Perception, 1995, volume 24, pages 937–968

All that glitters: a review of psychological research on the aesthetics of the golden section

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Received 10 February 1994, in revised form 20 March 1995

Abstract. Since at least the time of the Ancient Greeks, scholars have argued about whether the golden section—a number approximately equal to 0.618—holds the key to the secret of beauty. Empirical investigations of the aesthetic properties of the golden section date back to the very origins of scientific psychology itself, the first studies being conducted by Fechner in the 1860s. In this paper historical and contemporary issues are reviewed with regard to the alleged aesthetic properties of the golden section. In the introductory section the most important mathematical occurrences of the golden section are described. As well, brief reference is made to research on natural occurrences of the golden section, and to ancient and medieval knowledge and application of the golden section, primarily in art and architecture. Two major sections then discuss and critically examine empirical studies of the putative aesthetic properties of the golden section dating from the mid-19th century up to the 1950s, and the empirical work of the last three decades, respectively. It is concluded that there seems to be, in fact, real psychological effects associated with the golden section, but that they are relatively sensitive to careless methodological practices.

1 Introduction

1.1 *What is the golden section?*

Imagine that you are asked to divide a line so that the ratio of the shorter segment to the longer segment is the same as the ratio of the longer segment to the whole line. That is, if a represents the length of the shorter segment and b represents the length of the longer segment, find values for a and b such that

$$\frac{a}{b} = \frac{b}{a+b},$$

where $a+b = 1$. The division of a line that answers to this requirement has come to be known as the 'golden section'[(1)] of a line, and it is shown in figure 1.

It has often been claimed, since the time of the Ancient Greeks, and perhaps much earlier, that the golden section is the most aesthetically pleasing point at which to divide a line. Consequently, it has been incorporated, in a variety of ways, into many artworks and architectural designs through the ages. Establishing empirically the claims of aesthetic primacy for the golden section was among the very first topics of scientific psychological research as the new discipline emerged in the 19th century.

Figure 1. A line divided in the golden section. If the short segment is taken as 1, the long segment is ϕ in length. If the long segment is taken as 1, the short segment is ϕ' in length.

[(1)] It is variously called the golden section, golden number, golden ratio, golden proportion, and ϕ. Also, it has been so often confused with Aristotle's golden mean—another notion entirely—that this misnomer has acquired some legitimacy as well. I will use all but the last of these interchangeably, unless otherwise specified for special purposes.

Fechner first fixed his analytical gaze upon this task as early as the 1860s. Since that time it has been the focus of a number of research programs, winning the attention of structuralists, gestaltists, behaviorists, social psychologists, psychiatrists, and neuroscientists at various points in time.

The exact value of the golden section of a line is given by an irrational number approximately equal to 0.618. That is, with a line divided at slightly more than 60/40, the short segment will bear the same mathematical relation to the long segment as the long segment does to the whole line. By adding 1 to the golden section—1.618—one gets a closely related number widely known as ϕ. This number stands alongside of π and e as the most important irrational constants in mathematics. Constants such as π, e, and ϕ gain their importance from being the solutions to basic mathematical problems. In addition, they perhaps gain their special mystique from being irrational numbers; their exact values never being known. Just as π is the ratio of the circumference of any circle to its diameter, and e is the solution to the expression $\lim[1+(1/n)]^n$, the solution to the equation $x^2 = x+1$ is another of these long-standing problems in mathematics. Its exact solution is $(1 \pm \sqrt{5})/2$.

The positive solution is ϕ – approximately 1.618—and the negative solution is approximately -0.618 (exactly the negative of the golden section described above). For convenience, I shall denote the negative solution as $-\phi'$, and *its* negative (the golden section itself) as ϕ'.(2) A number of interesting relations hold between these two numbers. To begin with, ϕ' is equal to both $\phi-1$ and to $1/\phi$, or ϕ^{-1}. Thus, $\phi-\phi' = \phi\times\phi' = 1$. Just as ϕ^{-1} is equal to $\phi-1$, so ϕ^2 is equal to $\phi+1$, or $\phi^1+\phi^0$. Similarly, ϕ^3 is equal to $\phi^2+\phi^1$, and so on; $\phi^n = \phi^{n-1}+\phi^{n-2}$, for any value of n. A number of other interesting mathematical and geometrical properties are discussed below.

The origins of the names of these two numbers are matters of some dispute. Berlyne (1971) claims that the reference to gold "was adopted in large measure, because of vague associations with the 'golden mean'" (page 229). Kepler, however, referred to π and ϕ as the mathematical equivalents of gold and precious gems as far back as 1596. Fowler (1982) claims that the first use of "*goldne Schnitt*" appears in an 1835 mathematical text by Martin Ohm, and that its first titular appearance was in an 1849 book by A Wiegang. It was the publications of Adolf Zeising (1854, 1855, 1884), however, that did the most to popularize the name widely. Fowler also claims that the first English-language use of the term golden section was in James Sully's article on aesthetics in the 1875 edition of the *Encyclopaedia Britannica*.

The symbol ϕ, on the other hand, derives from the initial letter of the name of the great Greek architect and sculptor, Phidias. Phidias was a proponent of the aesthetic qualities of the golden section, going so far, according to legend, as to incorporate it into the basic dimensions of his most famous work, the Parthenon (Ogden 1937). Huntley (1970), however, says that the appellation, ϕ, was not adopted until "the early days of the present century" (page 25).

Whatever the truth of these various claims, it is certain that the golden section has been the focus of a great deal of interest for a very long time. It is the aim of this paper to review the results of that interest, with special emphasis on the reputed aesthetic properties of the golden section as explored in the empirical work of the last 130 years. There have been other, nonaesthetic, research programs associated with the golden section, however. Most developed of these is its relation to judgments of interpersonal relations (Adams-Webber 1985; Adams-Webber and Benjafield 1973; Benjafield 1985; Benjafield and Adams-Webber 1976). Other recent ones include its

(2) Note that this is the opposite of Huntley's (1970) notation, who denotes -0.618 ... with ϕ'. Because the negative number will be little used in the following discussion, I have chosen to denote 0.618 with ϕ' for the sake of clarity.

relation to the structure of ethical cognition (Lefebvre 1985); the cognitions of sufferers of anxiety, depression, and agoraphobia (Schwartz and Michelson 1987); consumers' perceptions of products and of retail environments (Crowley 1991; Crowley and Wiliams 1991); and even the proportion of wins a sports franchise needs to maintain fan support (Benjafield 1987). Nevertheless, this review is restricted to an aesthetic focus. It is intended to fulfill the needs of both historical and scientific researchers.

There are several different reasons one might be interested in a review of golden-section research. One reason, relevant to the history of psychology, is that, as mentioned above, studies of the golden section were among the very first empirical studies conducted in psychology; they take their place alongside Weber's and Fechner's psychophysical studies, Ebbinghaus's memory studies, and Helmholtz's studies of tone and color perception. Moreover, since that time the golden section has captured the interest of some of the most illustrious names in the discipline. A second reason is that the psychological phenomena associated with the golden section, despite repeated attempts to show them to be nonexistent or mere methodological artifacts, simply refuse to go away. Although many researchers have concluded that the effects are illusory, the more carefully conducted studies have fairly consistently shown that there is, in fact, a set of phenomena that require explanation, though no one has yet produced an explanation both adequate and plausible that has been able to stand the test of time.

The paper is structured as follows. First, some of the mathematical occurrences of the golden section are briefly examined. Second is a major section in which the empirical research conducted between the mid-19th century and about 1960 on the reputed aesthetic properties of the golden section is reviewed. In a third section the more-recent findings and theoretical explanations of psychologists are examined, with the focus particularly on the concerted efforts to show that golden-section effects are nothing but artifacts and on the stubborn resilience of the phenomenon many have tried to make go away. Finally, in a concluding section, I summarize the findings of this review.

