ISCI 1310
12/04/17

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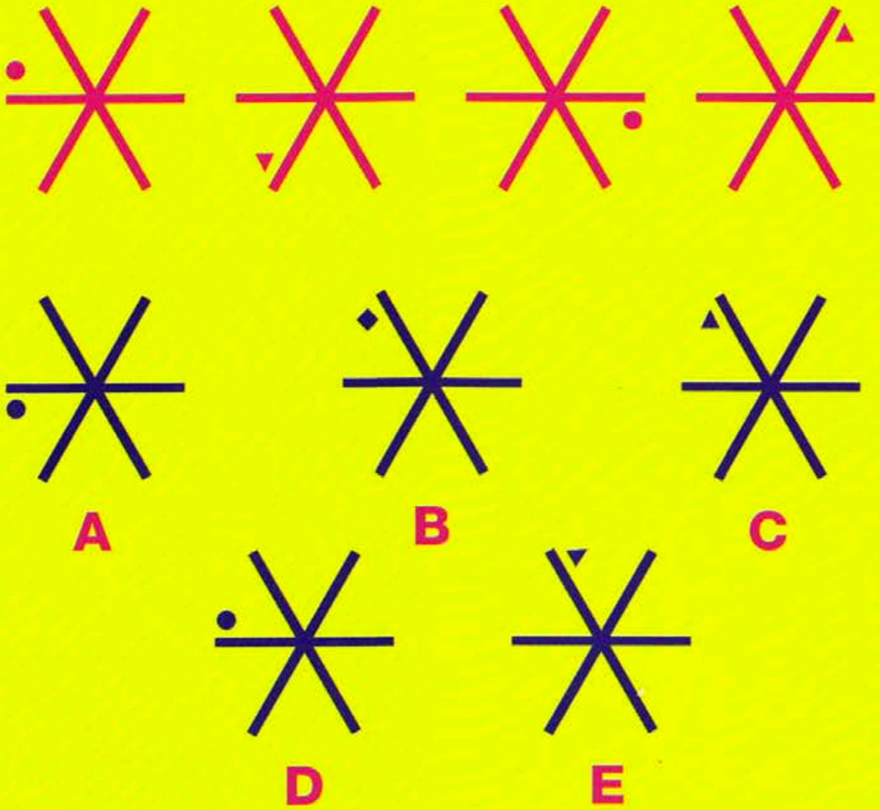
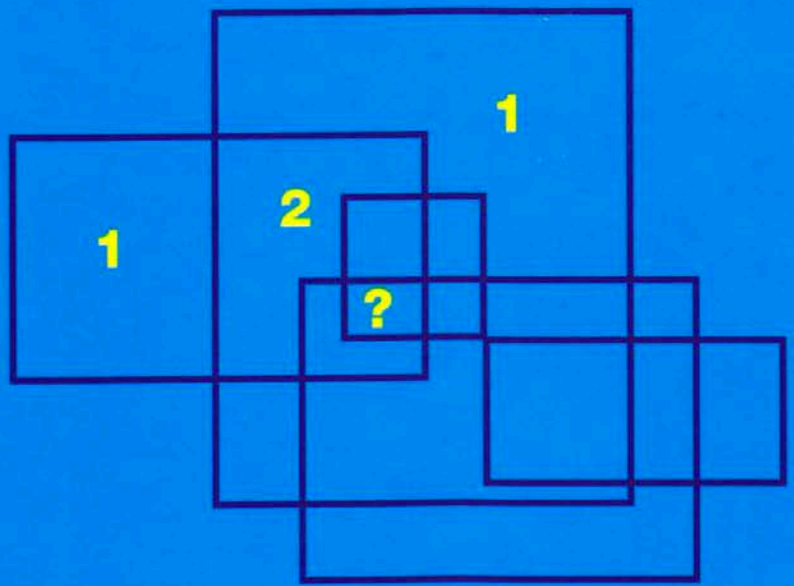
Relevant reading:
Wolfson ch.14.1, 31.4, 32.1-32.2, 36.4-36.5

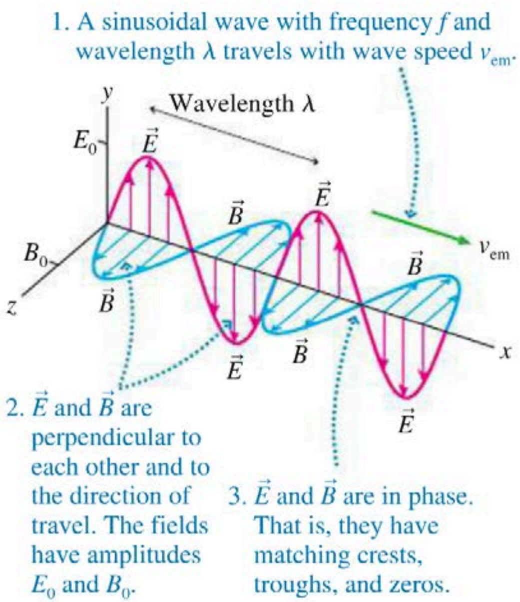
Which of the following comes next in the sequence?





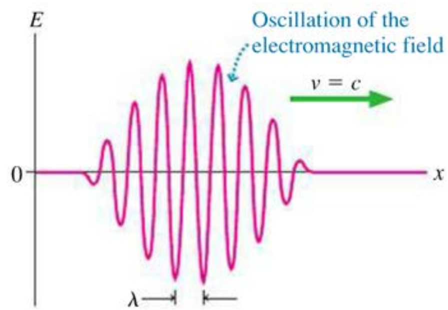
This figure has been partially filled with numbers according to a system. Can you work out the logic of the system and replace the question mark with a number?

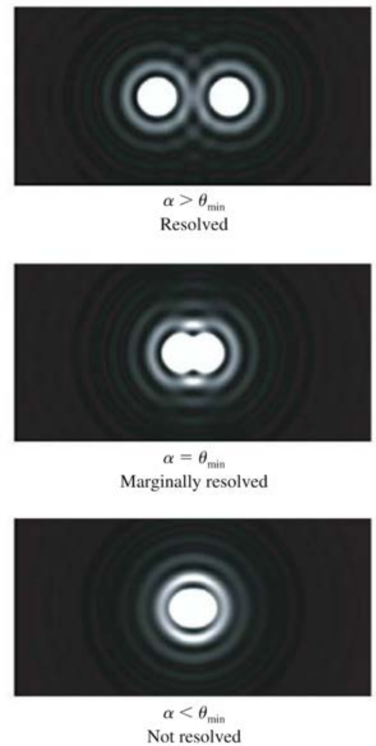
Light & “Wave-particle duality”

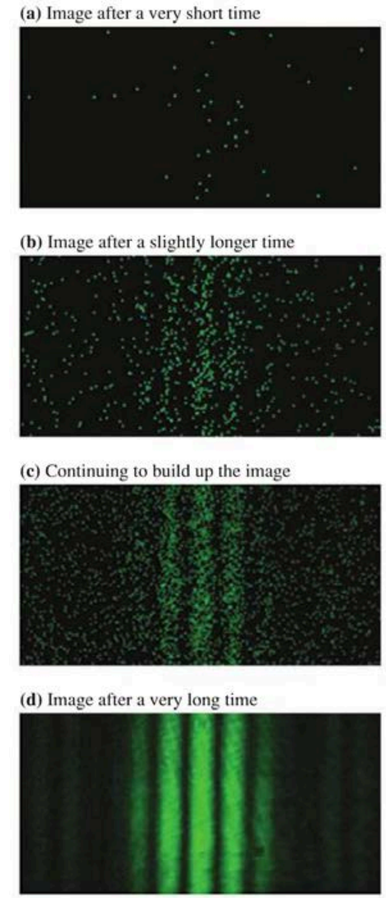


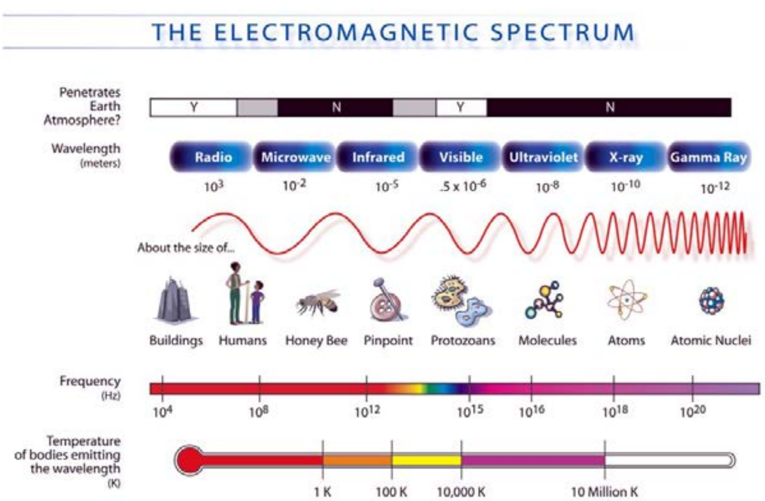
Wave-particle duality

A wave packet has wave-like and particle-like properties.









All this business about waves (e.g., wavelength, speed, diffraction, interference, matter acting like waves)

→ But what in fact is a “wave”?

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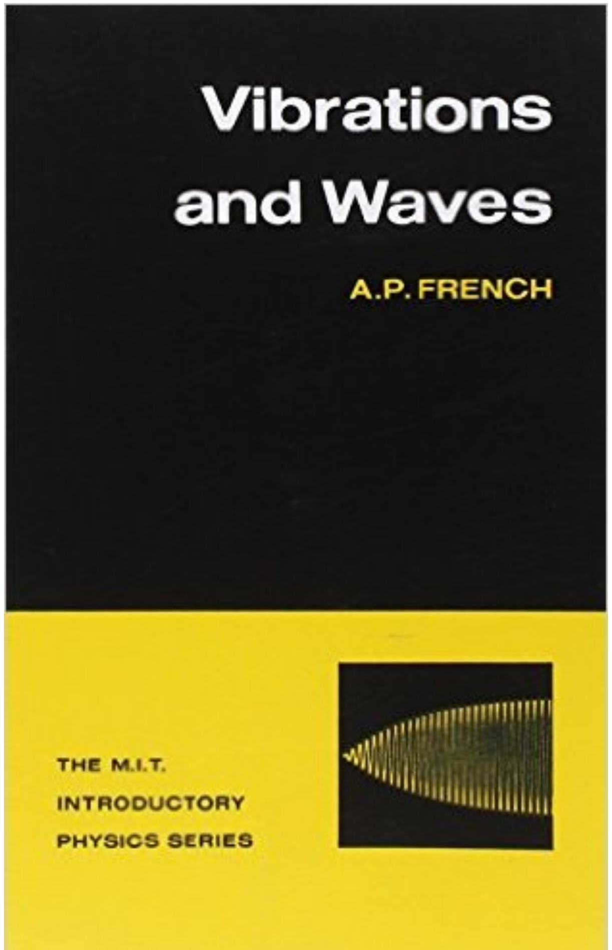
What is a "wave"?



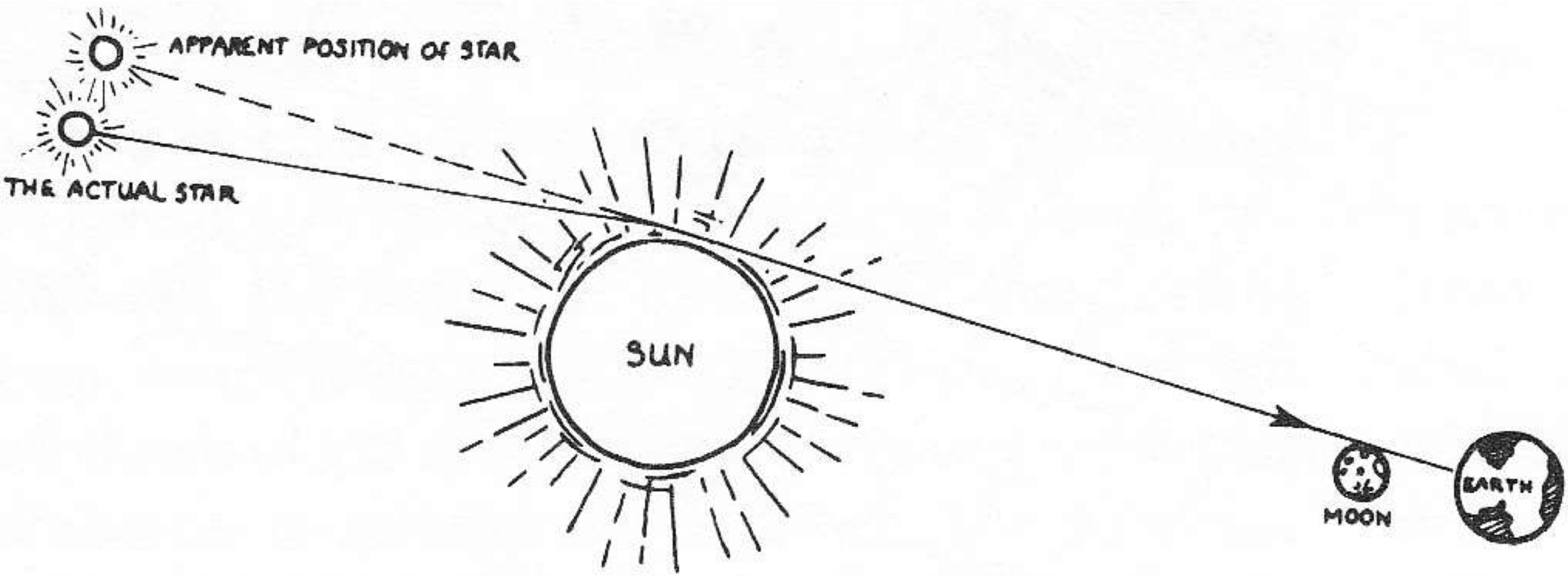
WHAT IS A WAVE?

FOR MANY PEOPLE—perhaps for most—the word "wave" conjures up a picture of an ocean, with the rollers sweeping onto the beach from the open sea. If you have stood and watched this phenomenon, you may have felt that for all its grandeur it contains an element of anticlimax. You see the crests racing in, you get a sense of the massive assault by the water on the land—and indeed the waves *can* do great damage, which means that they are carriers of energy—but yet when it is all over, when the wave has reared and broken, the water is scarcely any farther up the beach than it was before. That onward rush was not to any significant extent a bodily motion of the water. The long waves of the open sea (known as the swell) travel fast and far. Waves reaching the California coast have been traced to origins in South Pacific storms more than 7000 miles away, and have traversed this distance at a speed of 40 mph or more. Clearly the sea itself has not traveled in this spectacular way; it has simply played the role of the agent by which a certain effect is transmitted. And here we see the essential feature of what is called wave motion. A condition of some kind is transmitted from one place to another by means of a medium, but the medium itself is not transported. A local effect can be linked to a distant cause, and there is a time lag between cause and effect that depends on the properties of the medium and finds its expression in the velocity of the wave. All material media—solids, liquids, and gases—can carry energy and information by means of waves, and our study of coupled oscillators and normal modes has paved the way for an understanding of this important phenomenon.

Although waves on water are the most familiar type of wave, they are also among the most complicated to analyze in terms of underlying physical processes. We shall, therefore, not have very much to say about them. Instead, we shall turn to our old standby—the stretched string—about which we have learned a good deal that can now be applied to the present discussion.

