Modeling otoacoustic emission group delays in the lizard auditory periphery

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Work done in collaboration w/ Christopher Shera (HMS)
It is absolutely impossible to give too elementary a \textit{math} talk. Every talk I have ever attended in four decades of lecture-going has been too hard. There is therefore no point in advising you to make your talk clear and comprehensible.

\textbf{Tip #1 (of 2)}

But suppose you do remember why you got into your current line of research. If you succeed in conveying that early freshness and excitement to somebody else, your talk will be an unqualified success, even if you never manage to describe a single one of the splendid things you uncovered when the project was well under way.
**Question**: Can a series of (linear) coupled oscillators exhibit large group delays across a wide frequency range as seen in otoacoustic emissions (OAEs)?

**Means**: Use the (relatively simple) lizard ear as a basis for modeling the inner ear
Amplitude [dB]

Frequency [kHz]

2 dB
50 dB
40 dB
30 dB
20 dB
10 dB

\[ \Delta f_{\text{OAE}} \]

Shera and Zweig (1993)
SFOAEs

Lp=40 dB SPL, Ls=55 dB, fs=fp+40 Hz

Bergevin et al. (2008A)
Mammalian Cochlea Uncoiled

- Stapes
  - to Middle Ear
  - to Vestibular System
- Acoustic Energy
- Round Window
- Cochlear Partition
- Helicotrema
  - pliant & massive

C.D. Geisler (modified)
Not due to differences between mammals and non-mammals nor size

[cat and guinea pig data from Shera and Guinan, 2003]
Tuned Responses Take Time

Second Order System
(resonant frequency $\omega_o$)

$\Rightarrow$ External driving force at frequency $\omega$

$$x(t) = A(\infty) \left[ 1 - e^{(-t/\tau)} \right]$$

$$\tau = Q / \omega_o$$
Q and Phase Gradients Co-vary

Second Order System
(resonant frequency $\omega_o$)

\[ Q = \frac{\omega_o}{(2\pi \times \text{BANDWIDTH})} \]

\[ N = \frac{\omega_o \times \text{Phase Gradient}}{2\pi} \]  
(at $\omega_o$)

\[ Q \propto N \]

Shera, Oxenham and Guinan (2007)
Hypothesis: SFOAE group delays* reflect tuning mechanisms in the inner ear

* group delay = phase-gradient delay (for the purpose of the talk)
Tokay Gecko Auditory Nerve Fiber Responses

Manley et al. (1999)
Comparison of $N$ (SFOAE) to $Q$-value (ANF)

Bergevin et al. (2007)
Gecko Inner Ear

Wever (1978)
Coupled resonators
Assumptions

- middle ear delays are negligible

- papilla moves as rigid body ⇒ one-dimensional motion

- sinusoidal steady-state response

- exponential tonotopic map

- linearity
Question 1: suppression schematic
SFOAEs: Nonlinear suppression paradigm

**Step 1.** Present Probe Alone (emission is present)
FFT reveals magnitude and phase AT Probe Freq.

**Step 2.** Present both Probe & Suppressor tones (emission not present)
FFT reveals magnitude and phase AT Probe freq.

**Step 3.** Subtract phasors
SFOAE revealed!!
Assumptions II

- irregularity in damping along papilla

- OAE is complex difference between ‘smooth’ and ‘rough’ conditions
Motion of the papilla:

\[ m_p \ddot{y}_p = -k_p y_p - r_p \dot{y}_p + A(p_v - p_t) + \sum_{n} k_b^{(n)} (y_b^{(n)} - y_p) \]

Motion of an \((n'th)\) individual bundle:

\[ \ddot{y}_b^{(n)} = -\omega_{b,o}^{(n)} \left( y_b^{(n)} - y_p \right) - \gamma_b^{(n)} y_b^{(n)} \]
Sinusoidal steady-state:

\[ y_b^{(n)}(t) = Y_b^{(n)}(\omega)e^{i\omega t} \quad y_p(t) = Y_p(\omega)e^{i\omega t} \]

\[ Y_p k_b^{(n)} = Y_b^{(n)} \left[ -\omega^2 m_b^{(n)} + k_b^{(n)} + i\omega r_b^{(n)} \right] \]

\[ Y_p \left[ -\omega^2 m_p + k_p + i\omega r_p + \sum_n k_b^{(n)} \right] = A(p_v - p_t) + \sum_n k_b^{(n)} Y_b^{(n)} \]
Change of Variables (dimension-less):

\[ \chi = \frac{x}{L} \]

\[ \beta_n = \frac{\omega}{\omega_b^{(n)}} = \omega / \omega_b(\chi_n) \]

\[ \delta_{o}^{(n)} \equiv \frac{r_b^{(n)}}{\sqrt{k_b^{(n)} m_b^{(n)}}} = \frac{1}{Q} \]

\[ Y_b^{(n)} = \frac{Y_p}{[1 - \beta_n^2 + i\beta_n \delta^{(n)}]} \]

Roughness:

\[ \delta^{(n)} = \delta_{o}^{(n)} + \rho \]

(where \( \rho \) represents the irregularity)
Conservation of Mass:

\[ \dot{Y}_p A = U_{ow} \implies Y_p = \frac{U_{ow}}{i\omega A} \]

Impedance:

\[ I_c = \frac{p_v - p_t}{U_{ow}} \]

Impedance (smooth and rough conditions):

\[ I_o(\chi, \omega) = \frac{-\beta^2 + i\beta\delta_o}{1 - \beta^2 + i\beta\delta_o} \quad I(\chi, \omega) = \frac{-\beta^2 + i\beta\delta}{1 - \beta^2 + i\beta\delta} \]
Impedance (summing effect of all bundles):

\[ Z_o(\omega) = \int I_o(\chi, \omega) d\chi \quad Z(\omega) = \int I(\chi, \omega) d\chi \]

Finding the ‘Emission’:

Complex Ratio: \[ z = \frac{Z}{Z_o} \]

Reflectance: \[ R(\omega) \equiv \frac{z - 1}{z + 1} \] (ignoring higher order terms)
Comparison: Empirical Data and Model Results

150 oscillators
CF range 0.3-5 kHz
Q = 0.2 (uniform with frequency)
Different Irregularity Patterns ($\delta(x)$)
N Increases with Q (10 different ‘ears’)
Let Q Vary with Frequency (10 different ‘ears’)

$Q = 2N$ ?
Comparison of SFOAE Delays and ANF Tuning Across Species

SFOAE Delay

Emission frequency (kHz)

Sharpness of Tuning (Q_{ERB})

Characteristic frequency (kHz)

Bergevin et al. (2008B)
The ubiquitous heavy-handed concluding summary should be omitted; a talk should tell such a good story that a summary is uncalled for. Imagine War and Peace ending with a summary. There is no better way to make an audience happy than briskly finishing a talk five minutes earlier than it expected you to. Like this.

Tip #2 (of 2)

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Fini
Reflection of energy?

Transmission line with random irregular ‘sources’

Shera and Guinan (2007)
Pre-Existing Perturbations

Wave shifts when frequency increases

Pre-existing perturbation remains fixed in space

Phase changes at source

Cochlear Location

Wave-Induced Sources

Wave shifts when frequency increases

Induced source/perturbation moves with the wave

Wave phase stays constant at source

Cochlear Location

Shera (2003)

PLACE-FIXED

WAVE-FIXED
Tokay Gecko Auditory Nerve Fiber Responses

Manley et al. (1999)