1.2 *Instances of ϕ in mathematical and natural worlds*

Imagine that one of the segments of a line divided at the golden section is 'folded' at a 90° angle to the other, and a rectangle is formed on this L-shaped base, as in figure 2a. This figure is called the 'golden rectangle'. The golden rectangle can be constructed quite simply. Starting with a square, swing an arc, centered on the mid-point of the base of the square, from an upper corner of the square to a point collinear with the base. Build a rectangle on the new, longer base (see figure 2b). Just as it has

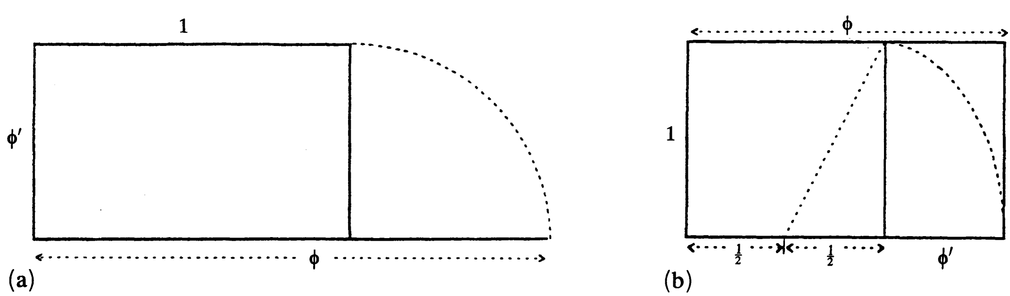

Figure 2. (a) A golden rectangle constructed by swinging vertical the short segment of a line divided in the golden section. (b) A golden rectangle constructed by swinging a line that runs from the midpoint of the bottom of a square to an upper corner to a point collinear with the bottom side.

been claimed that the golden section is the most aesthetically pleasing division of a line, so it has long been claimed that the golden rectangle is the most aesthetically pleasing of all rectangles.

The regular pentagon also contains many instances of ϕ and ϕ'. For instance, if the side of the pentagon is taken to be the unit, any diagonal of the pentagon has a length of ϕ. Also, the diagonal of any regular pentagon divides any other diagonal of the pentagon into a golden section (figure 3a). If a regular pentagram is constructed out of a regular pentagon, by extending its sides until they intersect each other (figure 3b), the result contains many instances of ϕ as well.

Each 'point' of the pentagram forms an isosceles triangle that is 72° at the base and 36° at the apex. This triangle is a rich source of examples of ϕ in its own right; so much so that it is known as the 'golden triangle' (figure 4). The ratio of the lengths of each of its legs to that of its base is ϕ. (It is important to note that the ratio of its height to its base is *not* ϕ, an erroneous assumption frequently made by some contemporary psychological researchers. The golden triangle has a height-to-base ratio of approximately 1.53:1.) Last, ϕ is hidden even in the component angles of the golden triangle: the cosine of 36° is $\phi/2$, and the cosine of 72° is $\phi'/2$.

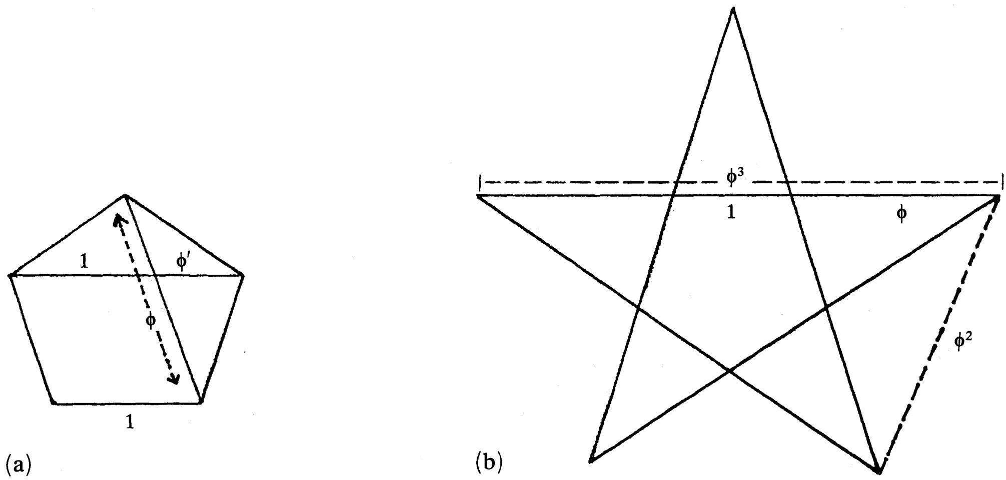

Figure 3. (a) The diagonals of a regular pentagon with side = 1 are of length ϕ, and divide each other in the golden section. (b) A pentagram constructed from a regular pentagon with sides = 1. The length of the sides of the five new triangles is ϕ, the distance between two adjacent triangle ends is ϕ^2, and the distance between the ends of two collinear triangles is ϕ^3. Each point is also a golden triangle.

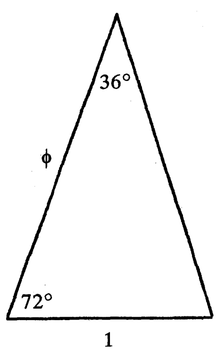

Figure 4. The golden triangle. If the base is taken as 1, its legs are ϕ in length. Its height-to-base-width ratio is 1.53:1, not ϕ as is often assumed.

There are many other geometrical instances of ϕ. The best-known reference is Huntley (1970). Another interesting account that includes links to contemporary work on fractals and tiling is Kappraff (1991). Not all the mathematical occurrences of the ϕ are geometric, however. Some are arithmetic as well. Most importantly, ϕ is the limit of the ratio of any two sequential numbers in any Fibonacci series (ie any series of integers in which $x_n = x_{n-1} + x_{n-2}$) such as 1, 1, 2, 3, 5, 8, 13, ... or, for that matter, $-2, 7, 5, 12, 17, 29, \ldots$.

The question remains, however, of what interest all this might be to the non-mathematician. One answer is that there appear to be many examples of ϕ in the natural world as well. Because ϕ is so intimately connected with the pentagon, as discussed above, pentagonal symmetry in nature has often been taken as a starting point for investigations of the 'natural reality' of ϕ. Flowers of many kinds exhibit pentagonal symmetry (Berlyne 1971, page 224; Ghyka 1946/1977, page 18, *n*) as do a wide variety of sea creatures (Ghyka 1946/1977, page 18; see also Hargittai 1992). Another connection between ϕ and the natural world is mediated by the logarithmic spiral, which can be easily constructed from a golden rectangle or triangle. The shell of the nautilus, the abalone, and the triton, for instance, show the same sort of geometric growth pattern characteristic of the logarithmic spiral (see Ghyka 1946/1977, pages 93–97). The logarithmic spiral is also characteristic of the patterns of growth found on pine cones, pineapples, and sunflowers. A third link between the natural world and ϕ is the frequency with which the Fibonacci series is seen in nature. Fibonacci (Leonardo of Pisa, ca 1175–1250) himself showed that the growth both of rabbit and of beehive populations can be modeled by a Fibonacci series, even though the reproductive principles of the two are quite different. The relation between Fibonacci numbers and phyllotaxis (the study of the arrangements of leaves on plants) is also widely cited (see Berlyne 1971, page 224; Ghyka 1946/1977, page 16; Huntley 1970, pages 161–163; Mitchison 1977 for competing accounts).

Last, there have been claimed to be a wide array of examples of the golden section itself in mammalian anatomy. These claims are more controversial than those previously described because they involve the dubious notion of 'ideal' proportions, and because the actual research is quite old and its details are difficult to establish. Such claims should not be viewed without a degree of skepticism in the absence of full and replicated scientific reports, but neither is there reason to dismiss them out of hand.