A traveling wave is a broad term, but in a general sense can be defined as occurring when a "condition of some kind is transmitted from one place to another by means of a medium, but the medium itself is not transported"

Examples of waves → EM waves (i.e., light)

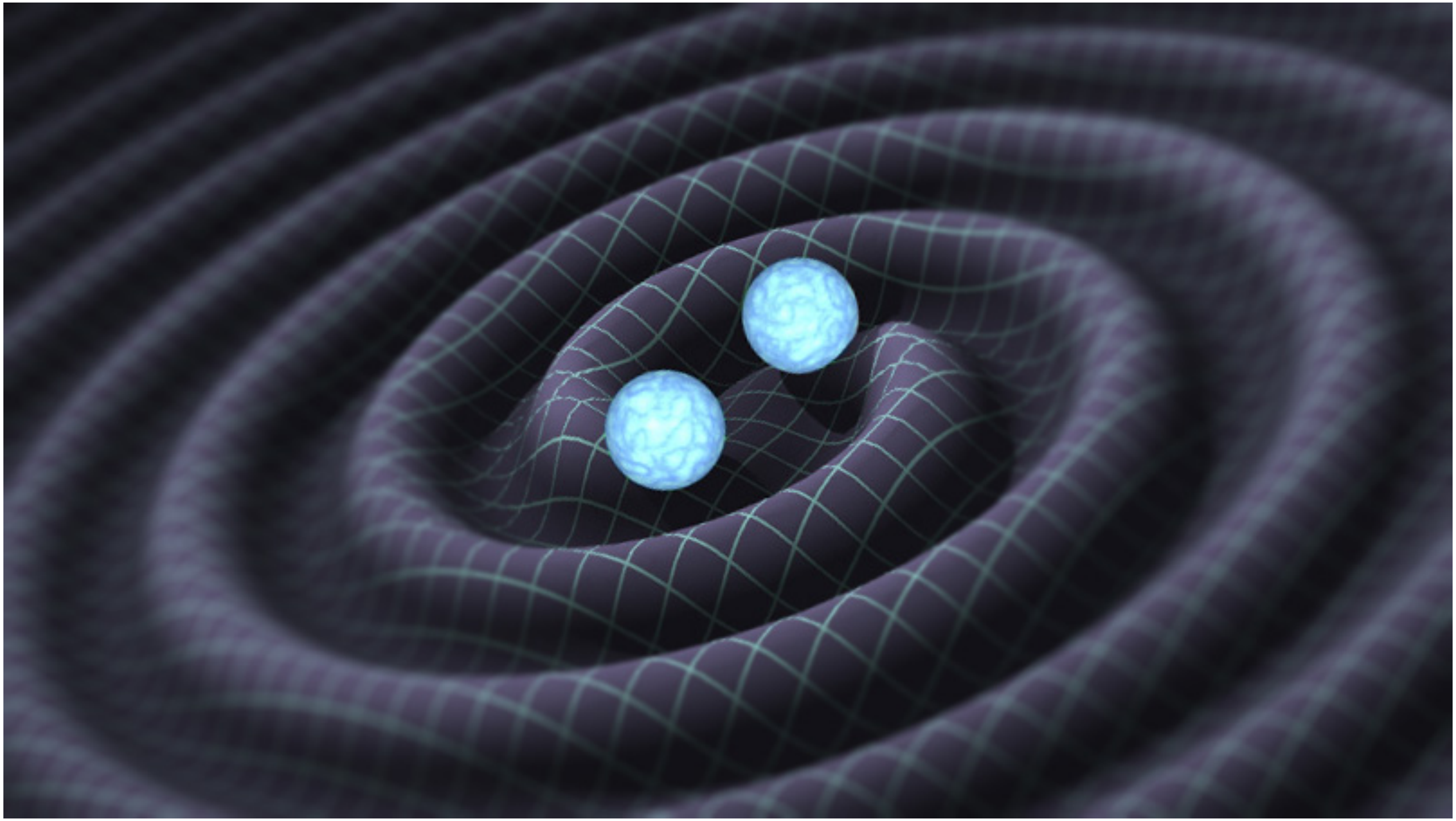


Reminder:
A testable prediction stemming from Einstein's theory of General Relativity

→ And it worked like a charm! Tested in Sept. 1919, Einstein became a rockstar afterwards!

von Baeyer

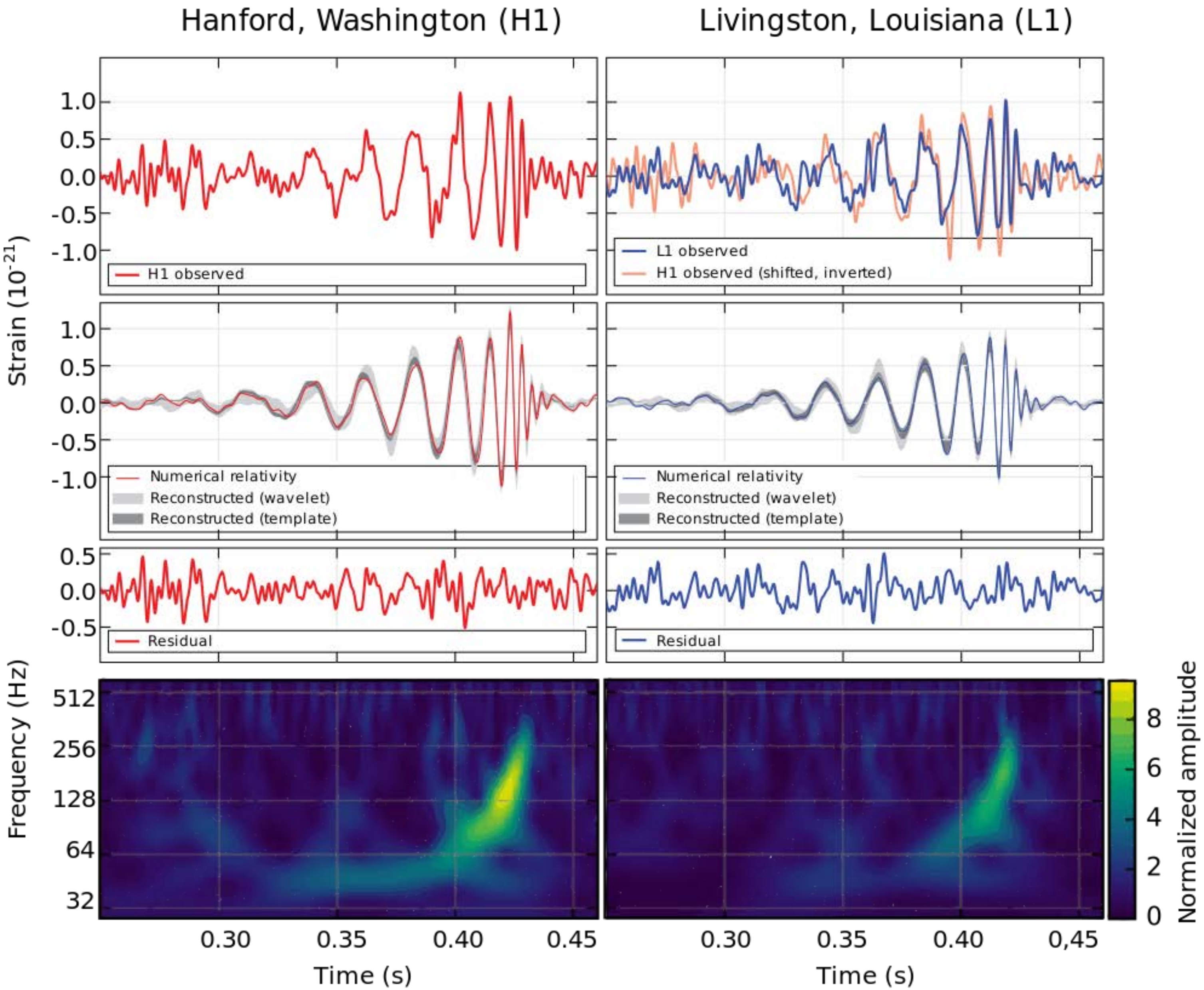
Examples of waves → Gravitational waves



Two black holes collide and form a ripple in spacetime (→ Gravitational Waves)

https://www.ligo.caltech.edu/

“The event” occurred on Sept.14, 2015



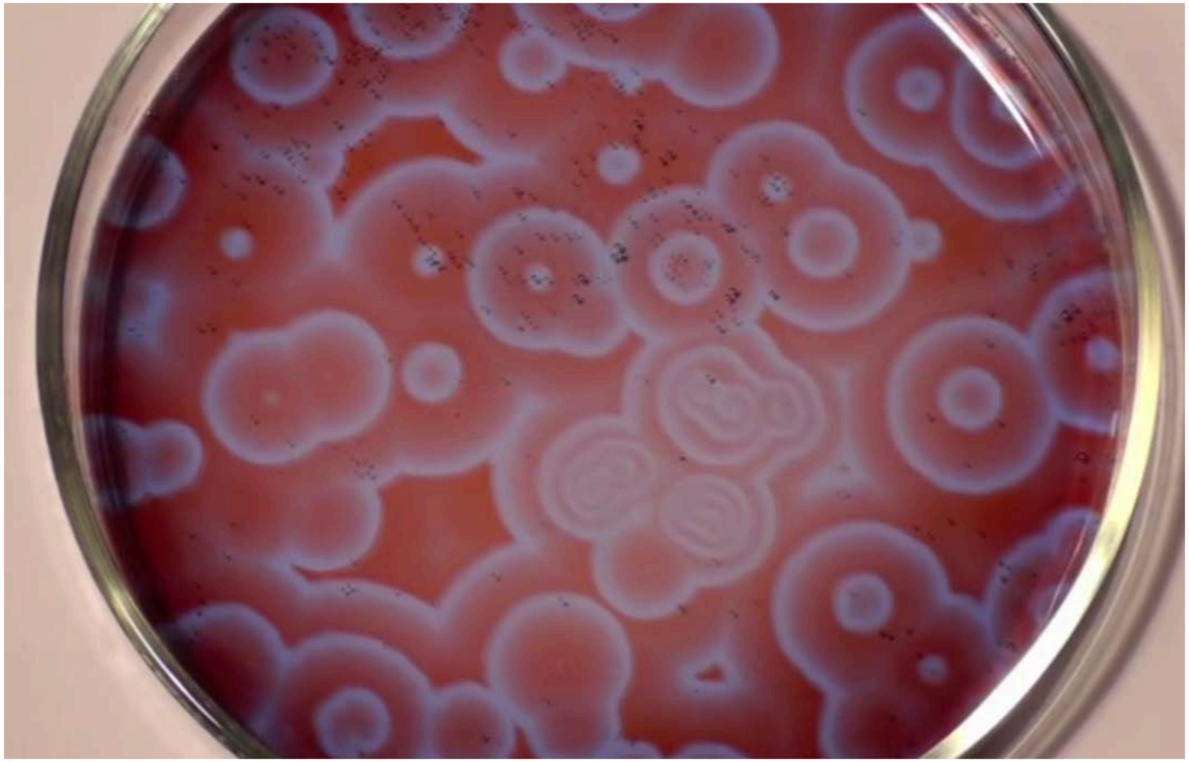
→ Can listen to this! (https://www.youtube.com/watch?v=TWqhUANNFXw)

https://en.wikipedia.org/wiki/First_observation_of_gravitational_waves

Examples of waves → Chemical waves

- "BZ reaction" = *Belousov–Zhabotinsky reaction*

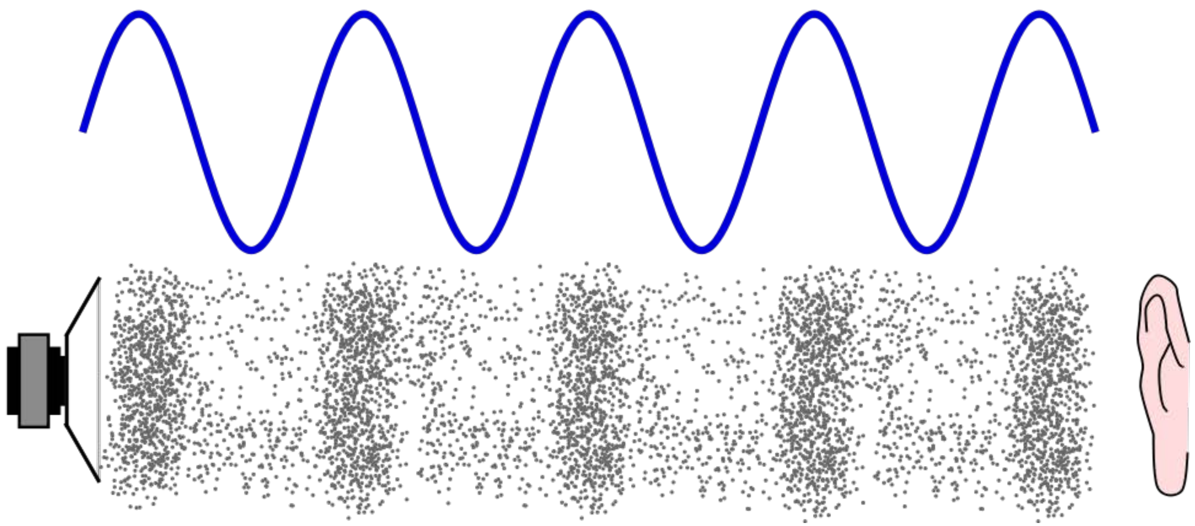
"... is one of a class of reactions that serve as a classical example of non-equilibrium thermodynamics, resulting in the establishment of a nonlinear chemical oscillator. The only common element in these oscillating is the inclusion of bromine and an acid."



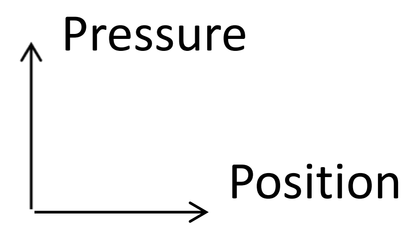
https://www.youtube.com/watch?v=3JAqrRnKFHo

https://en.wikipedia.org/wiki/Belousov%E2%80%93Zhabotinsky_reaction

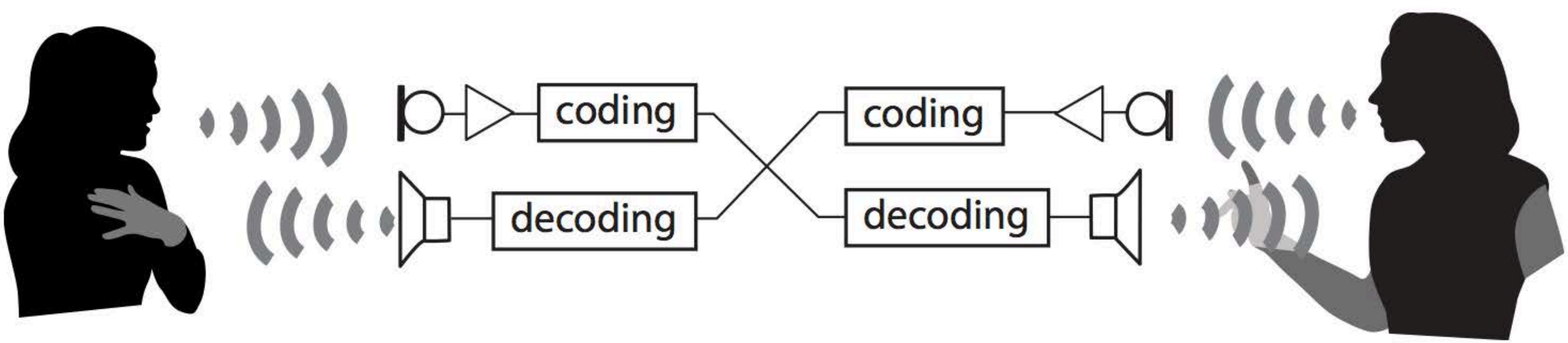
Examples of waves → Sound waves



Snapshot in time



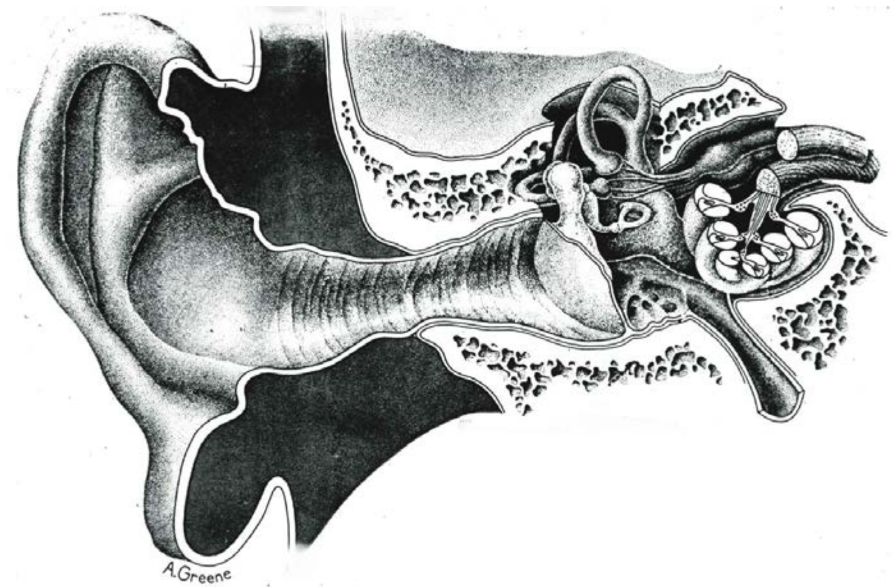
The “speech chain”



