$

$$\frac{dC_{ES}^o}{dt} = \alpha_3 C_S^o C_E^o - \beta_3 C_{ES}^o$$

$$\frac{dC_S^i}{dt} = \frac{dC_E^i}{dt} = \beta_1 C_{ES}^i - \alpha_1 C_S^i C_E^i$$

$$\frac{dC_S^o}{dt} = \frac{dC_E^o}{dt} = \beta_3 C_{ES}^o - \alpha_3 C_S^o C_E^o$$

$$\frac{dC_{ES}^o}{dt} = \alpha_2 C_{ES}^i - \beta_2 C_{ES}^o$$

$$\frac{dC_E^o}{dt} = \alpha_4 C_E^i - \beta_4 C_E^o$$

$$\frac{dC_{ES}^i}{dt} = \beta_2 C_{ES}^o - \alpha_2 C_{ES}^i$$

$$\frac{dC_E^i}{dt} = \beta_4 C_E^o - \alpha_4 C_E^i$$