1.3 *The history of the golden section*

The Egyptians are often credited with all manner of arcane mathematical knowledge (see eg Ghyka 1946/1977, pages 60–68). Regarding the golden section specifically, it seems clear that they had an estimate of ϕ accurate within 0.5%, and that some religious buildings might have had ϕ explicitly incorporated into their designs. None of this seems to have been achieved with the assistance of abstract mathematics, however. Egyptian mathematicians seem to have been akin to Lévi-Strauss's (1962/1966, pages 16–36) *bricoleurs*—concrete scientists who develop the applied knowledge they require without raising it to the level of a full theoretical discipline.

Some of this knowledge was transmitted from Egypt to Greece, and there can be no doubt that the Classical Greeks possessed knowledge of, and a fascination with, the golden section (contrary to Gardiner 1994). Euclid discussed it extensively, calling it the "division into mean and extreme ratio" (Berlyne 1971, page 222). Proclus (ca 410–485) reported that the Greek geometers of the Platonic schools called it simply *hê tomê*, 'the section' (Ghyka 1946/1977, page 4). The aesthetic valuation of the golden section was transmitted to the Romans as well, and its beauty extolled by Vitruvius himself (Watts and Watts 1986).

The golden section also had a great influence on medieval and Renaissance architects and artists at least partly because of its ease of construction (Coldstream 1991, page 33). Arches and apses of 'golden' proportions can be inscribed by people who have minimal knowledge of engineering or geometry. A tool as simple as a rope, fixed at one end, can be used to inscribe the arc to be built upon (for a hint as to how, see figure 2b). By the 16th century, interest in the golden section was so widespread that Luca Pacioli di Borgo published a treatise on it entitled *De divina proportione* in 1509, illustrated by Leonardo da Vinci. By the end of the century, a young mathematics instructor named Johannes Kepler would write:

> "Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel" (cited in Huntley 1970, page 23).

The neo-Platonism of the age supported, and was supported by, the interest in the myriad and fascinating properties and uses of the golden section. With the rise of empiricism, however, in the 17th century, interest in such matters came to be actively discouraged. It was not until the 19th century that significant interest in the golden section (and formal logic, for that matter) was revived. The most significant figure in the 19th-century revival of interest in ϕ was, as mentioned above, Adolf Zeising. It was primarily his work that stimulated G T Fechner to select the golden rectangle as one of the first stimulus objects ever to be used in a scientific psychological study.

2 Early experimental work on the golden section

In the 19th century, interest in the golden section began to turn experimental. Fechner began the movement with his attempt to develop an aesthetics 'from below'; to discover empirically what people actually find aesthetically pleasing, and build a theory to account for these data. Some of his earliest research in this regard was focused on the golden section. He was followed in this quest by many of the German 'Fathers' of psychology. By the last decade of the century, the American laboratories were fully in operation and their occupants had entered into the fray as well. Although Fechner considered aesthetic questions to be primarily psychophysical, by the end of the century the focus of the debate shifted to the primarily structuralist question of what combination of 'psychical elements' combine to give rise to the complex 'aesthetic feeling'.

2.1 *19th-century research and theory*

2.1.1 *The Germans.* Fechner was the first experimentalist to study systematically the aesthetic properties of the golden section. His first major treatise on the topic was *Zur experimentalen Aesthetik* (1871). Here Fechner first described his three famous methods of investigation: (1) the method of choice (*Wahl*), in which subjects choose, from among a number of alternatives, the item that they like (or dislike) the most; (2) the method of production (*Herstellung*), in which subjects are asked to draw, or otherwise create, an object of a certain kind that has features or proportions they find most agreeable (or disagreeable); and (3) the method of use (*Verwendung*), in which the experimenter examines preexisting objects of the kind being studied, and determines whether they conform to certain hypotheses about the determination of aesthetic pleasure.

Fechner (1876) reported the results of experiments conducted with his methods. For some reason, many myths and half-truths have been promulgated concerning Fechner's exact procedure, and many criticisms have been advanced on the basis of these erroneous beliefs. Some of the most common and influential errors are noted in the description of Fechner's method below.

Employing his method of choice, Fechner presented subjects with a set of ten rectangles—white, on a black table [pages 193-194; both Eysenck and Tunstall (1968) and Berlyne (1970) have it reversed]—ranging in proportion from 1:1 to 2.5:1. All the rectangles had equal areas. This was done to control for the possibility that the sheer size of the figures affects people's preferences.

Quite a difference of opinion has arisen on the question of whether Fechner presented his rectangles in vertical or horizontal orientation. Lalo (1908, page 43) and Zusne (1970, page 400) both suggest that the rectangles were presented in the horizontal orientation. By contrast, Farnsworth (1932, page 479), Eysenck and Tunstall (1968, page 3), and Berlyne (1971, page 226) believed the vertical position to have been the standard one. In fact, Fechner (1876, page 194) reports that he placed the rectangles before his subjects "*kreuz und quer*" (willy-nilly). It is unclear whether they were willy-nilly with respect to orientation, or just with respect to their relative proportions (ie not shown in order of relative size).

Of the ten rectangles Fechner used, the golden rectangle ranked seventh, in order of long-to-short ratio. That is, there were six figures with lower such ratios, and three with larger such ratios. This is contrary to the oft-repeated claim (eg Godkewitsch 1974) that the golden rectangle was at the center of his distribution. Also, the rectangles were shuffled and placed in a randomly distributed array anew for each subject. In short, Fechner employed a fair approximation of a contemporary randomized procedure (by spatial arrangement, and possibly by orientation) in order to ensure that the effects of extraneous variables canceled each other out. What is more, he did so long before psychologists (inasmuch as there then were psychologists) considered such matters to be important.

Fechner first had his subjects choose among the ten rectangles the one they found most pleasing. If they were unable to choose a single one they preferred most, Fechner allowed them to select more than one, dividing the score among the various preferred figures (ie $\frac{1}{2}$ each if two were chosen, $\frac{1}{3}$ if three were chosen, etc). Second, he had many of his subjects choose the rectangle they found least pleasing. This is contrary to the frequent claim (eg Godkewitsch 1974) that Fechner had his subjects rank order the rectangles, from most-preferred to least-preferred. Fechner's subjects gave a total of 347 responses (it is not clear from Fechner's report exactly how many subjects were used). Of those, 35.0% expressed a preference for the golden section. An additional 20.6% expressed a preference for the 1.5:1 rectangle (the next least elongated), and 20.0% were for the 1.77:1 rectangle (the next most elongated). Preferences dropped off quickly on both tails of the distribution. By contrast, none of the subjects selected the golden rectangle as the least pleasant and only 1.4% selected rectangles immediately adjacent to it. The least favored rectangles were the 2.5:1 with 40.9% of selections, and the square with 31.9% of selections. Fechner took these results to confirm the hypothesis that the golden rectangle is the most preferred. Just as the details of the procedure are often misreported, so are those of the results. Mahon and Battin-Mahon (1984), for instance, report that, "over 75% of [Fechner's subjects] chose rectangles of one particular dimension [sic]" (page 549).

Fechner also studied preferences for ellipses, though the results were not published until 1894, when Witmer presented them. Of nine ellipses with major-to-minor axis ratios ranging from 1:1 to 2.5:1, Fechner found that 42% of his subjects preferred the 1.5:1 ellipse, while 16.7% preferred the adjacent 'golden' ellipse. All other ellipses were preferred by even smaller percentages of the subject sample.

Recall that the method of choice was only one of three approaches Fechner took to experimental aesthetics. Employing the method of use, Fechner collected data from some 20000 paintings in twenty-two museums and art galleries, in order to see if great works of art tended to be framed in golden proportions. Contrary to his prediction, however, the golden rectangle did not characterize the height-to-width ratio of the paintings. Vertical paintings, on average, displayed a 5:4 ratio, horizontal paintings a 3:4 ratio. It is not clear what bearing this finding really has on the claim that the golden proportion is particularly pleasing, however. As Arnheim (1983, page 60) has argued, there are many "visual forces" at work in paintings that may cause the aesthetically ideal picture frame to deviate from the golden rectangle and, thus, the experiment was ill-conceived from the outset.