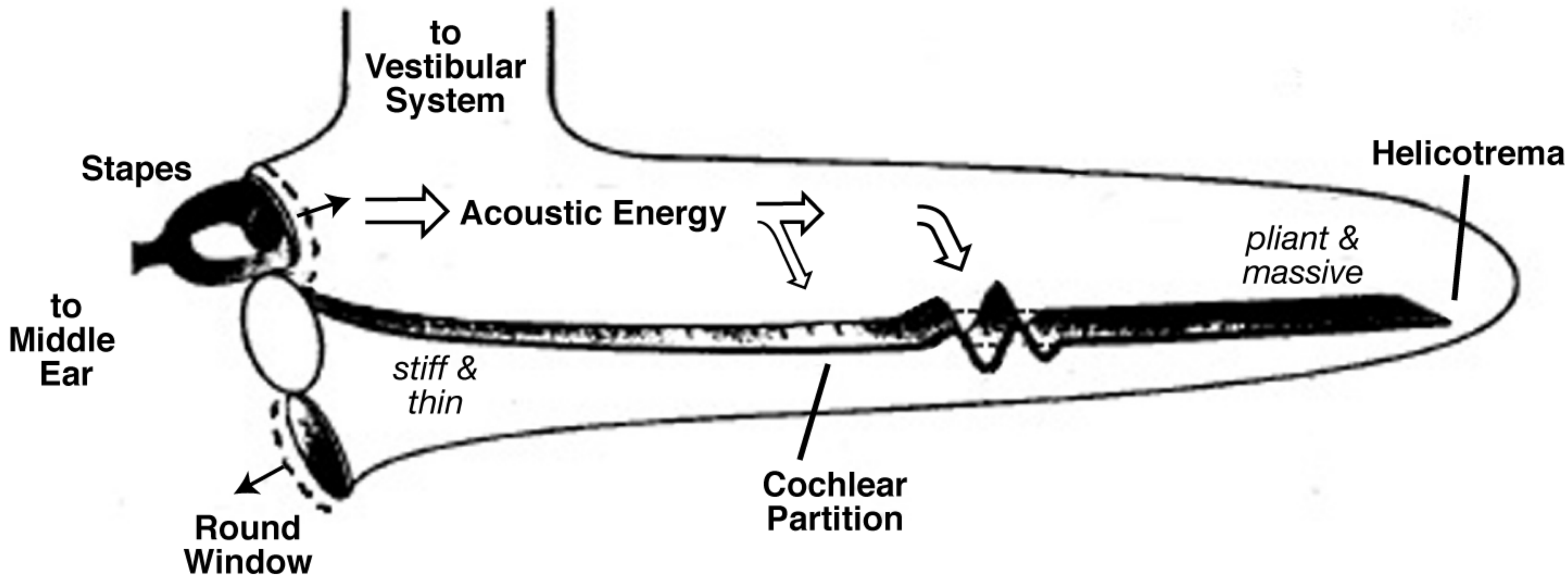
→ You create waves when you speak!

Pulkki & Karjalainen (2015)

Examples of waves → Cochlear waves

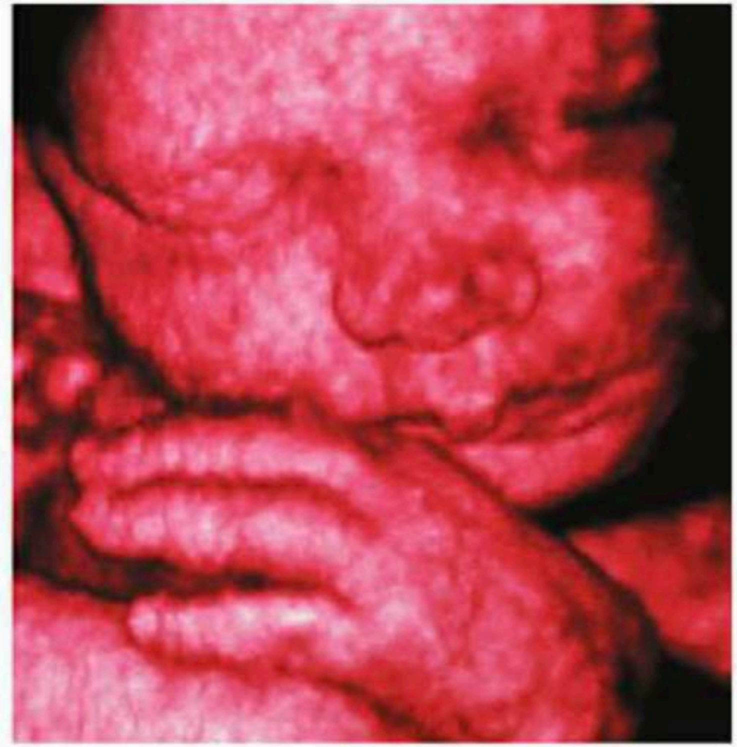


Basilar membrane traveling waves



Geisler (1996)

<u>Examples of waves</u> → Seeing babies



This ultrasound image is an example of using high-frequency sound waves to "see" within the human body.

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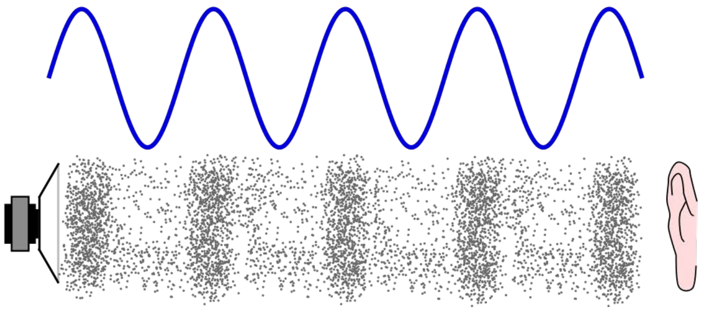
→ Modern ultrasound can image in 3-D

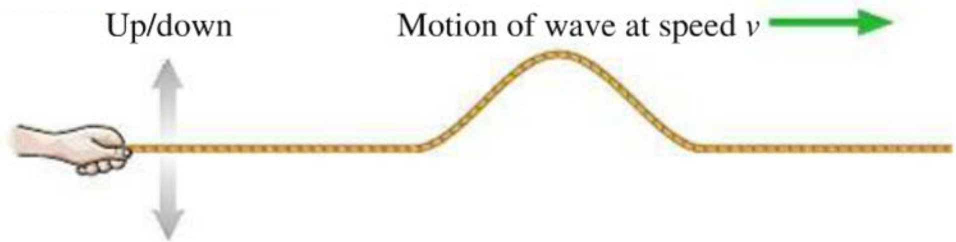
Types of waves

- So now we have seen some examples of waves, but more generally, what “types” of waves are there?

Longitudinal/Compression



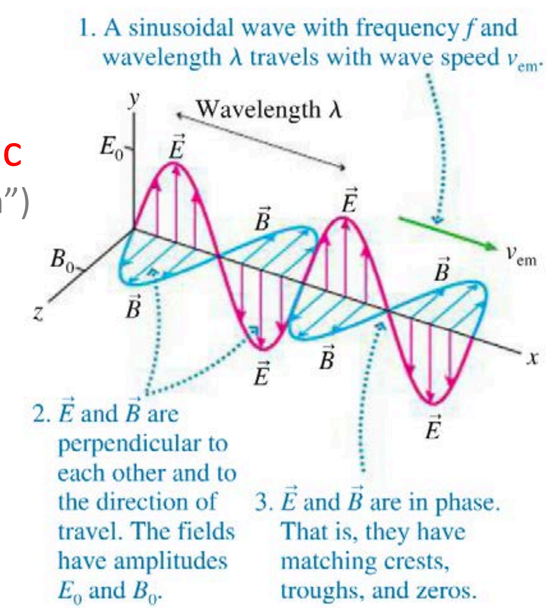
Transverse



Standing

Electromagnetic

(Note: No “medium”)



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<u>Question</u>

J. Acoust. Soc. Am. **123**, 3507 (2008);

Acoustic communication in Panthera tigris: A study of tiger vocalization and auditory receptivity revisited

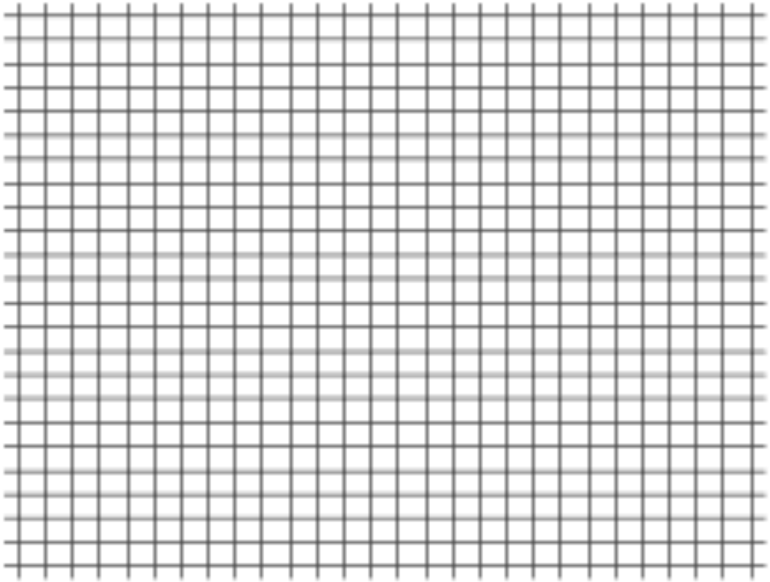
Edward Walsh[1], Douglas L. Armstrong[2], Julie Napier[3], Lee G. Simmons[4], Megan Korte[5] and Joann McGee[6]

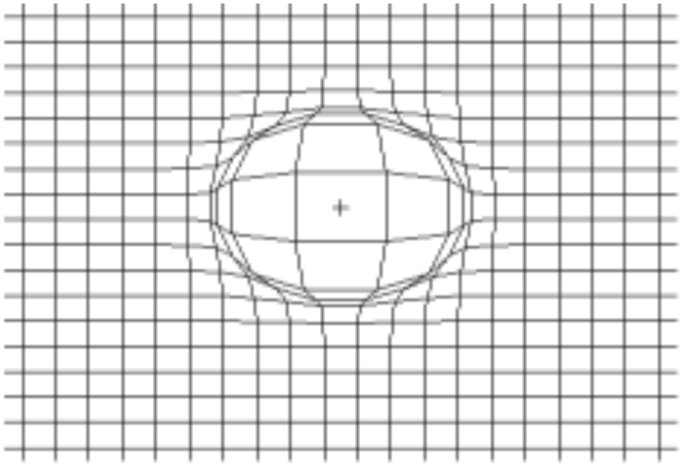
Preliminary findings reported at the 145th meeting of the Society suggested that confrontational tiger roars contain energy in the infrasonic portion of the electromagnetic spectrum. This discovery generally supported the proposition that free ranging individuals may take advantage of this capability to communicate with widely dispersed conspecifics inhabiting large territories in the wild. Preliminary ABR findings indirectly supported this view suggesting that although tigers are most sensitive to acoustic events containing energy in the 0.3 to 0.5 kHz band, they are most likely able to detect acoustic events in the near-infrasonic and infrasonic range based on the assumption that felid audiograms exhibit uniform shapes. In this study, the spectral content of territorial and confrontational roars was analyzed and relevant features of ABR based threshold-frequency curves were considered in relation to the acoustical properties of both roar types. Unlike the confrontational roar, infrasonicenergy was not detected in the territorial roar; however, like the confrontational roar, peak acoustic power was detected in a frequency band centered on˜ 0.3 kHz. In addition, ABR recordings acquired in a double walled sound attenuating chamber recently installed at the Henry Doorly Zoo suggest that acoustic sensitivity is significantly underestimated under "field" conditions.

→ What is wrong here?

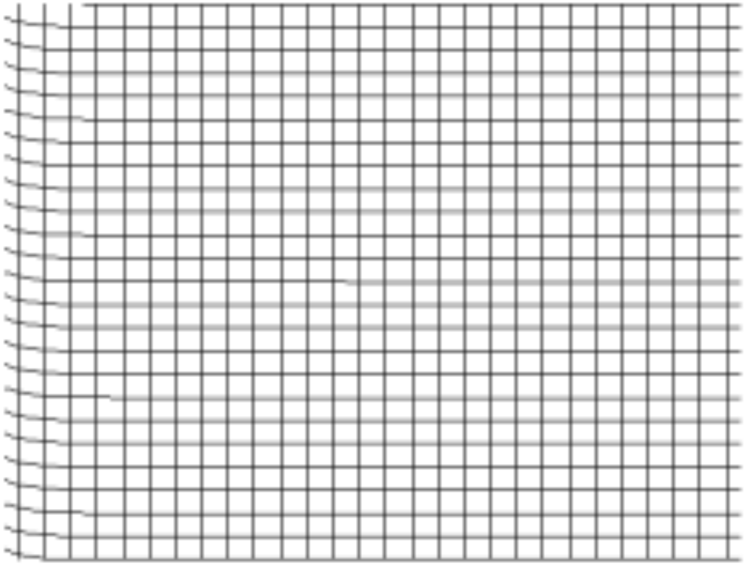
Types of waves

Longitudinal wave





Transverse wave



https://en.wikipedia.org/wiki/Longitudinal_wave
https://en.wikipedia.org/wiki/Transverse_wave

Types of waves

A **transverse wave** is a wave in which the displacement is *perpendicular* to the direction in which the wave travels. For example, a wave travels along a string in a horizontal direction while the particles that make up the string oscillate vertically. Electromagnetic waves are also transverse waves because the electromagnetic fields oscillate perpendicular to the direction in which the wave travels.

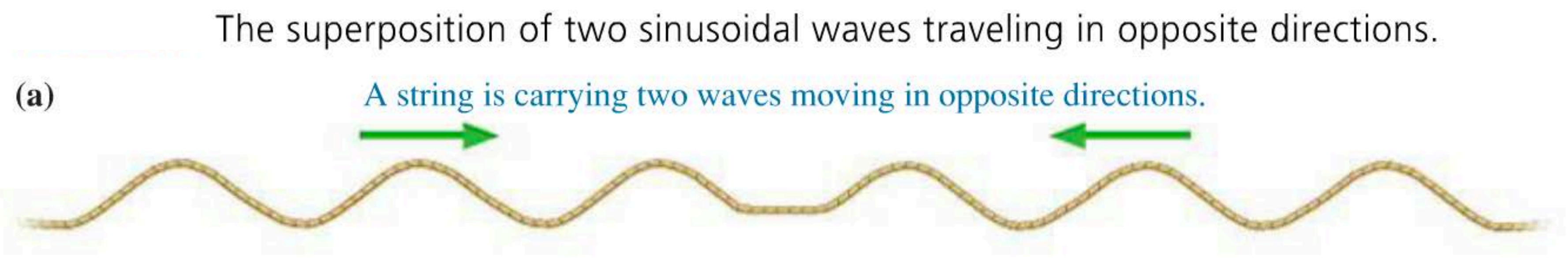
In a **longitudinal wave,** the particles in the medium move *parallel* to the direction in which the wave travels. Here we see a chain of masses connected by springs. If you give the first mass in the chain a sharp push, a disturbance travels down the chain by compressing and expanding the springs. Sound waves in gases and liquids are the most well known examples of longitudinal waves.

<u>Question</u>

→ What kind of waves are ocean waves?

Aside: Standing waves

➢ Consider that in 1-D, there can be two waves on a string: one going *forward* and one going *backward*



➢ Their combination leads to interference (or *superposition*)

➢ Sometimes the waves interfere (i.e., add up) *constructively*, other times it is *destructively*

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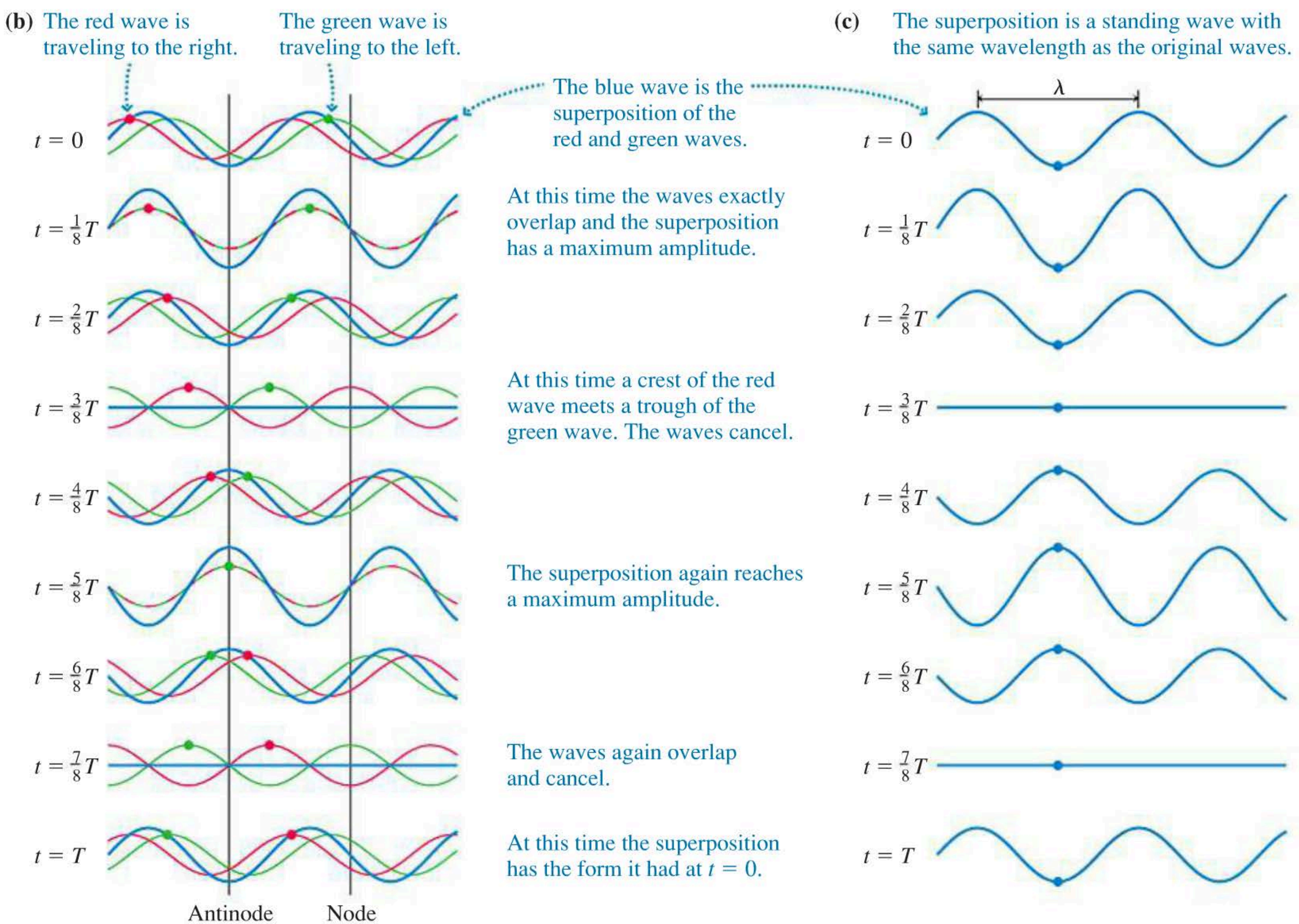
Aside: Standing waves



→ A bit hard to see via a static picture....

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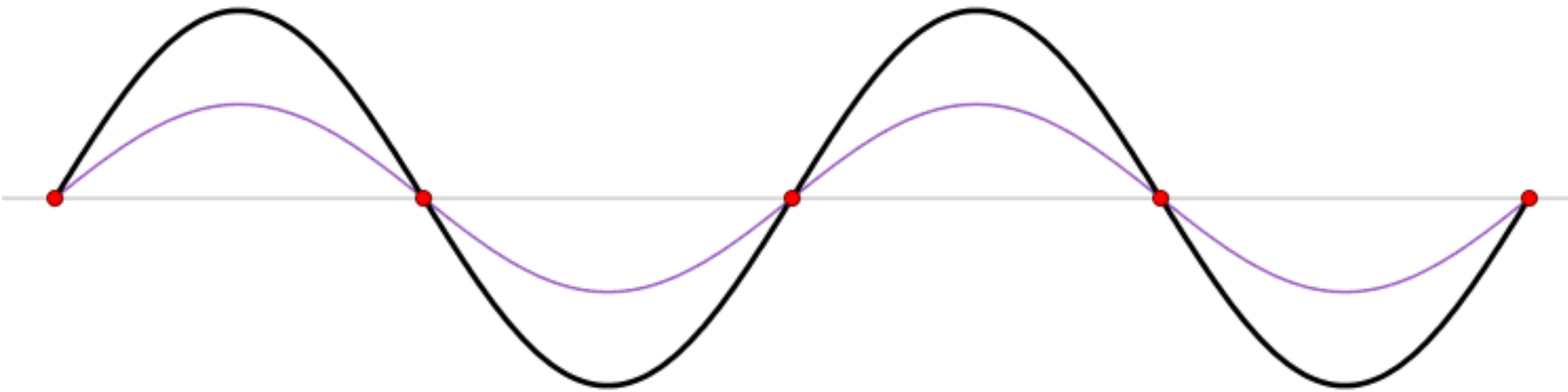
Aside: Standing waves

... but is much more readily apparent via a movie



Blue is the left-going wave
Red is the right-going wave
Black is the sum of the two (i.e., the "standing" wave)

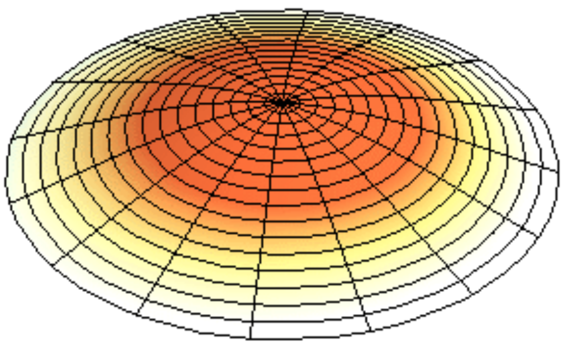
Note: Locations where the amplitude stays zero are called ***nodes***

https://en.wikipedia.org/wiki/Standing_wave

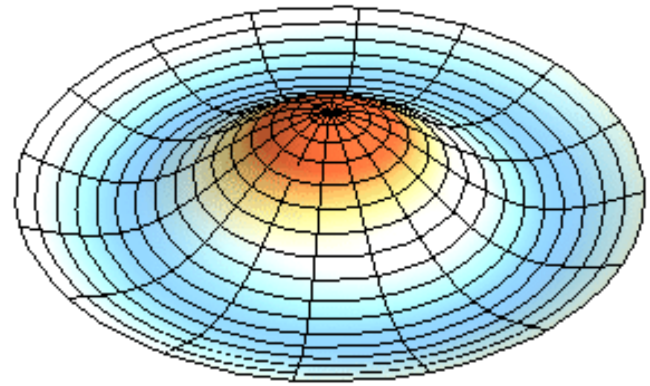
Aside: Standing waves

- Standing waves can arise in 2-D as well (e.g., drumhead)

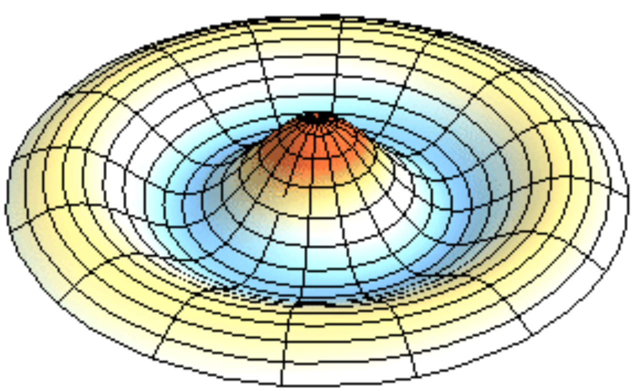
(0,1) mode



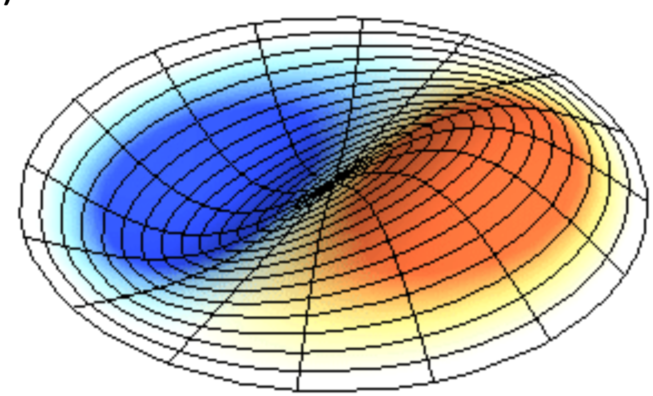
(0,2) mode



(0,3) mode



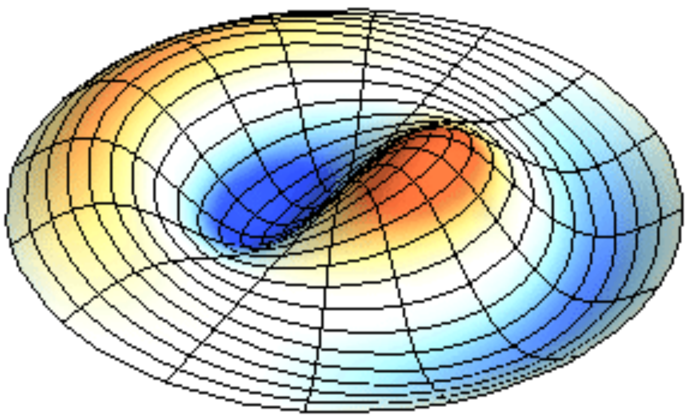
(1,1) mode



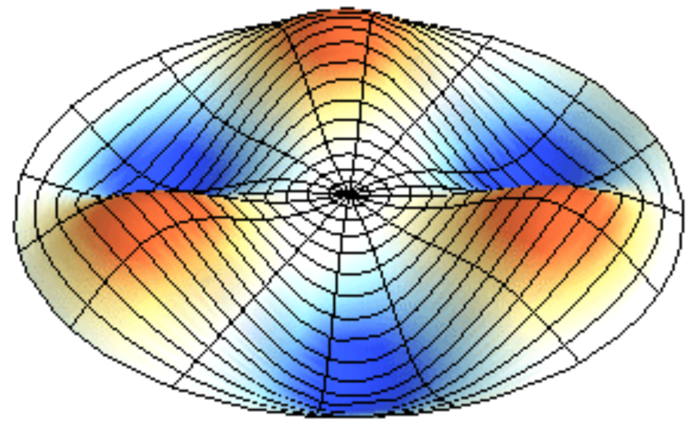
http://www.acs.psu.edu/drussell/demos/membranecircle/circle.html

Aside: Standing waves

(1,2) mode



(3,1) mode

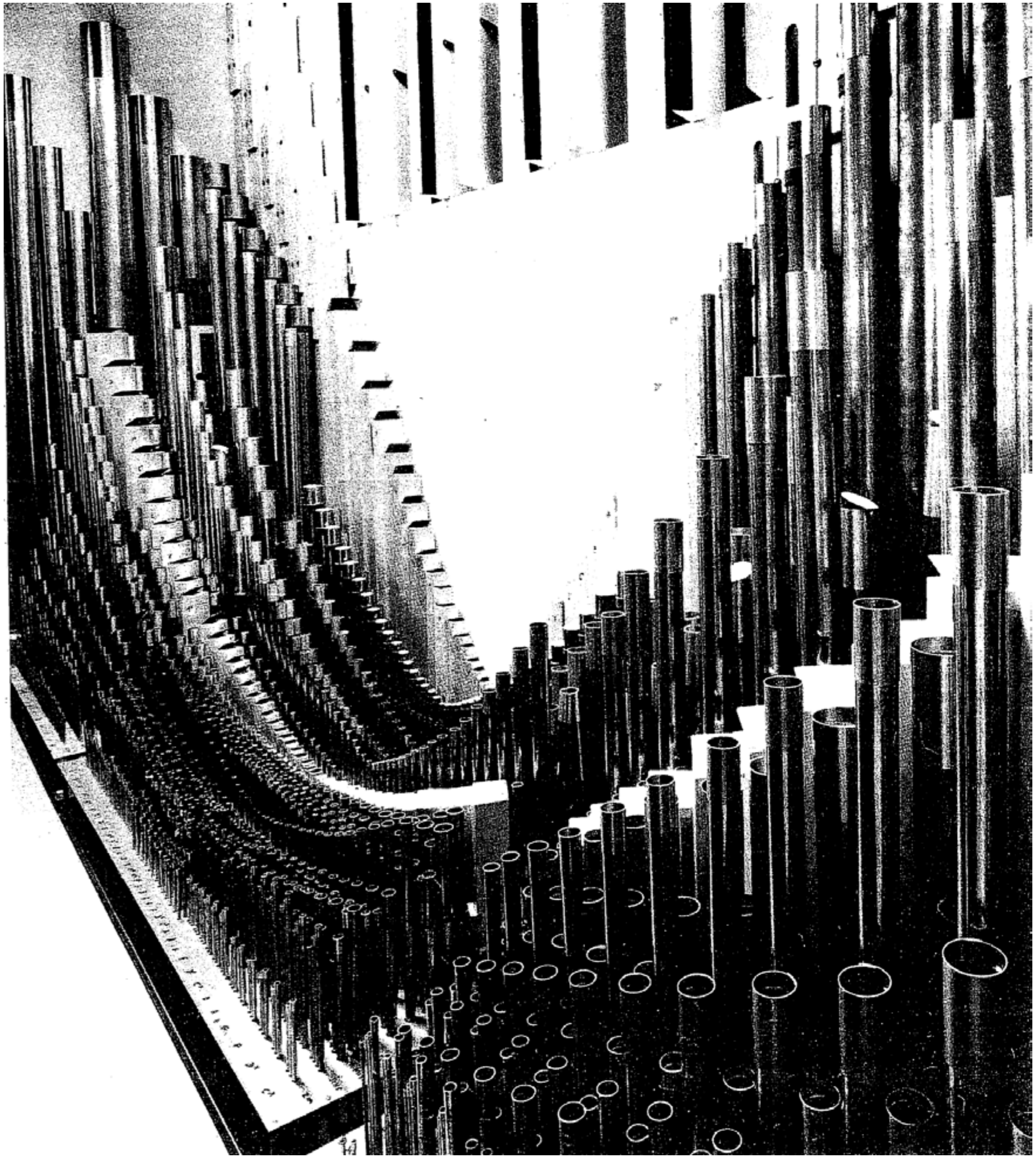


→ Note clear presence of nodes (chief characteristic of standing waves)

http://www.acs.psu.edu/drussell/demos/membranecircle/circle.html

Aside: Standing waves

INTERIOR VIEW OF ORGAN at the Sydney Opera House shows some of its 26 ranks of pipes, most of which are of metal but some of which are of wood. The length of the speaking part of each pipe doubles at every 12th pipe; the pipe diameter doubles at about every 16th pipe. Through long experience master organ builders arrived at the proportions necessary for achieving balanced tone quality.