Having uncovered some evidence that the golden rectangle is the most preferred, Fechner was left with the task of explaining *why* people prefer it. On this question, he only said, "if you ask me, I simply say, I do not know" (cited in Haines and Davies 1904, page 249). It seems that no plausible explanation even occurred to him.

Because Fechner's results lie at the heart of empirical claims in favor of a special status for the golden section, many detractors have attempted to cast doubt on Fechner's results, often by casting aspersions on his character or competence. Among the most critical is McManus (1980), who has written:

> "Whilst it is not possible to accuse Fechner of direct, nefarious alteration of his experimental results so that his data fit with his prior theories, we may *speculate* as to how much Fechner, consciously or subconsciously, produced experimental circumstances which would tend to give him his desired results" (page 506, italics added).

Of course, speculation, pure and simple, is just what this is, and McManus proclaims it with no more evidence than the well-known facts that Fechner spent part of his career investigating the possibilities of life after death, mental life in plants, and the nature of angels. This is a plain ad hominem slur, and in the absence of any evidence such insinuations are best ignored.

Moreover, none of the legitimate criticisms that McManus provides of Fechner's method were of concern to any psychologist of Fechner's time. These were (1) that the averaging of rankings is a procedure that may bias in favor of the middle of the stimulus range, (2) that his subjects were not randomly selected, and (3) that the subjects may have been cognizant of Fechner's hypothesis. With respect to point (3) the significant problems that 'demand characteristics' pose for the psychological experiment, though occasionally alluded to early in the century (eg Pfungst 1905/1911; A H Pierce 1908), did not become a major focus of concern until the 1960s (eg McGuigan 1963; Orne 1962; Rosenthal 1964). Concerning point (2), the criticism that, "Fechner's subjects were not selected at random" (McManus 1980, page 506) casts more doubt on McManus's own appreciation of common research practice than on Fechner's methods. Random selection of subjects has *never* been considered an imperative of actual psychological research, though it may be desirable in principle. Even today, virtually no psychologist randomly selects human subjects [see Rosenthal (1965) on the differences between volunteer subjects and randomly chosen subjects]. Randomization of *treatments* is another matter, but Fechner only had one treatment: "pick your favorite rectangle". In any case, the randomization of treatments would not become a widespread research practice until half a century after Fechner's work (see, eg Fisher 1925). Last, regarding point (1), the finding that averaging certain kinds of rankings biases in favor of the middle of the range was not published until the 1970s (Godkewitsch 1974), more than a century after Fechner's work (although as early as 1903 Angier expressed concern that an average may correspond to no single subject's preference; a distinct, though somewhat related, point). More importantly, Fechner did not have the subjects rank the rectangles, only pick out their favorite (and sometimes their least favorite). The problems associated with the averaging of rankings do not bear against such a method.

Witmer[3] (1894) replicated Fechner's investigations, and extended them to triangles and other shapes as well. Unlike Fechner, Witmer presented his figures serially, one at a time, to his subjects. He found that the rectangle most preferred bore a 1:1.651 side-to-side ratio, a figure quite close to the golden section. He found that the most-preferred ellipse, however, bore a major-to-minor axis ratio somewhat smaller than the golden section, as had Fechner. The most-preferred isosceles triangle bore a height-to-base ratio of 0.41:1, considerably less than the golden section. Witmer only investigated triangles with heights smaller than their bases, all much shorter than the golden rectangle.

Like Fechner, Witmer had little to say concerning the causes of the preferences he had discovered, other than to say it was the "right middle between too small and too large differences in form" (1894, page 262). Although by the last decade of the 19th century there was a fair bit of data in support of the alleged preference for the golden rectangle, there was precious little in the way of psychological theory to explain it. Würzburg psychologists Oswald Külpe (1893/1895) suggested a psychophysical account. He argued that the golden section "is simply a special case of the constancy of the relative sensible discrimination, or of Weber's law" (pages 253-254). That is, because the golden section embodies equal *proportional* increases in length—from the short segment, to the long segment, to the combined whole segment—he surmised that "we have in the pleasingness of the golden section simply the pleasingness of *apparently equal differences*. It represents, so to speak, a *symmetry of a higher order*" (page 254, all italics added).

Wundt tried to provide a general theoretical account of aesthetic preference, from which preference for the golden rectangle in particular might be derived. He considered the experience of aesthetic pleasure to be a composite feeling, about which he wrote:

> "The *optical feeling of form* shows itself first of all in the preference of regular to irregular forms and then in the preference among different regular forms of those which have certain *simple* proportions in their various parts. The most important of these proportions are those of symmetry, or 1:1, and of the golden section, or $x+1:x = x:1$.... The fact that symmetry is generally preferred for the horizontal dimensions of figures and the golden section for the vertical, is *probably due to associations, especially with organic forms, such as that of the human body.* This preference for regularity and certain simple propositions can have no other interpretation than that the *measurement of every single dimension is connected with a sensation of movement* and an accompanying sense-feeling which enters as a partial feeling into the total optical feeling of form. The total feeling of regular arrangement that arises at the sight of the whole form, is thus modified by the relation of the different sensations as well as of the partial feeling to one another. As secondary components, which also fuse with the total feeling, we may have here too *associations and their concomitant feelings*" (Wundt 1897/1969, pages 166-167, all italics added).

In short, Wundt's position was that aesthetic preference is a function of associations with familiar objects and movements. He evaded the trap of radical relativism, however, by claiming that the crucial associations involve the proportions of the body itself, shared by all humans.

2.1.2 *The Americans.* The same year that Witmer's articles appeared, the experiments of Edgar Pierce (1894) were published. Though these were the first English-language reports of experiments on the golden section, Pierce's work has been routinely misconstrued (see eg Berlyne 1971; Godkewitsch 1974; Woodworth 1938), or just ignored (see eg McManus 1980; Valentine 1962; Zusne 1970), by reviewers of

[3] Witmer was actually an American, studying with Wundt. Because his work was conducted in Germany, and published in German, however, I have included him among the Germans.

golden-section research. This may be because the main object of his research was the aesthetic feeling associated with *equality*.

E Pierce (1894) showed his subjects three parallel vertical lines 10 cm long by 5 cm wide. Two of the lines were fixed at 60 cm apart. The third could be moved by the subjects between the other two. Six subjects were asked to choose the "most agreeable" position for the middle line. Pierce reported that "every one ... chose as most agreeable a position for the third line roughly corresponding to the golden section" (page 485). Unfortunately, no raw data or quantitative analysis were provided.

Pierce found that as he increased the number of fixed vertical lines, the subjects tended increasingly to place the moveable line at a position equidistant from the two fixed lines nearest to it. For instance, subjects would be shown fixed lines at 0 cm, 20 cm, and 60 cm, and asked to place a fourth between the latter two. They tended to place it at 40 cm. Pierce interpreted this result as reflecting a need for balance between variety and simplicity; a time-honored aesthetic theory dating back to the Ancients. A display of three lines is so simple, it was argued, that it needs the variety of an unequal division; a display of four, five, or six lines, by contrast, has so much variety that it requires the simplicity of equal division.

A simpler, more realistic, interpretation is that the equal spacing of three or more fixed lines created a demand characteristic that the apparent pattern—equal spacing of lines—be completed. Pierce's subjects agreeably obliged. If the line had been placed in successive, embedded golden-section relations, subjects would have likely conformed to that pattern as well. In any case, his initial discovery of a preference for a golden section division of the space between two lines stands. Nevertheless, Pierce concluded that equality, rather than ϕ, is the aesthetically more important division.

The battle between advocates of the primacy of equality, and the advocates of the primacy of ϕ—a debate that I shall later argue is moot—continues to the present day. Others have attempted to synthesize these apparently contrary preferences into a single theory. One of the first among these was Titchener, who wrote:

> "The most pleasing division of a simple visual form was, originally, the symmetrical division. Symmetry is repetition with reversal: the two hands, two eyes, two halves of a circle, etc., are symmetrical. The proportion of parts, in a symmetrical figure, is accordingly that of equality, 1:1. At a higher level of aesthetic development, the symmetrical division is replaced by what is known as the golden section" (1899, page 324).

Thus, the preference for the golden section is, according to Titchener, a product of some maturational process. Whether Titchener thought it to be epigenetic, or environmentally mediated, or both is not clear from his presentation.

The statement of his development hypothesis is followed by a description of an experiment the student is asked to conduct:

> "Prepare a long series of simple geometrical figures—crosses, ovals, rectangles, etc.—varying the proportions little by little throughout the series. Lay them before the observer, and let him pick out the most pleasing. The first few chosen will be figures whose proportions are in the near neighborhood of the golden section; the last will, in all probability, be symmetrical" (Titchener 1899, page 324).

This seems contrary to his hypothesized developmental progression from equality to ϕ. He does not explain why symmetry, a proportion that was once the most pleasing, should sink, in adulthood, all the way to the level of the most displeasing. Moreover, more-recent research described below has shown this claim to be false.

Another popular theoretical suggestion was that eye movements are responsible for some aspects of visual aesthetic preference, and for some aspects of perception in general. This explanatory paradigm was first used by experimental psychologists to explain the horizontal-vertical illusion. Wundt for instance, argued that this illusion is the result of the different kinds of eye movements needed to scan a vertical, as opposed to a horizontal, extension. Helmholtz gave a similar explanation of the apparent incongruence of horizontally and vertically striped squares (both cited in Künnapas 1955).

Following this line of research, E Pierce (1896) investigated the "function of the eye-movements in relation to the aesthetic consciousness" (page 271). As before, he had subjects arrange parts of various visual displays of lines and shapes to their liking. He also rotated each display 90°, and had the subjects arrange the same parts to their liking again. As expected, the responses differed according to which orientation the display was in. Pierce also had his subjects do these tasks while lying horizontally. He reasoned that the eye movements necessary to arrange the lines while sitting would be identical to those needed to arrange the same display, rotated 90°, while lying down. Thus, if the subjects were consistent in their preferences, they should arrange the display while sitting in the same way as they do the 90°-rotated display while lying down. He found this to be the case, and concluded that eye movements, in conjunction with associations to features of the experimental situation (such as having to lie down), "determined a general way of apperceiving the object" (page 280).

This line of research (viz in which the role of eye movements was emphasized), however, was soon to come under attack. Zusne, for instance, reports that experiments by Stratton (1902, 1906) "showed that symmetry has no particular effect on eye movements either, so that the assumption about a pleasurable effect produced by an even, bilateral distribution of eye movements about an axis of symmetry could not be supported" (Zusne 1970, pages 397-398). These findings have been confirmed and extended in more-recent research (see Norton and Stark 1971 for a review).

2.2 *Early-20th-century research and theory*

As structuralism gradually gave way to behaviorism generally, so did it give way with respect to the golden section in particular. Behaviorism, however, had fairly little to say about the golden section, other than preference for it being a matter of previous environmental reinforcement—a claim for which there was little evidence—or that the phenomenon is altogether artifactual. The 'other' response to 19th-century psychology, the Gestalt movement, however, was not so metaphysically hamstrung as behaviorism, and was able to put forth a more sophisticated analysis of the matter. With the 20th century also came the start of a trend that has come to dominate scientific discussions of the golden section: criticism on the basis of method. It may well be that there has been, in fact, more discussion of the procedures and analyses of golden-section research than there has been of the golden-section phenomenon itself.

Angier (1903) was among the first to criticize the procedure of averaging across subjects' rankings. As he explained, "such a total average may fall wholly without the range of judgments of every subject concerned, and tell us nothing about the specific judgments of any one" (page 542). By contrast, Angier asked his nine subjects to divide a horizontal line "at the most pleasing place" 72 times each. Although the mean proportion was very near the golden section (0.600), only two of the nine subjects chose the golden section with great regularity. Angier argued that this confirmed his belief that the golden section is a mathematical abstraction, not a universal aesthetic ideal.

This critique has since become a favorite with golden-section detractors, but it is not clear that the argument is valid. The standard argument for modern sampling procedures is that each subject's score is contaminated with 'error'—variation due to 'extraneous' factors. Because such 'error' is assumed to be randomly distributed, the effect of sampling is to set 'errors' against each other so that the resulting mean score is a measure of the 'pure' effect of the independent variable. One can, of course, reject this position, but it operates as a basic assumption in much of experimental psychology. To argue against it in the case of the golden section alone is to be tendentious, at best. Thus, conclusive evidence that people pick the golden section, even if only on average, is as legitimate a psychological finding as, say, that people consistently pick a certain colour as 'pure' red, *on average*, even if not everyone makes precisely the same choice (given certain procedural precautions against artifact discussed below). In any case, the discovery of a distribution in which the *modal* preference coincides with the mean evades the thrust of the critique. Angier's findings notwithstanding, most researchers have found just such a distribution.

As with Pierce, it seems that Angier's aesthetic agenda was behind his rapid dismissal of the golden section. In two-thirds of his paper he argues for special aesthetic status for symmetry, and a physiological theory supporting a preference for symmetry that would be difficult to adapt to the golden section. According to Angier, symmetry is preferred because it gives rise to "a corresponding equivalence of bilaterally disposed organic energies, brought into equilibrium because acting in opposite directions", which, in turn, produces "a feeling of balance, which is, in symmetry, our aesthetic satisfaction" (Angier 1903, page 551). Angier's physiological approach seems to derive from William James's then-popular theory of emotion. Like James's theory, it holds the dubious implication that people would lose much of their aesthetic sensibility if they lost bodily sensation.

Following Angier, Haines and Davies (1904) criticized Fechner's and Witmer's methodological practices. They claimed that Fechner had presented all his rectangles in a single orientation, and, thus, that relative preferences for rectangles in other orientations were not known. Even if Fechner did effectively randomize orientation, however, he did not manipulate it as an independent factor in aesthetics. Haines and Davies also criticized Fechner for having presented all his rectangles together in the same display, so that each was not in the same position relative to the subject's line of vision. Although serial presentation has problems of its own, Witmer, by contrast, had presented his figures individually and serially, thereby evading Haines and Davies' criticism of Fechner. Still, they claimed that "distractions were still incident in [Witmer's] method" (page 254). The precise nature of these distractions was not specified, however. To address these problems, Haines and Davies presented a great number of rectangles to their subjects individually, asking them either to "accept" or to "reject" each by picking it up or by pushing it away. Unfortunately, they did not report the exact question put to their subjects [though it might have been "favourable or unfavourable?" (see page 255)], nor did they specify the order in which the rectangles were presented.

Four sets of rectangles were used. Each set was based on a 'standard' side of 80 mm, 90 mm, 100 mm, and 120 mm, respectively. The other side of each varied from 25 mm in length to 5 mm less than the length of the 'standard' side. The intervals between rectangle sizes were 2.5 mm for the 80 mm and 90 mm rectangles (eg 80×25, 80×27.5, 80×30 ...), and 5 mm for the 100 mm and 120 mm figures (eg 100×25, 100×30, 100×35 ...). In all there were twenty-one 80 mm rectangles, twenty-four 90 mm rectangles, fifteen 100 mm rectangles, and nineteen 120 mm rectangles. One group of eleven subjects was shown all of the 80 mm, 100 mm, and 120 mm figures. A second group of twelve was shown only the 80 mm and 90 mm rectangles. No justification for this procedure was given.

Presumably because of the growing opposition to the use of means in such experiments, Haines and Davies's results were presented entirely in the form of raw data.