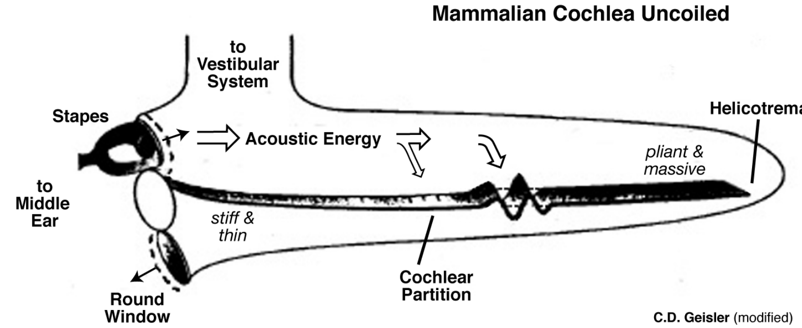
The Physics of Organ Pipes

The majestic sound of a pipe organ is created by the carefully phased interaction of a jet of air blowing across the mouth of each pipe and the column of air resonating inside the pipe

by Neville H. Fletcher and Suszanne Thwaites

Scientific American (1983)

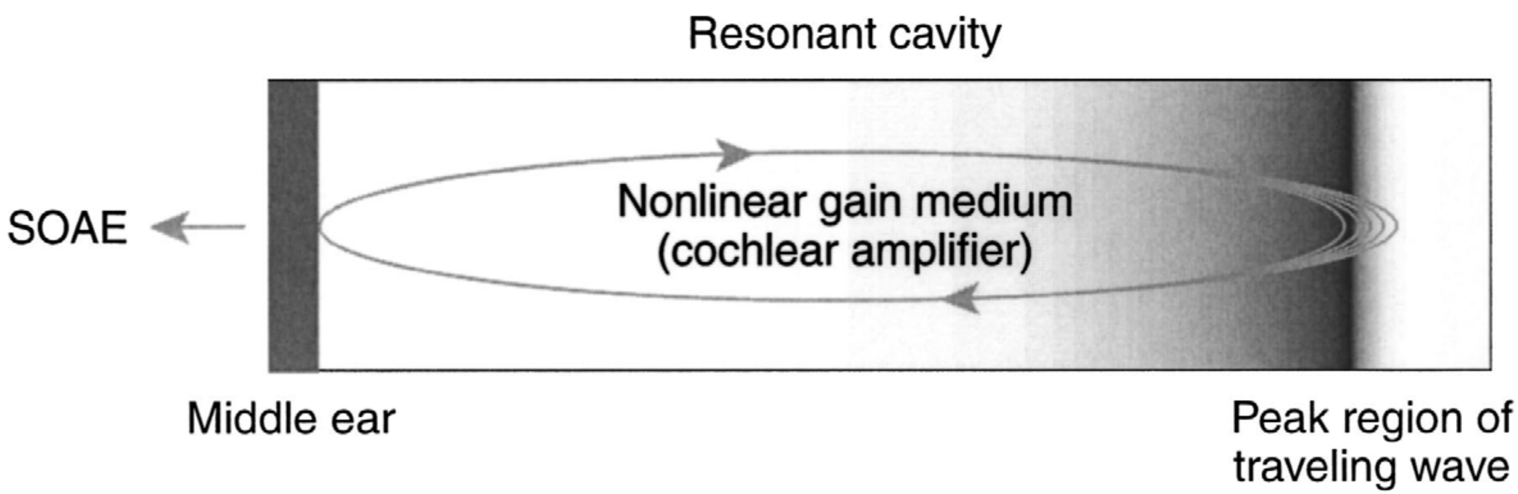
Aside: Cochlear standing waves



Mammalian spontaneous otoacoustic emissions are amplitude-stabilized cochlear standing waves

Christopher A. Shera[a)]
Eaton-Peabody Laboratory of Auditory Physiology, Massachusetts Eye and Ear Infirmary, 243 Charles Street, Boston, Massachusetts 02114 and Department of Otology and Laryngology, Harvard Medical School, Boston, Massachusetts 02115

J. Acoust. Soc. Am. **114** (1), July 2003



"Taken together, the results imply [...] the cochlea acting as a biological, hydromechanical analog of a laser oscillator."

→ Inner ear acts like a laser!

Further aside:

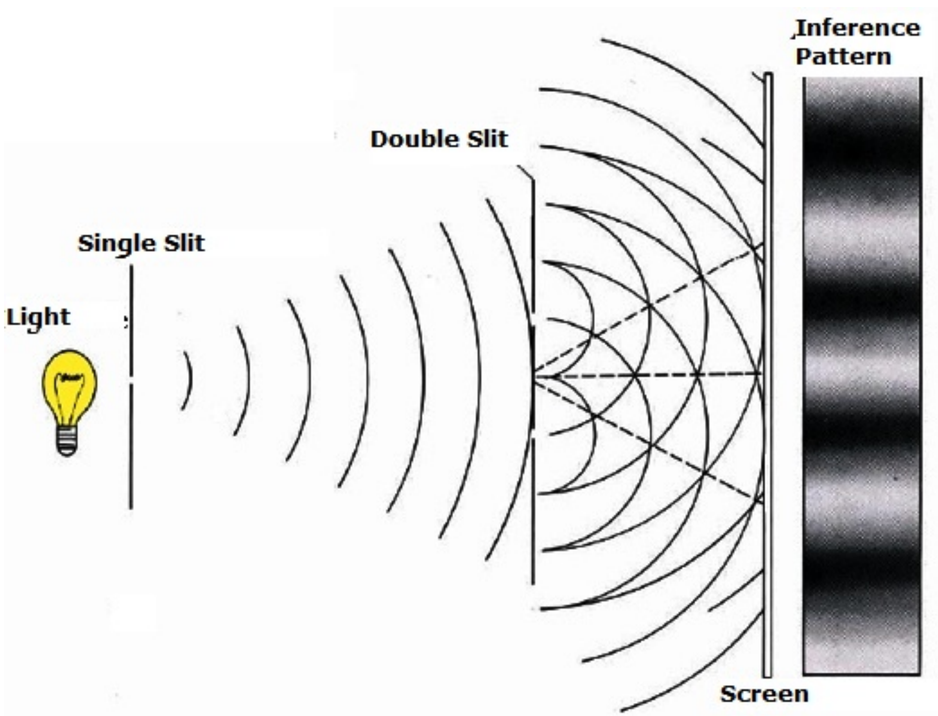
$$R[1 + RR_{\text{stapes}} + (RR_{\text{stapes}})^2 + \cdots] = R\sum_{n=0}^{\infty}(RR_{\text{stapes}})^n$$

→ Reflections back and forth can be described via a ***geometric series*** (something you'll see soon re integral calculus!)

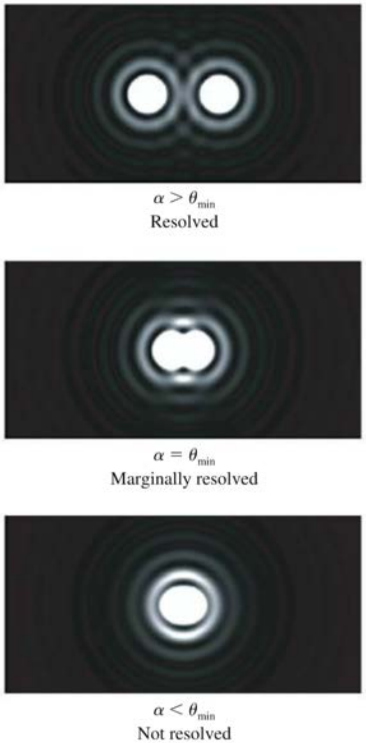
<u>Interference & Diffraction</u>

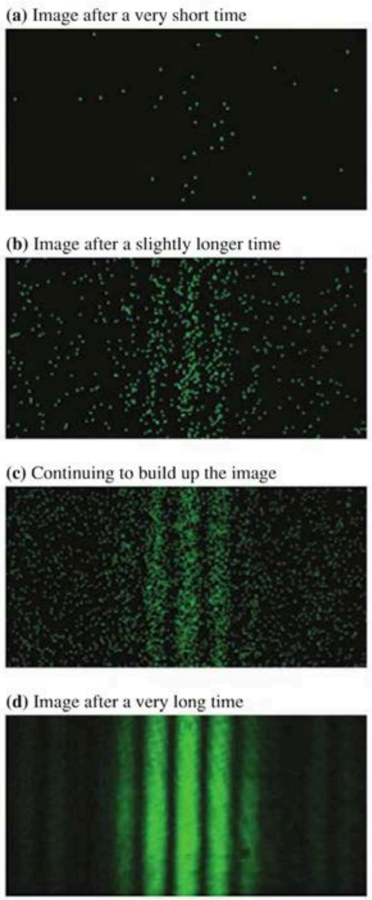
<u>Review</u>: Sometimes the waves interfere (i.e., add up) *constructively*, other times it is *destructively*

→ Same idea applies here re the "double slit" experiments previously described



http://www.mysearch.org.uk/website1/html/546.Double-Slit.html

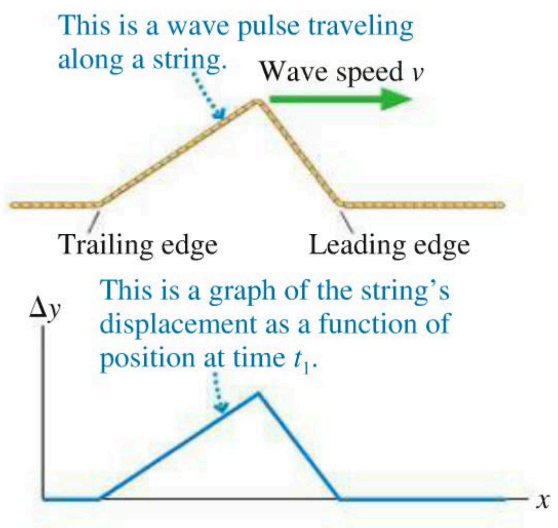




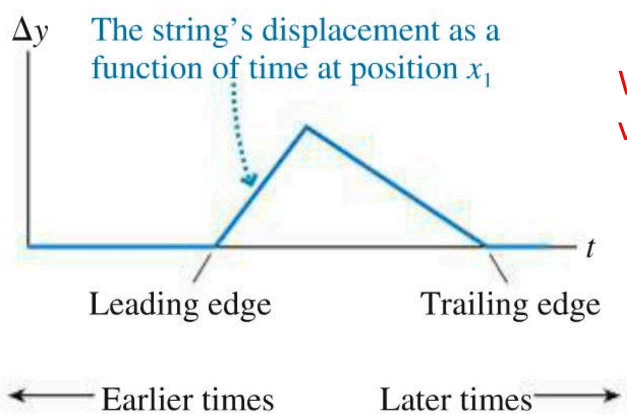
<u>Note</u>: Relevant concept here is known as ***Huygens-Fresnel principle***

How to describe a wave?

A snapshot graph of a wave pulse on a string.

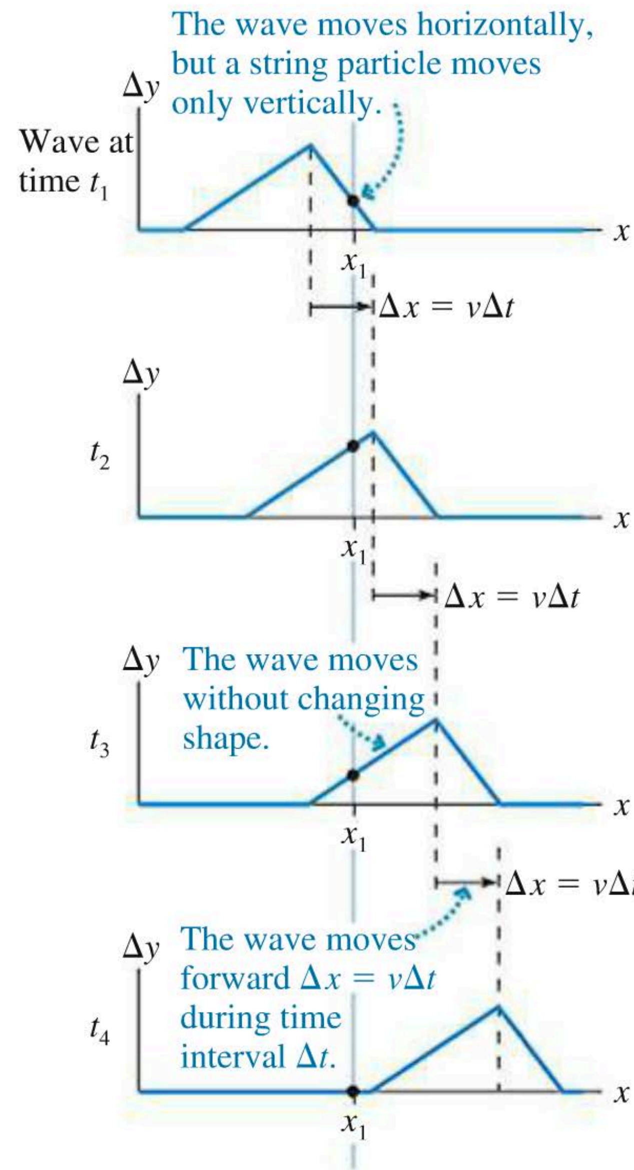


Wave height versus position



Wave height versus time

A sequence of snapshot graphs shows the wave in motion.



Note: Wave shape appears "flipped" between the two....

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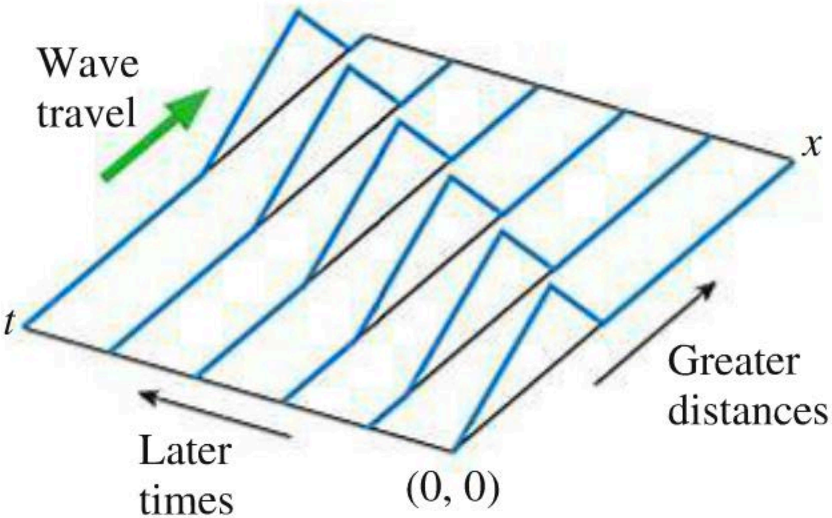
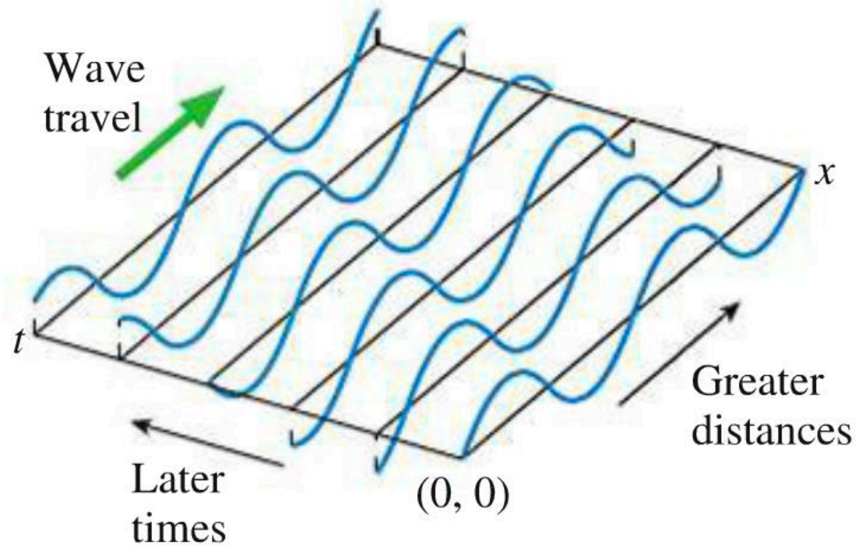
How to describe a wave?

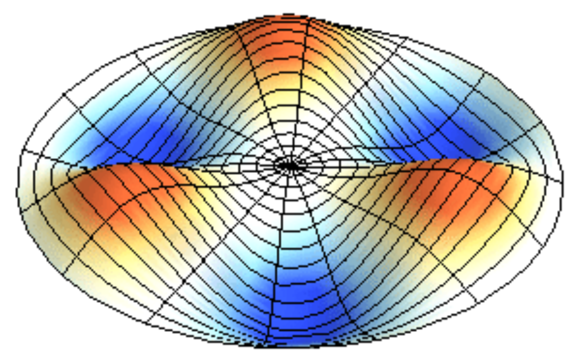
An alternative look at a traveling wave.



A sinusoidal wave moving along the x-axis.



→ Clearly, the displacement of a wave (standing or not) *depends upon both space and time*



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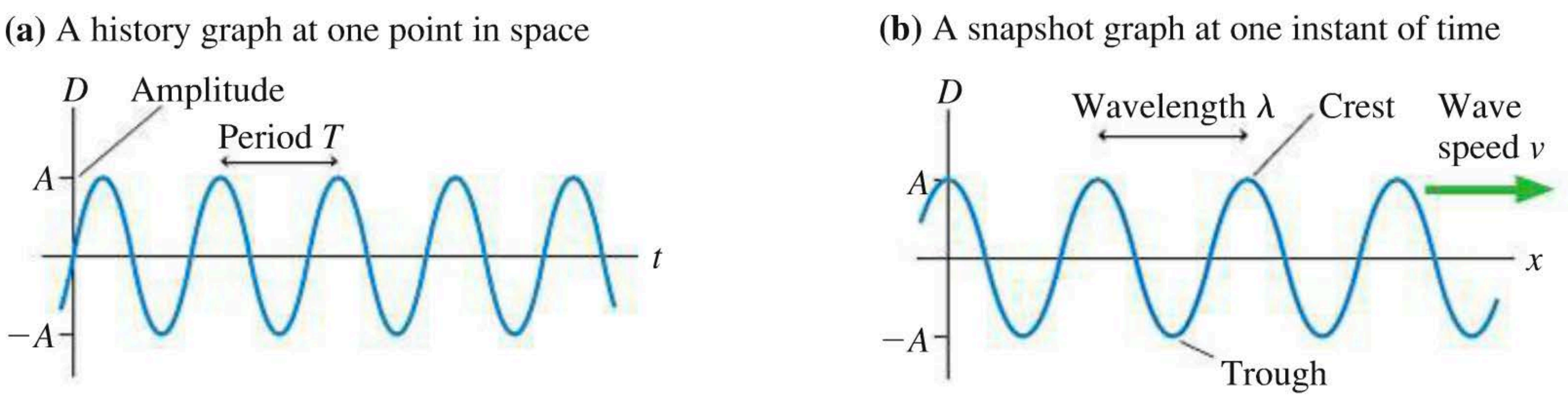
<u>"Wave math"</u> → Multivariable functions

History and snapshot graphs for a sinusoidal wave.



Time snapshot

Space snapshot

- Can take "snapshots", either in time or space

→ We need some additional mathematical tools to deal w/ this new reality....

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"Wave math" → Multivariable functions

- Multivariate functions be important for the various types of systems you will see throughout science

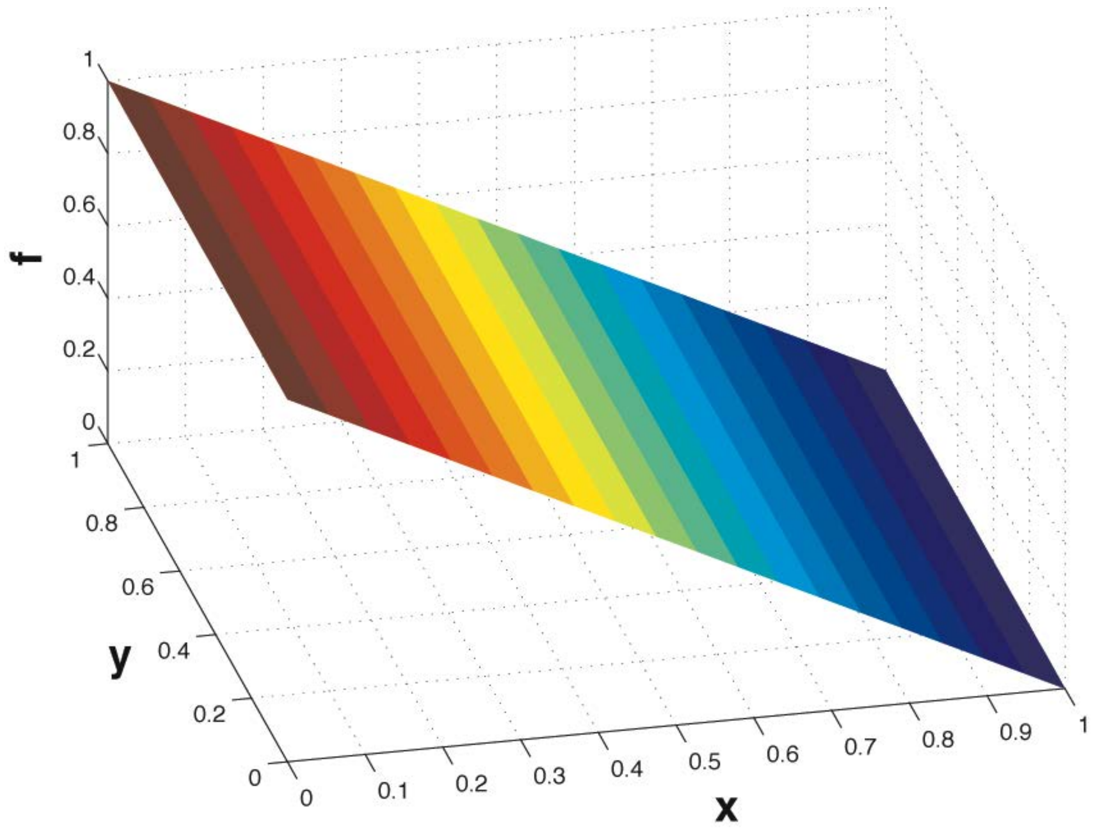
e.g. concentration of a solute in a solution (*c*) depends upon both spatial location (*x*) and time (*t*)

$$f = f(x, y)$$

f - dependent variable

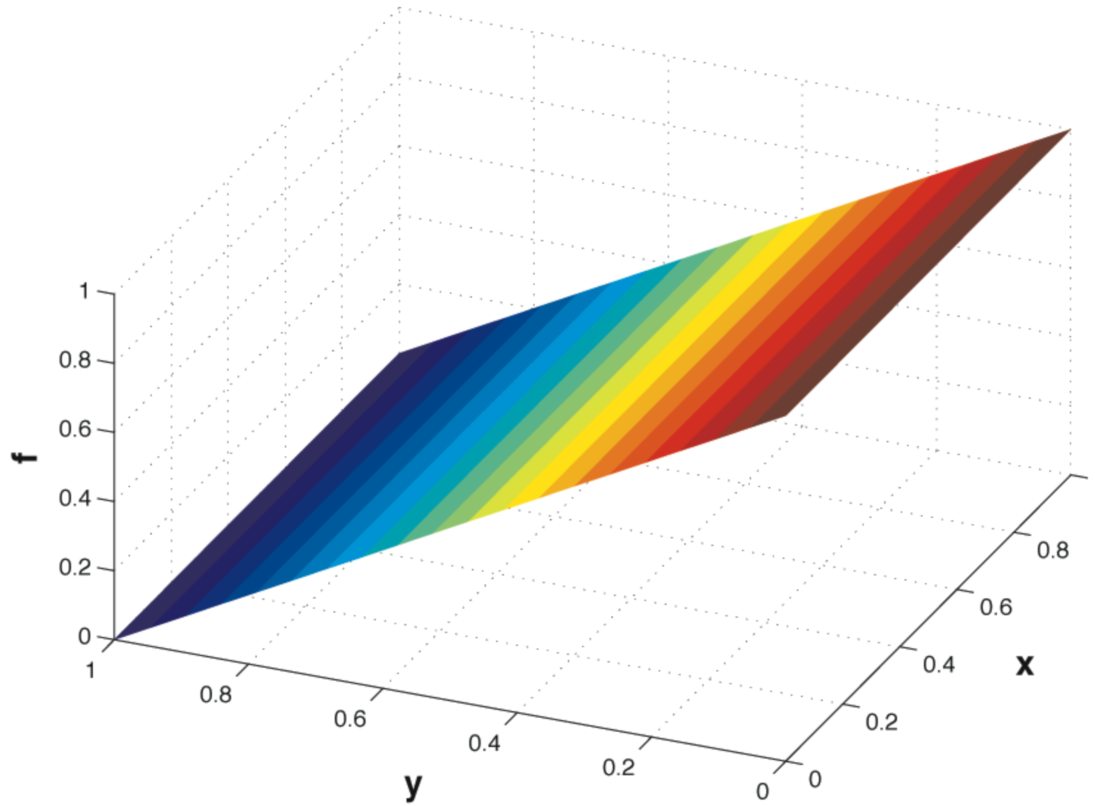
x, y - independent variables

"Wave math" → Multivariable functions



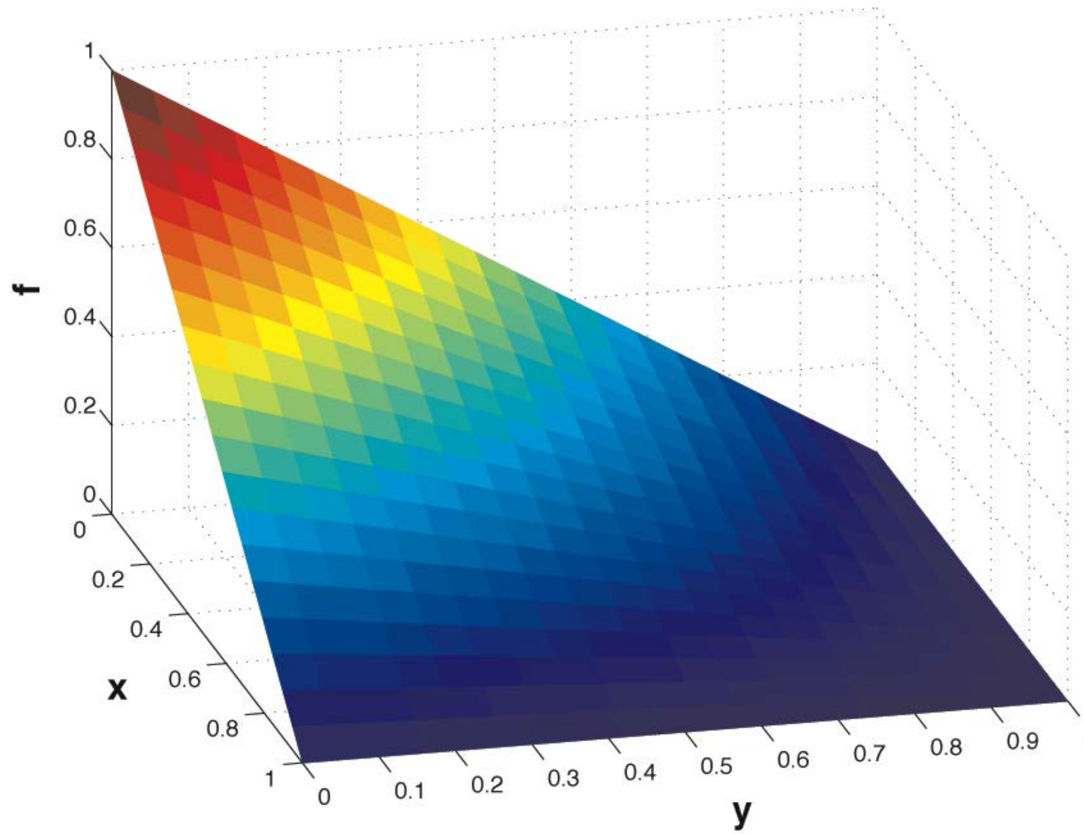
$$f(x,y) = (1 - x) = f(x)$$

“Wave math” → Multivariable functions



$$f(x, y) = (1 - y) = f(y)$$

"Wave math" → Multivariable functions



Biological Context

f - reaction rate [*mol/s*]

x, y - concentration of inhibitor agents [*mol*]

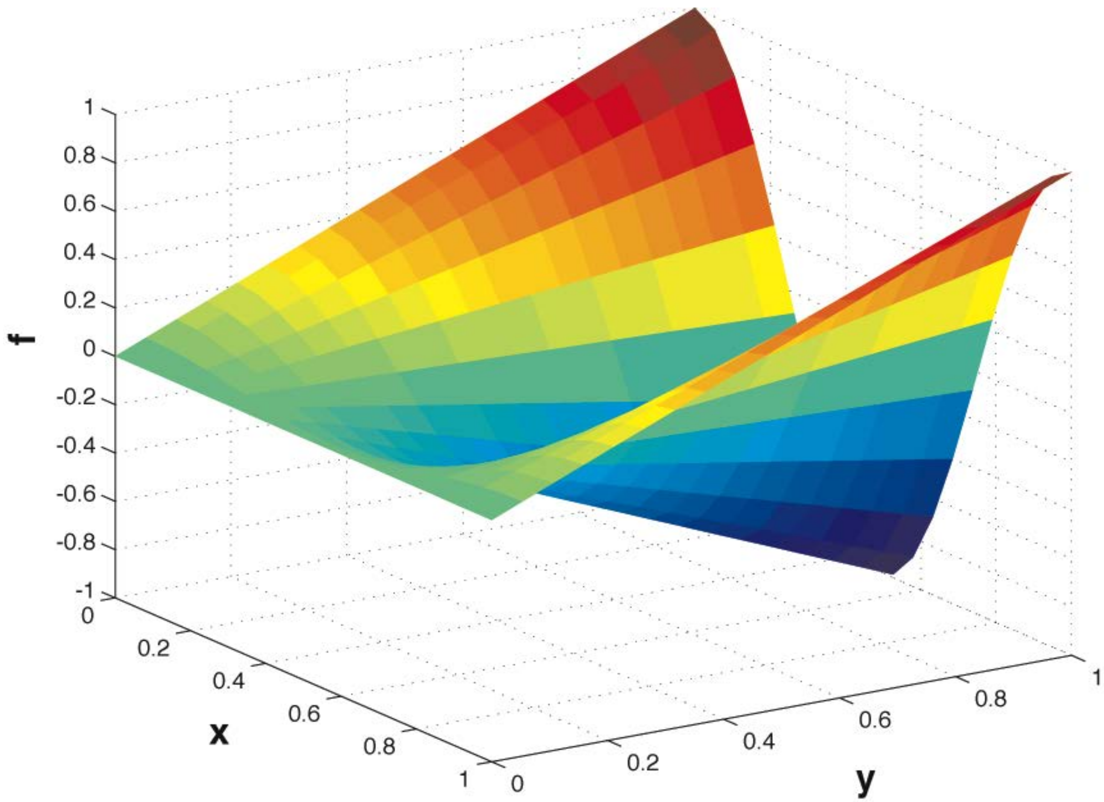
~~$$f(x,y) = (1-y)(1-x)$$~~

don't forget about units!