By doing a proportional analysis of their data, one can find that, of the 605 trials (fifty-five rectangles × eleven subjects) conducted on the first group, 130 (21.5%) resulted in 'acceptance' of the given rectangle. Of the 130 'accepted' rectangles, sixteen (12.3%) bore side-to-side proportions within the range 0.58 – 0.66, a range closely approximating ϕ'. Of the fifty-five rectangles presented, seven (12.7%) bore proportions within this range as well. Thus, no effect for 'goldenness' was found. Of the 552 trials (forty-six rectangles × twelve subjects) conducted on the second group, seventy (12.7%) were 'accepted'. Of those seventy, twelve (17.1%) bore side-to-side proportions within the range 0.58 – 0.66. Of the forty-six rectangles presented, however, only six (13.0%) were within this range. With the high number of trials, this difference (17.1% – 13.0%) is, no doubt, statistically significant, showing an effect for 'goldenness'. It is, however, a quite small effect given the claims that have often been made for the golden rectangle.

Haines and Davies also had subjects produce their own preferred rectangles. Hundreds of trials were conducted over a period of months. No clear overall preference came to light, although the data of some individual subjects were clustered tightly in a variety of ranges. They also analysed the introspections of their subjects on the reasons for their preferences. The results of this experiment were relatively inconclusive as well.

The research of the French aesthetician Lalo (1908) is among the most cited in the golden-section literature. Lalo's procedure was essentially a replication of Fechner's, asking subjects which, of a simultaneously presented set of ten horizontally oriented rectangles they liked most and least. Although he was familiar with the American research of the day, Lalo criticized the serial-presentation procedure for being too great a tax on the memory (page 44). Lalo found that 30.3% of his subjects preferred the golden rectangle, whereas Fechner had found 35.0%. This was the modal response in both experiments. Lalo also found that an additional 18.3% preferred the two rectangles adjacent to the golden, whereas Fechner had found 40.6% preferred these. Thus, Fechner's main findings were essentially replicated, though at somewhat lower levels of agreement. An interesting twist, however, was that substantially more of Lalo's subjects preferred the rectangles at the extremes of the range than did Fechner's; 11.7% chose the square (3.0% for Fechner) and 15.3% chose the 2.5:1 rectangle (1.5% for Fechner). Despite the preferences of these anomalous groups, 49.1% of Lalo's subjects selected one of these two as the worst rectangle (63.5% for Fechner). None of the subjects chose the golden rectangle as the worst.

The next major study was by famed learning theorist E L Thorndike (1917), who presented sets of twelve rectangles of varying proportions, twelve triangles, twelve crosses, and twelve line patterns to groups of subjects ranging from one hundred to two hundred and fifty in number. I will discuss only the rectangle and triangle results here. The subjects were asked, about each type of figure, the rather colloquial question, "which do you like the looks of most?". After choosing their favorite, they were asked to choose their second favorite, and so on, until they reached their least-favorite figure.

The rectangles were all the same height (so that total area was not controlled), all shown in the vertical orientation, and shown all at once in order of proportion from narrowest to widest. Their height-to-width ratios ranged from 1.3:1 to 3.75:1. None of the rectangles garnered overwhelming support, but the golden rectangle and the next two most-elongated rectangles (1.8:1, 2.0:1) were ranked first, second, or third (of twelve rectangles) by between 35% and 45% of the subjects. Conversely, they were each ranked among the three least favorite by only 7% of the subjects. The 1.8:1 rectangle had the best mean ranking.

The triangles used by Thorndike all had bases of the same size (thus, total area was not controlled). They were all shown in the vertical orientation, and were shown all at once in order of proportion from shortest to tallest. Their height-to-base-width ratios ranged from 1.1:1 to 3.3:1. The 1.4:1 and 1.5:1 triangles were ranked among the three most-preferred figures by 60.0% and 68.4% of the subjects, respectively. The 1.6:1 triangle had the best mean ranking, however. These results do not strongly favor any hypothesis about which proportions are most preferred. The procedure was so casual, however, that it cannot be relied on in any case.

Some fourteen years later, another psychologist, C O Weber (1931), looked at the question of the golden section. He used the method of paired comparisons to find the preferred rectangle of sixty-eight subjects. The subjects were tested a second time after a two-week interval. The nine rectangles used were the square, $\sqrt{2}:1$, $\phi:1$, $\sqrt{3}:1$, $\sqrt{4}:1$, $\sqrt{5}:1$, and four rectangles interpolated between these (there was no rectangle between the $\phi:1$ and the $\sqrt{3}:1$). They were arranged in order of proportion, but their overall areas were kept constant (each 20 cm^2). In all, 44% of the comparisons resulted in selection of the $\phi:1$, $\sqrt{3}:1$, or 1.871:1 rectangle in both experimental sessions. Although preferred only 15% of the time, the 1.871:1 was the most-highly preferred rectangle in both sessions. Analyses of individual responses, however, showed that the preferences of far more subjects shifted from broader to narrower rectangles over the two-week lag than in the other direction. In sum, this study showed general but ambivalent support for the idea that rectangles in the region around the golden rectangle are more preferred by more people. The specific support craved by strong advocates of the golden section was lacking, however.

The very next year, Farnsworth (1932) reported that for five years he had been conducting rectangle preference tests in his laboratory classes, and that "the golden section had always received first or second place in the preference scale" (page 479). These findings led him to conduct a formal experiment on rectangle preference by means of two quite different procedures. In the first, he presented twenty-two subjects with all possible pairs of nine neutral-grey cardboard rectangles with side-to-side ratios ranging from 1:1 to 2.5:1, in both the horizontal and the vertical orientation. The golden rectangle in the vertical position was the most preferred. The 1.5:1 in the horizontal position was second.

In Farnsworth's second procedure, he tachistiscopically presented sixty-six subjects with black rectangles on a white background. Again, the method of paired comparisons was used, but the stimuli were quite a bit smaller than those used in the first procedure (14.44 cm^2 vs 1162.88 cm^2). No strong preferences were found at all, and Farnsworth concluded that "the preference status of the golden section is a function of the size and color of the rectangle" (page 481). The rectangles in the second procedure were so small, however, as to likely be indiscernible (or, at least, aesthetically indistinguishable) to the subjects, resulting in an essentially flat distribution.

Davis (1933) is one of the few researchers to have had subjects simply draw their preferred rectangle. He had three hundred and ten of them do this, after they had first paused to "visualize what seem[ed] to them, at that moment, the rectangle of the most pleasing proportions" (page 298). He repeated the procedure after 40 min. The resulting distribution of rectangles had three clear modes, at ratios of 1.72:1, 2.02:1, and 2.22:1. These are almost precisely the ratios of the $\sqrt{3}:1$, $\sqrt{4}:1$, and $\sqrt{5}:1$ rectangles. Davis reported that the golden rectangle was drawn by only 3% of the subjects, but its proximity to the $\sqrt{3}:1$ rectangle made it difficult to disentangle the two.

After a lull of over a decade, the late 1940s saw a flurry of new interest in the possible effects of age upon the appeal of the golden rectangle. Thompson (1946) presented four hundred subjects—one hundred each of preschoolers, 3rd-graders, 6th-graders, and college students—with twelve rectangles bearing ratios ranging from

0.25 to 0.75, all presented in the horizontal orientation. The rectangles were black against a white background, and were all of the same height (ie total area was not controlled). The subjects were asked, "which one do you like best?" After they had selected one, and it was removed from the display, they were asked, "Now which one do you like best?" (page 51).

The preschoolers exhibited no particular preference. The 3rd-graders and 6th-graders, Thompson reported, preferred the wider rectangles. The college students, by contrast, preferred the rectangles with ratios of 0.55, 0.60, and 0.65 above the others, these figures garnering median rankings of approximately 2.8, 3.5, and 3.6, respectively. Interestingly, since wide rectangles more closely approximate squares than others, the results supported Titchener's belief that equality is the early aesthetic tendency, and only later does the preference for the golden section develop. Thompson's discussion of his results is revealing in this regard. Although he admitted that the college students exhibited a preference for rectangles near the golden, he puzzled over the fact that this "cultural transmission" had been effected "non-verbally". That is, since the majority of his subjects "had never even heard of the golden section" (page 57), they must have been inculcated with a cultural norm supporting it *without their knowledge*. The discussion suggests that it was literally inconceivable to Thompson, writing in the midst of the behaviorist heyday, that such a preference might be innate, but that it does not come to full fruition until adulthood.

Following on Thompson's study, Shipley et al (1947) worried that the apparent preference of 3rd-grade and 6th-grade subjects for 'wide' rectangles was confounded with the overall areas of the rectangles. Consequently, they showed a hundred child subjects and a hundred college-age subjects two sets of six rectangles. One set was like Thompson's (viz all of equal height but varying in width) except that there were only half as many, ranging in proportion from 0.25 to 0.75. The second set bore the same distribution of proportions, but were all of equal area. Like Thompson's, the rectangles used by Shipley et al were black and presented in groups on a white background, and they were all presented in the horizontal orientation. The subjects were asked to rank their preferences. The results showed that the adults preferred the 0.65 rectangle over the others, though the preference was not very strong (median ranking of approximately 4.7 for both the equal-height and equal-area series). The children generally preferred the wider rectangles, but the effect was much more pronounced in the equal-height series (median rank = 1.5) than in the equal-area series (median rank = 4.7).

Soon after, Nienstedt and Ross (1951) investigated the preferences of people aged 61–91 years as compared with those of younger adults. They presented four series of rectangles—one smaller and one larger series of twelve rectangles, half of equal heights and half of equal areas—to one hundred college-age subjects and fifty elderly adults. Again the rectangles were black, presented on a white background, bore ratios ranging from 0.25 to 0.75, and were all presented in the horizontal orientation. Again the subjects were asked to select the one they liked best, after which the one selected was removed, and they were asked which of those remaining they liked best, etc.

The college-age subjects preferred the 0.65 rectangle among both the smaller and the larger series, equal-height and equal-area. The median rankings of this rectangle were near 4.0 for all series. Substantial drops in preference were observed for the 0.75 rectangle in all series (median ranks ranged from about 6.0 to 7.5). The elderly subjects, by contrast, generally preferred wider rectangles, though not as strongly or consistently as had the child subjects in the studies of Thompson and of Shipley et al. For the larger series, median preference ranks were low (indicating high preference) and virtually flat across the rectangles with ratios of 0.55, 0.65, and 0.75. For the smaller equal-area series, preference ranks rose somewhat (indicating less preference) after a peak at 0.65. For the smaller equal-height series, the best ranking went to the 0.75 rectangle. Thus, again we find a series of studies in which there is general, if not overwhelming, support for the hypothesis that the golden rectangle is preferred by people on average. More recent investigations, however, have cast doubt on the data-analytic procedures used in these experiments (see next section).

Figures other than the rectangle were rarely investigated with respect to the golden section. Austin and Sleight's (1951) study is one exception. They criticized earlier research on triangle preference, by Witmer (1894) and by Thorndike (1917), for failing to control for what they called the "central tendency of judgment". That is, the earier researchers put the triangles with height-to-base-width ratios of ϕ' (in the case of Witmer) or ϕ (in the case of Thorndike) in the centers of their distributions, and this was thought artificially to bias subjects toward it. The criticism is essentially identical to the one Godkewitsch would run against rectangle research more than 20 years later. To minimize this hypothesized effect, Austin and Sleight showed fifty-two subjects all possible pairs of twelve triangles and asked them to select which of each pair they preferred. The same subjects were retested a week later. Although the triangles with height-to-base-width ratios of either ϕ or ϕ' were not among those used in the experiment, those most preferred ranged from 1:1 to 1.75:1. At first glance, this range seems too low to lend support to the hypothesis that ϕ has some sort of psychological salience, but recall that the 'golden triangle' is not so-called because it has a ϕ or ϕ' ratio between its height and the width of its base; the ratio of its leg length to its base is ϕ. The golden triangle has a height-to-base-width ratio of only 1.53:1, very near the peak of Austin and Sleight's range of preferred triangles.

There was quite a bit of data concerning the golden rectangle by the 1950s, but still precious little in the way of theory. A major exception to this trend was to be found in the gestaltists, particularly in the person of Rudolf Arnheim, who wrote:

> "When a square is divided into two halves, the whole pattern prevails over its parts because the 1:1 symmetry of the square is simpler than the shapes of the two 1:2 rectangles. Even so, we can manage at the same time to single out the two halves without much effort. If we now divide a 1:2 rectangle in the same manner, the figure breaks apart quite readily because the simplicity of the two squares imposes itself against the less compact shape of the whole. If, on the other hand, we wish to obtain a particularly coherent rectangle, we may apply our subdivision to the rectangle of the golden section .... Traditionally and psychologically, this proportion of 1:0.618 ... has been considered particularly satisfying because of its combination of unity and dynamic variety. Whole and parts are nicely adjusted in strength so that the whole prevails without being threatened by a split, but at the same time the parts retain some self-sufficiency" (1954/1974, pages 70-71).

Thus, according to Arnheim, it is the unreasonable tension between two competing Gestalt organizations that leads to our fascination with the golden rectangle. Interestingly, however, this tension would be maximized with the $\sqrt{2}:1$ rectangle, rather than with the golden. This is because when one divides a $\sqrt{2}:1$ rectangle in half, the result is two, smaller, $\sqrt{2}:1$ rectangles. Thus one receives no information as to whether such a subdivision results in a more 'stable' figure because the exact same problem is posed again. The same division of a golden rectangle results in two 1.23:1 rectangles, figures much closer to the optimally stable square. Thus, although Arnheim's theory was perhaps the most plausible up to that time, it implies that the $\sqrt{2}:1$, rather than the golden, is the rectangle most preferred.

3 Modern theories and findings

Several new theories of the preference for the golden section have been proposed in the last three decades. The dominant trend in late-20th-century research, however, has been to show that there is, in fact, no effect at all; that the golden section is, at least psychologically, nothing more than a bit of ancient 'metaphysical speculation' (in the pejorative sense) that refuses to go away. The reason interest in the golden section refuses to fade away is that the phenomenon itself refuses to go away. It seems to occur most reliably in the best-designed, best-controlled studies.

3.1 *Influential theories of the 1960s*

3.1.1 The perimetric hypothesis. Stone and Collins (1965) observed that, "with a little imagination, [the] binocular visual field can be seen to possess an outline form which is not too unlike that of a rectangle" (page 504). They found that a rectangle drawn around the outside of the visual field has a height-to-width ratio of approximately 0.768, and that a rectangle drawn fully inside the visual field has a height-to-width ratio of approximately 0.565. The average of these two rectangles is about 0.665, a value relatively close to ϕ'. They went on to suggest most cautiously that this fact

> "provides the substance for a hypothesis which attempts to explain why rectangles having proportions similar to that of the golden section are generally regarded as having the most pleasing appearance .... It would not be difficult to construct a theory based on either *imprinting* or adaptation-level" (page 505).

As far back as Woodworth (1938), it had been observed that there are differences between the rectangle preferences of American and European subjects, and it had been argued that these belie any thought of a universal preference for the golden rectangle. Stone and Collins, by contrast, suggested that if such preferences are a function of the shape of the individual's visual field they might be due to the different cheek and nose shapes of the two respective populations.

Schiffman (1966) dubbed Stone and Collins's idea the 'perimetric hypothesis', and attempted to gather evidence in its support. He reasoned that, if the shape of the binocular visual field were responsible for the preference for the golden rectangle, then people should, by the same token, generally prefer horizontally to vertically oriented rectangles. To test the hypothesis, thirty-six subjects were asked to draw "the most aesthetically pleasing rectangle". In support of his hypothesis, thirty-five (97%) drew a rectangle in the horizontal orientation. The mean height-to-width ratio, however, was 0.525, far from the golden rectangle. The median value was 0.500.