$$f(x,y) = k(1-x)(1-y)$$

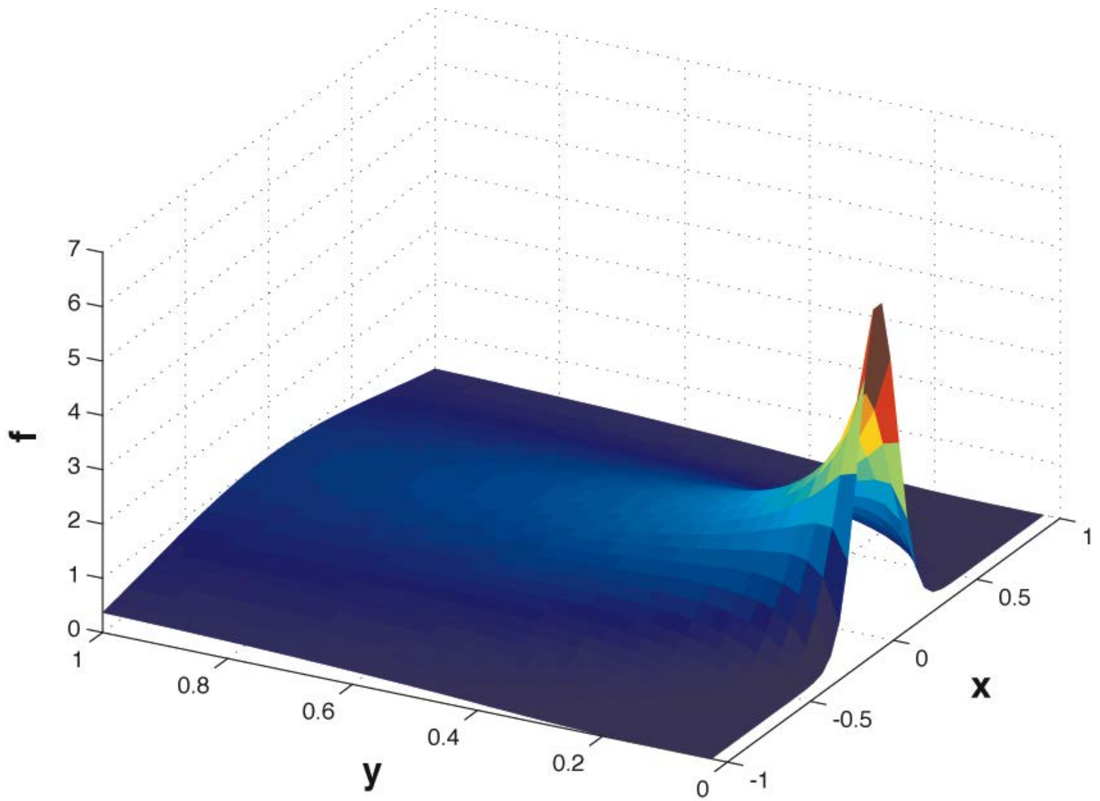
$$[k] = \frac{1}{mol \cdot s}$$

“Wave math” → Multivariable functions



$$f(x, y) = y \cos(2\pi x)$$

“Wave math” → Multivariable functions

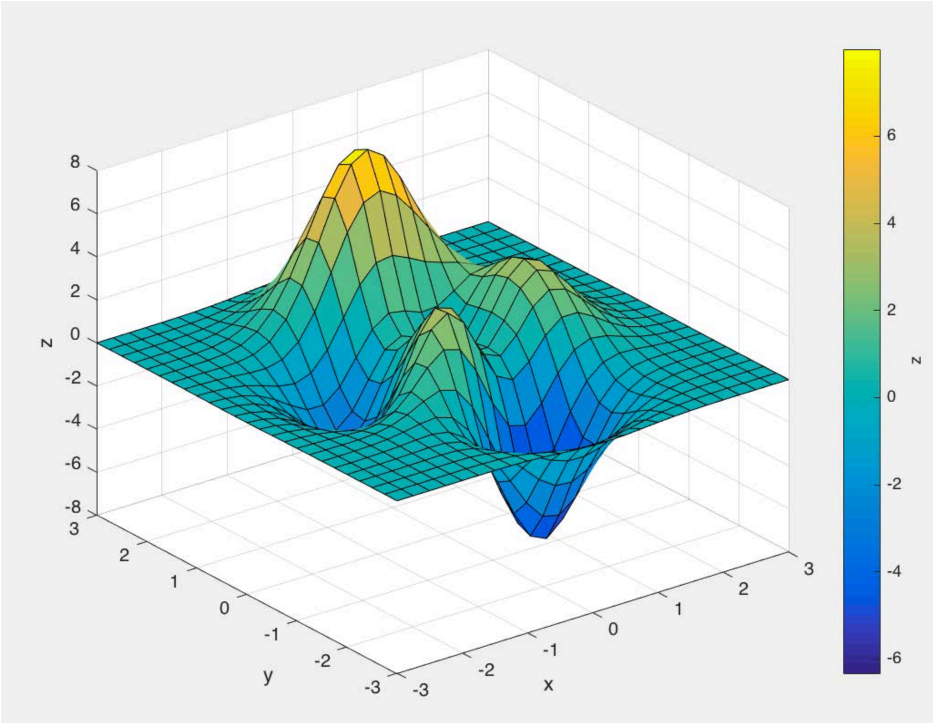


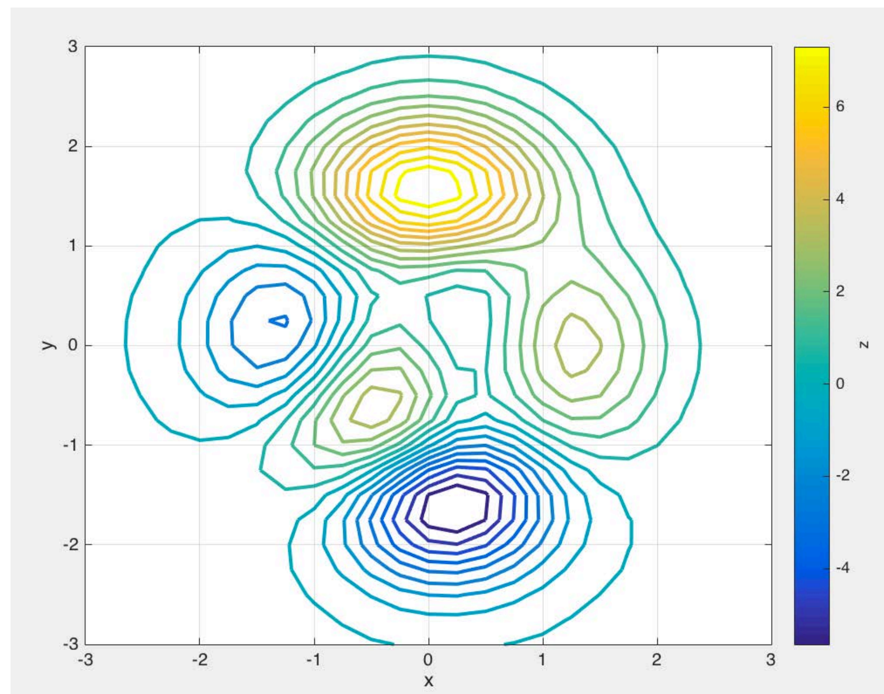
$$f(x,y) = \frac{1}{\sqrt{y}} e^{-x^2/y}$$

Solution to diffusion equation (or *heat eqn.*)

```
% ### EXcreate3D.m ###      11.04.16

% code to demonstrate/fiddle w/ 3-D plots in Matlab

% Notes
% o once you make the plot, in the figure window, look up top for the icon
% that is a cube w/ a circular arrow around it ("Rotate 3D"). Click on that
% and then have some fun....
% o type "help peaks" to see where/how the multivariable function (i.e., z)
% is generated

clear
% -----------
N= 25;          % # of axis points in xy-plane
M= 20;          % # of lines for contour plot
% -----------

% ====
% surface plot
[X,Y,Z] = peaks(N);     % use built-in Matlab function to create "z"
figure(1); clf;
surf(X,Y,Z);    h= colorbar;
xlabel('x'); ylabel('y'); zlabel('z'); ylabel(h,'z');

% ====
% contour plot
figure(2); clf;
contour(X,Y,Z,M,'LineWidth',2);    h= colorbar; grid on;
xlabel('x'); ylabel('y'); zlabel('z'); ylabel(h,'z');
```

EXcreate3D.m





Aside → Differentiating multivariable functions

Derivative (definition) for a function of a single variable

$$\frac{dg(x)}{dx} = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$

For multi-variable function, keep one variable constant

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x,y)}{\Delta x}$$

note difference in notation

y is effectively held constant here

→ "Partial derivative"

Aside → Differentiating multivariable functions

- Can take partial derivative with respect to partial derivative

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

- Simplified notation:

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}(f_x) = (f_x)_y = f_{xy}$$

Aside: Examples

Ex.1 $$\frac{\partial z}{\partial x} \quad \text{for} \quad z(x,y) = (xy)^{1/2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2}\, y^{1/2}\, x^{-1/2}$$

Ex.2 $$\frac{\partial}{\partial b}\left(\frac{\partial \phi}{\partial a}\right) \quad \text{for} \quad \phi(a,b) = ab^2 + 3a^2e^b$$

Notation: $\frac{\partial}{\partial b}\left(\frac{\partial \phi}{\partial a}\right) = \frac{\partial}{\partial b}(\phi_a) = (\phi_a)_b = \phi_{ab}$

$$\frac{\partial}{\partial b}\left(\frac{\partial \phi}{\partial a}\right) = 2b + 6ae^b$$

First find ϕ_a (assume b is constant): $\phi_a = b^2 + 6ae^b$

Now differentiate with respect to b : $\phi_{ab} = \frac{\partial}{\partial b}(\phi_a) = 2b + 6ae^b$

Note: In this case $\phi_{ab} = \phi_{ba}$ which is true for continuous functions (i.e. order of differentiation doesn't matte

"Wave math" → Partial differential equations

ODE ('ordinary'): considers a function of one variable and how it changes with respect to that variable

ex.

$$\frac{dc}{dt} = kc \qquad \text{where} \quad c = c(t) \text{ and } k = \text{ const.}$$

PDE ('partial'): considers a function of more than one variable and how its various partial derivatives are related

ex.

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} \qquad \text{where} \quad c = c(x,t) \text{ and } D = \text{ const.}$$

"Wave math" → The wave equation

- A wave's dependence upon space and time are interrelated via a PDE commonly referred to as the *wave equation*

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

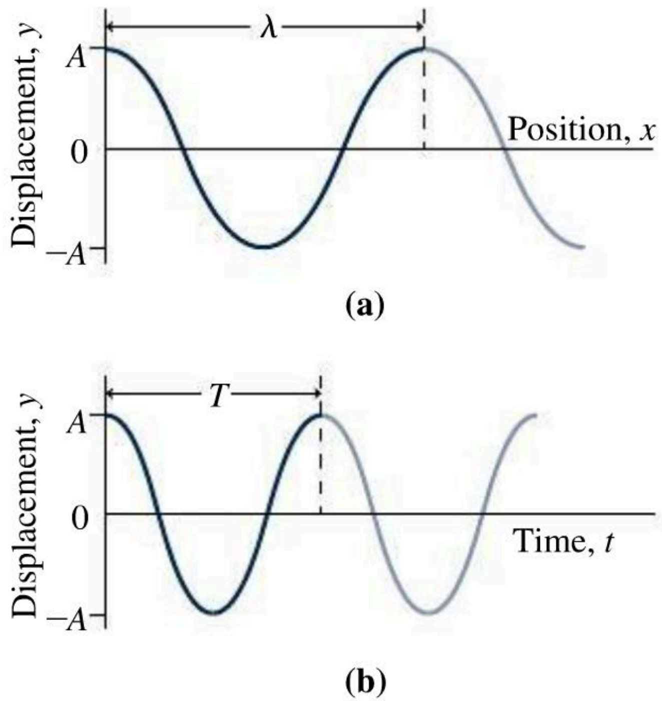
Note: One can readily derive this via combining Newton's 2nd Law and conservation of mass

$$y(x, t) = A\cos(kx \pm \omega t)$$

Possible solution to wave eqn. ("sinusoidal wave")

$$k = \frac{2\pi}{\lambda} \qquad \text{(wave number)}$$

$$v = \frac{\lambda}{T} = \frac{2\pi/k}{2\pi/\omega} = \frac{\omega}{k}$$

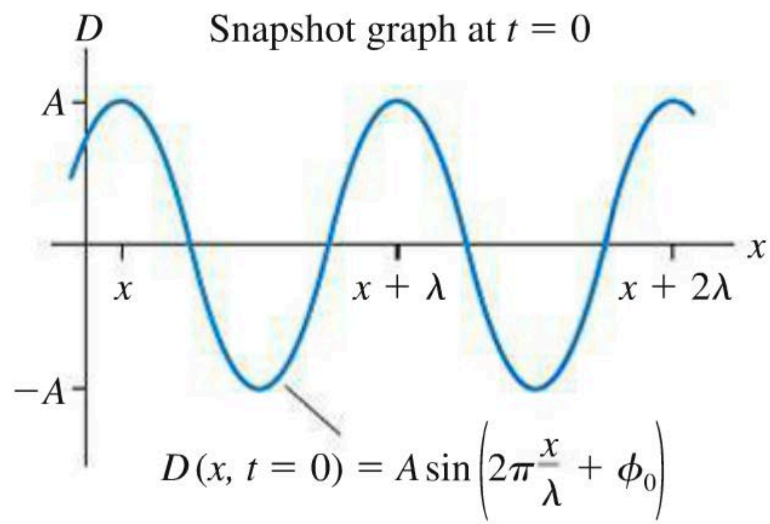


Wolfson

“Wave math” → Multivariable functions (REVISITED)

- Let’s consider a sinusoidal wave traveling to the right

$$D(x, t = 0) = A\sin\left(2\pi\frac{x}{\lambda} + \phi_0\right)$$



Eventually the wave repeats itself:

$$D(x + \lambda) = A\sin\left(2\pi\frac{(x + \lambda)}{\lambda} + \phi_0\right) = A\sin\left(2\pi\frac{x}{\lambda} + \phi_0 + 2\pi\text{ rad}\right)$$

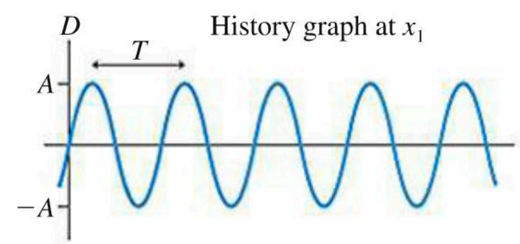
$$= A\sin\left(2\pi\frac{x}{\lambda} + \phi_0\right) = D(x)$$

Note:

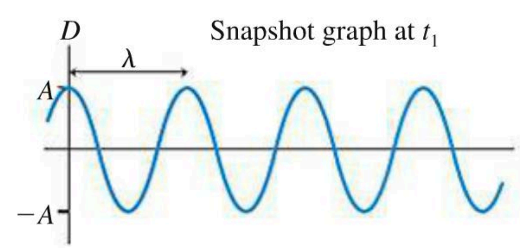
$$\sin(a + 2\pi\text{ rad}) = \sin a.$$

But the wave is “in motion”, so we can rewrite as:

$$D(x, t) = D(x - vt, t = 0)$$



If x is fixed, $D(x_1, t) = A\sin(kx_1 - \omega t + \phi_0)$ gives a sinusoidal history graph at one point in space, x_1. It repeats every T s.



If t is fixed, $D(x, t_1) = A\sin(kx - \omega t_1 + \phi_0)$ gives a sinusoidal snapshot graph at one instant of time, t_1. It repeats every λ m.

Knight

"Wave math" → Multivariable functions (REVISITED)

$$D(x,t) = D(x - vt, t = 0)$$

So we rewrite as:

$$D(x,t) = A\sin\left(2\pi\frac{x - vt}{\lambda} + \phi_0\right) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi_0\right)$$

$$D(x,t) = A\sin(kx - \omega t + \phi_0)$$

(sinusoidal wave traveling in the positive x-direction)

Relevant derived quantities:

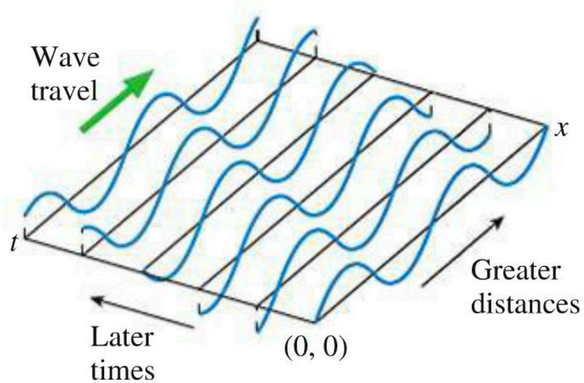
$$v = \lambda f = \lambda/T$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{\lambda}$$

$$v = \lambda f = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k}$$

A sinusoidal wave moving along the x-axis.



Knight

"Wave math" → Standing waves (REVISITED)

$$D_R = a\sin(kx - \omega t)$$ Right-going wave

$$D_L = a\sin(kx + \omega t)$$ Left-going wave

Note: The difference here is the sign. For "bonus" credit, look up **d´Alembert's formula**

Via superposition: $$D(x, t) = D_R + D_L = a\sin(kx - \omega t) + a\sin(kx + \omega t)$$

Relevant trig identity: $$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

Rewriting:

$$D(x, t) = a(\sin kx\cos\omega t - \cos kx\sin\omega t) + a(\sin kx\cos\omega t + \cos kx\sin\omega t)$$
$$= (2a\sin kx)\cos\omega t$$

$$D(x, t) = A(x)\cos\omega t$$

$$A(x) = 2a\sin kx$$

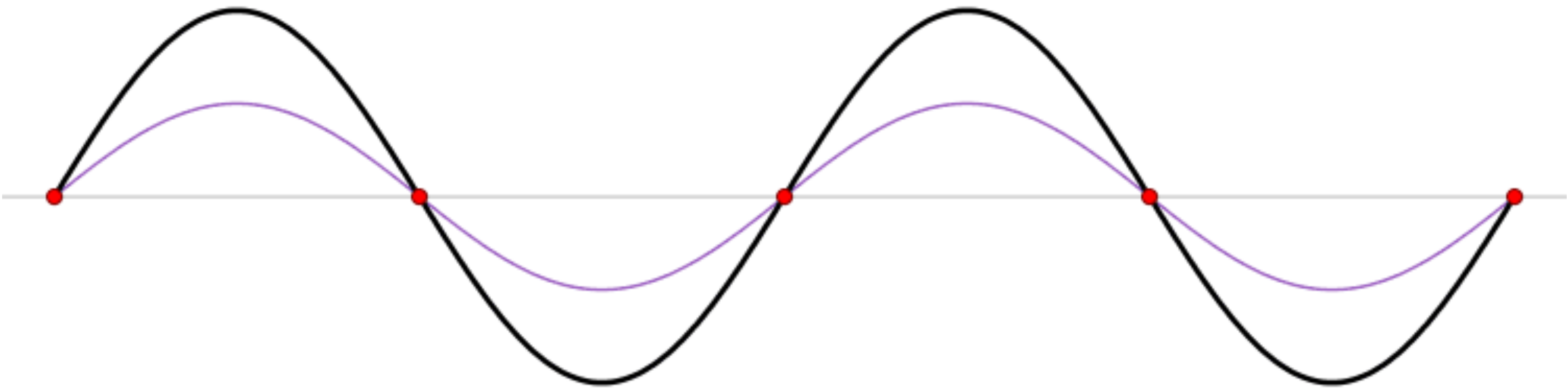
Note: A standing wave is not a traveling wave per se(!)

Knight

Standing Waves (REVISITED)

$$D(x, t) = A(x) \cos \omega t$$

$$A(x) = 2a \sin kx$$



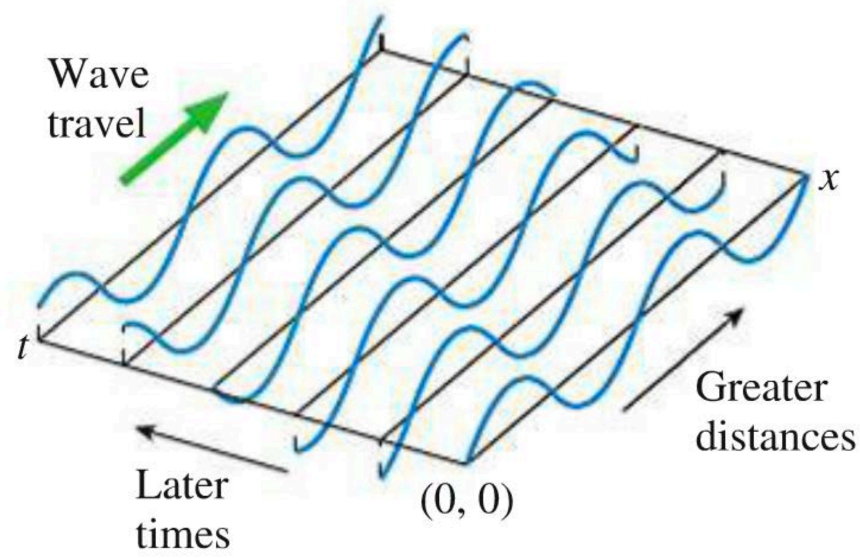
Blue is the left-going wave
Red is the right-going wave
Black is the sum of the two (i.e., the "standing" wave)

Note: Locations where the amplitude stays zero are called ***nodes***

https://en.wikipedia.org/wiki/Standing_wave

“Wave math” → Wave speed

A sinusoidal wave moving along the x-axis.



- For sinusoidal waves (w/ period T and wavelength λ), there is a straightforward means to determine the wave’s velocity

$$v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T}$$

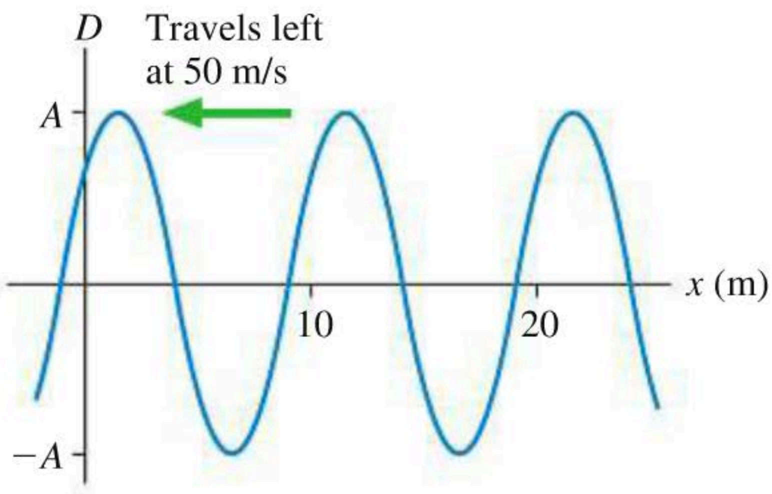
$$v = \lambda f$$

$$\lambda = \frac{v}{f} = \frac{\text{property of the medium}}{\text{property of the source}}$$

Ex. Try talking after sucking on a helium balloon....

Question

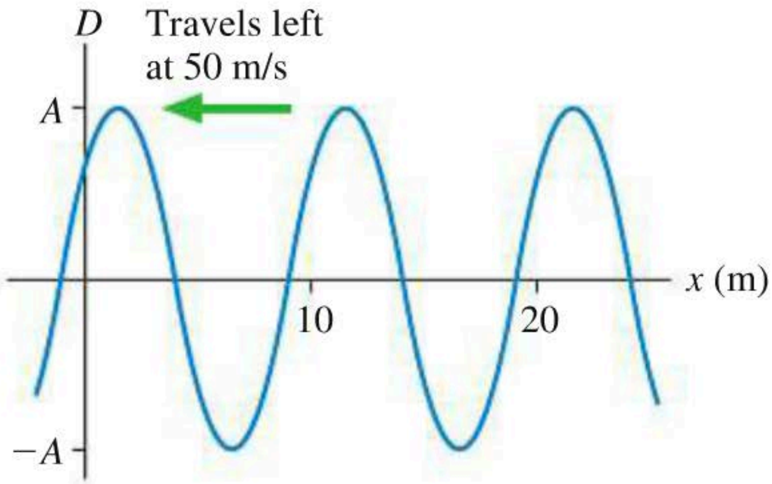
STOP TO THINK 20.3 What is the frequency of this traveling wave?



a. 0.1 Hz
b. 0.2 Hz
c. 2 Hz
d. 5 Hz
e. 10 Hz
f. 500 Hz

Question SOL

STOP TO THINK 20.3 What is the frequency of this traveling wave?

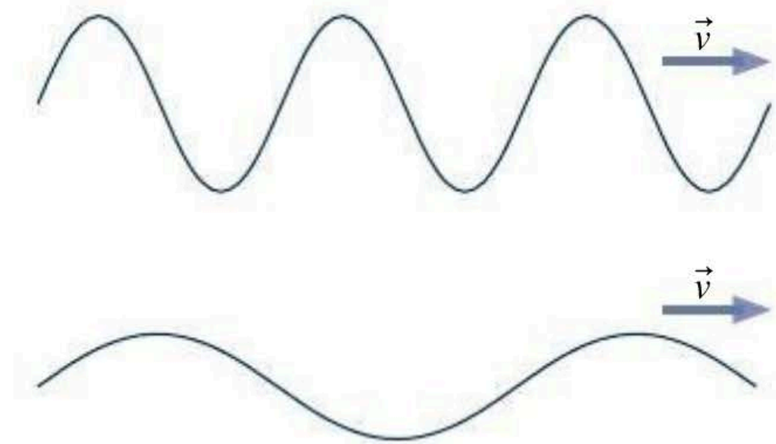


a. 0.1 Hz
b. 0.2 Hz
c. 2 Hz
d. 5 Hz
e. 10 Hz
f. 500 Hz

D

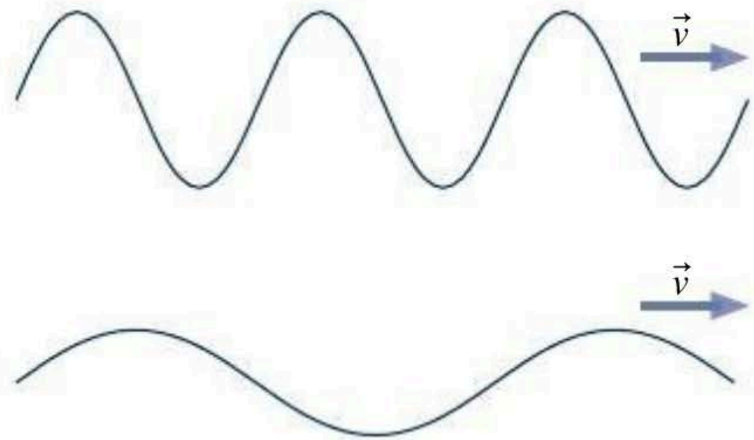
<u>Question</u>

GOT IT? 14.2 The figure shows snapshots of two waves propagating with the same speed. Which has the greater (1) amplitude, (2) wavelength, (3) period, (4) wave number, and (5) frequency?



Question SOL

GOT IT? 14.2 The figure shows snapshots of two waves propagating with the same speed. Which has the greater (1) amplitude, (2) wavelength, (3) period, (4) wave number, and (5) frequency?



(1) upper wave; (2) lower; (3) lower; (4) upper; (5) upper

And now back to light as a wave....

➢ EM waves are a bit special in that they are not entirely consistent w/ our definition of a wave....

Review: A traveling wave is a broad term, but in a general sense can be defined as occurring when a "condition of some kind is transmitted from one place to another by means of a medium, but the medium itself is not transported"

➢ We will need to develop further mathematical tools and physical concepts (e.g., electric fields, magnetism) to properly understand, classically at least, EM waves

Maxwell's equations

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$	Gauss's law
$\oint \vec{B} \cdot d\vec{A} = 0$	Gauss's law for magnetism
$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_{\text{m}}}{dt}$	Faraday's law
$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \frac{d\Phi_{\text{e}}}{dt}$	Ampère-Maxwell law

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ (Lorentz force law)

- **Gauss's law:** Charged particles create an electric field.
- **Faraday's law:** An electric field can also be created by a changing magnetic field.
- **Gauss's law for magnetism:** There are no isolated magnetic poles.
- **Ampère-Maxwell law, first half:** Currents create a magnetic field.
- **Ampère-Maxwell law, second half:** A magnetic field can also be created by a changing electric field.
- **Lorentz force law, first half:** An electric force is exerted on a charged particle in an electric field.
- **Lorentz force law, second half:** A magnetic force is exerted on a charge moving in a magnetic field.

→ Buried in all this is an even more basic notion: ***Oscillations***

Knight

Ex.

‖ Ships measure the distance to the ocean bottom with sonar. A pulse of sound waves is aimed at the ocean bottom, then sensitive microphones listen for the echo. FIGURE P20.45 shows the delay time as a function of the ship's position as it crosses 60 km of ocean. Draw a graph of the ocean bottom. Let the ocean surface define $y = 0$ and ocean bottom have negative values of y. This way your graph will be a picture of the ocean bottom. The speed of sound in ocean water varies slightly with temperature, but you can use 1500 m/s as an average value.

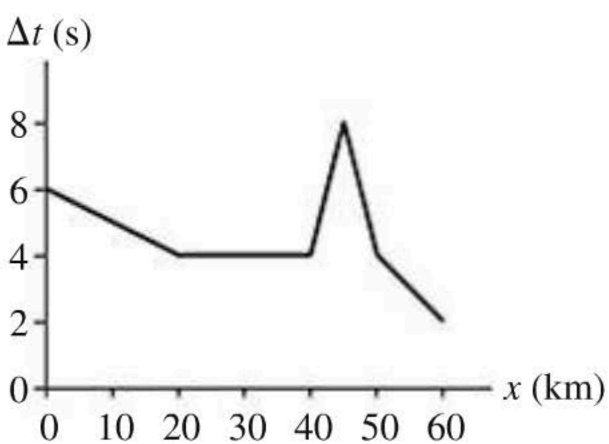


FIGURE P20.45

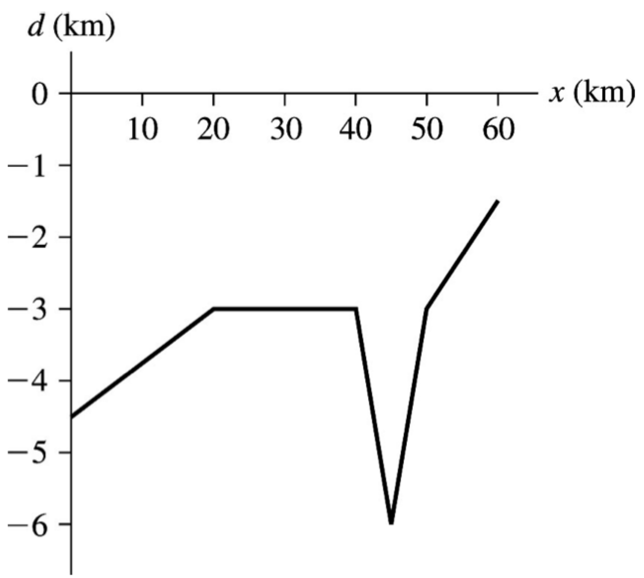
Ex. (SOL)

Solve: Δt is the time the sound wave takes to travel down to the bottom of the ocean and then up to the ocean surface. The depth of the ocean is

$$2d = (v_{\text{sound in water}})\Delta t \Rightarrow d = (750\ \text{m/s})\Delta t$$

Using this relation and the data from Figure P20.45, we can generate the following table for the ocean depth (d) at various positions (x) of the ship.

x (km)	Δt (s)	d (km)
0	6	4.5
20	4	3.0
40	4	3.0
45	8	6.0
50	4	3.0
60	2	1.5



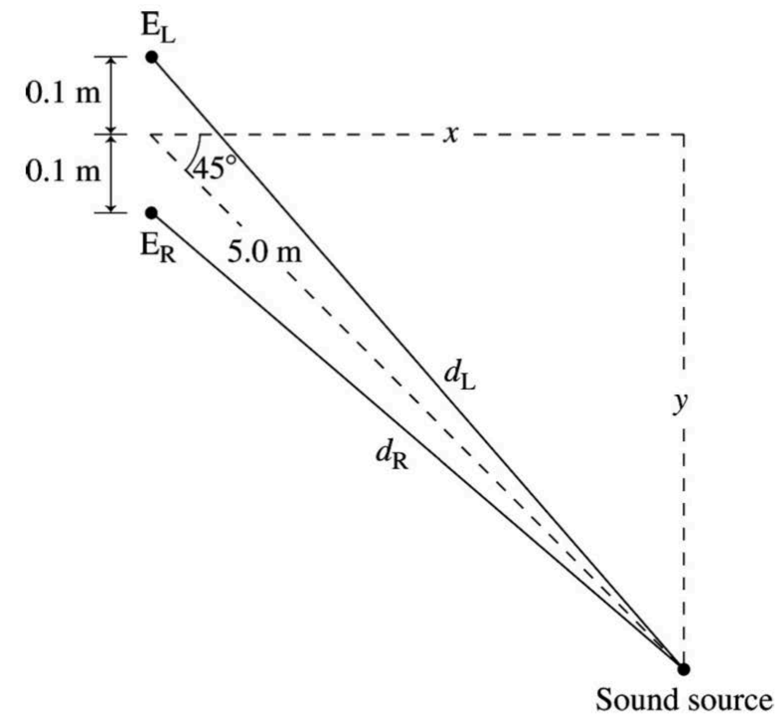
Ex.

|| One cue your hearing system uses to localize a sound (i.e., to tell where a sound is coming from) is the slight difference in the arrival times of the sound at your ears. Your ears are spaced approximately 20 cm apart. Consider a sound source 5.0 m from the center of your head along a line 45° to your right. What is the difference in arrival times? Give your answer in microseconds. **Hint:** You are looking for the difference between two numbers that are nearly the same. What does this near equality imply about the necessary precision during intermediate stages of the calculation?

Ex. (SOL)

Model: Assume a room temperature of 20°C.

Visualize:



Solve: The distance between the source and the left ear (E_L) is

$$d_L = \sqrt{x^2 + (y + 0.1\text{ m})^2} = \sqrt{[(5.0\text{ m})\cos 45°]^2 + [(5.0\text{ m})\sin 45° + 0.1\text{ m}]^2} = 5.0712\text{ m}$$

Similarly $d_R = 4.9298$ m. Thus,

$$d_L - d_R = \Delta d = 0.1414\text{ m}$$

For the sound wave with a speed of 343 m/s, the difference in arrival times at your left and right ears is

$$\Delta t = \frac{\Delta d}{343\text{ m/s}} = \frac{0.1414\text{ m}}{343\text{ m/s}} = 410\ \mu\text{s}$$

Ex.

‖ A wave on a string is described by $D(x, t) = (3.0\text{ cm}) \times \sin[2\pi(x/(2.4\text{ m}) + t/(0.20\text{ s}) + 1)]$, where x is in m and t is in s.

a. In what direction is this wave traveling?

b. What are the wave speed, the frequency, and the wave number?

c. At $t = 0.50$ s, what is the displacement of the string at $x = 0.20$ m?

Knight

Ex. (SOL)

Model: This is a sinusoidal wave.

Solve: **(a)** The displacement of a wave traveling in the positive x-direction with wave speed v must be of the form $D(x, t) = D(x - vt)$. Since the variables x and t in the given wave equation appear together as $x + vt$, the wave is traveling toward the left, that is, in the $-x$ direction.

(b) The speed of the wave is

$$v = \frac{\omega}{k} = \frac{2\pi/0.20\text{ s}}{2\pi\text{ rad}/2.4\text{ m}} = 12\text{ m/s}$$

The frequency is

$$f = \frac{\omega}{2\pi} = \frac{2\pi\text{ rad}/0.20\text{ s}}{2\pi} = 5.0\text{ Hz}$$

The wave number is

$$k = \frac{2\pi\text{ rad}}{2.4\text{ m}} = 2.6\text{ rad/m}$$

(c) The displacement is

$$D(0.20\text{ m}, 0.50\text{ s}) = (3.0\text{ cm})\sin\left[2\pi\left(\frac{0.20\text{ m}}{2.4\text{ m}} + \frac{0.50\text{ s}}{0.20\text{ s}} + 1\right)\right] = -1.5\text{ cm}$$