Schiffman (1969) later replicated this procedure with one hundred and fifteen male subjects. Nearly 90% of them drew horizontally oriented rectangles. Again, however, the mean height-to-width ratio fell far short of ϕ', attaining a value of 0.489. The median was again 0.500. In a second experiment, reported in the same paper, Schiffman showed twenty-five male subjects all possible pairs of six rectangles of various dimensions, both in the horizontal and in the vertical orientation. The rectangles had ratios of 0.318, 0.418, 0.518, 0.618 (ϕ'), 0.718, and 0.818. There was no significant preference for any one rectangle, though there was a trend in favor of those near the middle of the range. Contrary to his earlier findings, Schiffman also found a moderate preference (57%) for vertically oriented rectangles. In a third experiment, twenty male subjects were involved and the 0.818 rectangle was dropped from the procedure. Again, no preference was found, though there was a trend in favor of the narrower (ratios < 0.5) rectangles. There was also a very slight preference (51%) for vertical rectangles. Not only had the perimetric hypothesis been refuted, but it seemed as though the very phenomenon of preference for the golden rectangle had been lost utterly.

Hintz and Nelson (1970) continued the search for evidence for the perimetric hypothesis. They reasoned that if rectangle preference were a function of the shape of the visual field, then there should be a correlation between the individual's preferred rectangle and that person's own visual field dimensions. Beginning with fourteen rectangles that ranged in short-to-long side ratios from 0.10 to 1.00, Hintz and Nelson went through a painstaking stimulus-comparison procedure to determine the rectangle preference of each of twenty subjects. They then measured the dimensions of each subject's binocular field. The correlation coefficient between the preferred rectangles and the binocular field measurements was 0.279 ($p > 0.05$). The rectangles that the subjects preferred were closer to the golden (median preferred ratio = 0.558, modal preferred ratio = 0.600) than Schiffman had found. Their intensive selection procedure likely made their findings more reliable as well. Hintz and Nelson also had twenty subjects draw their preferred rectangles freehand. As is consistent with Schiffman's early findings, 80% of these were in the horizontal orientation (median ratio = 0.545, modal ratio = 0.570).

Following up further on the perimetric hypothesis, Plug (1976) found preferences "in the vicinity" of golden-section dimensions not only for rectangles, but also for diamond and pear shapes, both in horizontal and in vertical orientation. Because these shapes are very different from the shape of the binocular visual field, he concluded that "the visual field hypothesis cannot be accepted as it stands" (page 10). Because he used figures with a much wider range of ratios than others (1:1 – 8.5:1), what he counted as "in the vicinity" of the golden section was much wider than most would accept as good confirmatory evidence (see also Plug 1980).

Hintz and Nelson (1971) also tested whether the golden rectangle might be aesthetically preferred haptically, that is, if felt with the fingers rather than seen. If so, such a finding would seem inconsistent with the perimetric hypothesis (although they did not explicitly say so). Hintz and Nelson compared two groups of blind subjects (congenital and late) with blindfolded and nonblindfolded sighted subjects. Hintz and Nelson's haptic rectangles were inscribed on paper; fourteen figures with equal areas, having ratios ranging from 0.10 to 1.00. The subjects were first presented with rectangle ratios of 0.30, 0.60, and 0.80, and asked which they preferred most, next most, and least. Rectangles falling between the first and second preference were then presented repeatedly, three at a time, until the range had been narrowed to only five figures. These five were then presented in all possible combinations of three. By means of this extensive, painstaking procedure, the preference of each subject was determined. The modal preference for the congenitally blind was the rectangle with a ratio of 0.10, but their median preference was the 0.50 (the 2:1 rectangle). The late-blind subjects' modal preference, however, was 0.60, and their median preference was 0.615, both numbers being quite close to the golden rectangle. Blindfolded subjects had modal and median preferences of 0.600 and 0.575, respectively, while the nonblindfolded sighted subjects had modal and median preferences of 0.600 and 0.558, respectively. Unlike the data distributions of many earlier experiments on visual preference, the haptic data showed a quite pronounced peak near ϕ'. It was concluded that "data obtained from late blind and sighted subjects generally confirmed the haptic aesthetic value of the golden section. However, preference of congenitally blind subjects call into question the existence of the golden section as a haptically satisfying simple figure" (Hintz and Nelson 1971, page 221).

3.1.2 *Personality factors.* Another idea introduced to golden-section research in the latter half of the 20th century was that personality traits might be linked to people's aesthetic preferences. Eysenck and Tunstall (1968), with forty male subjects, replicated earlier findings that the vertically oriented rectangle with the greatest mean ranking is the golden. They also found that the preference for the golden rectangle actually increased upon asking subjects to rank the rectangles a second time. Interestingly, the preference for the golden rectangle was considerably stronger among introverts than among extroverts. The introverts' mean ranking for the golden rectangle was 5.7, the next highest being the 0.69 rectangle with a mean ranking of 6.7.

The extroverts actually preferred the 0.75 rectangle, ranking the golden rectangle second, on average (mean ranks 5.4, 6.3, respectively).

3.1.3 *Cultural factors*. Berlyne (1969) showed twenty Canadian subjects examples of Western art from the 18th, 19th, and 20th century, as well as classical Chinese, Indian, and Japanese artworks, and asked them to locate the major subdivision of each work along its longer dimension. Although the mean of the modal divisions was 0.390—very close to $1-\phi'$, 0.382—Berlyne concluded that there was no evidence for a "privileged status" (page 1) for the golden section. This study, however, was confounded by factors such as the relative external proportions of the picture being used (ie overall shape), as well as the proportions internal to each work (ie geometric arrangement of objects). Because Berlyne did not identify the paintings he used, it is impossible adequately to evaluate the validity of his results.

Berlyne was inclined toward the belief that the golden section was a cultural, rather than a deep psychological, matter. In a cross-cultural test of his belief (Berlyne 1970), he compared the preferences of thirty-three Canadian high-school girls and forty-four Japanese high-school girls. He found the Asians preferred square-like rectangles, on average. Analysis of the subjects' first choices, rather than mean rankings, told a somewhat different story. Both groups selected the square as their first choice (27% and 20%, respectively). The Canadians' preferences dropped off markedly for rectangles with near-equal proportions, but peaked again at the 1.5:1 rectangle, about 18% selecting it as most preferred. Japanese subjects displayed roughly even first-choice percentages across the near-square rectangles, but their preferences dropped off markedly after the 1.5:1 rectangle. The golden rectangle was selected first by only about 9% of Canadians, and by about 5% of Japanese.

This result—the square being preferred to the golden rectangle—is found fairly frequently, as we have seen, and is often cited as evidence against the golden rectangle. It is doubtful, however, that it would bother the 'Pythagorean' psychologist, who claims only that forms with simple (or otherwise 'interesting') ratios will be most preferred. The square is, surely, to be included among such figures, as are the $\sqrt{2}:1$, $\sqrt{3}:1$, and $\sqrt{4}:1$ (or 2:1) rectangles.

3.1.4 *Information theory*. Berlyne's (1971) review of golden-section research up through the 1960s is probably the most complete in the psychological literature (though even it skims much of the material). He evaluated several then-current theories. The perimetric theory he rejected, on the basis of Hintz and Nelson's (1970) observation that individuals' rectangular preferences are not correlated with the shapes of their actual visual fields. He dismissed, as well, the 'harmony' theories of Zeising (1884), and a number of other mathematically inclined aesthetic theorists, on the ground that it is preposterous to believe that we "compare the longer line to the sum of the two [segments]" (page 228), though he admitted that experimental evidence on this point is lacking. Piaget (1961) had once suggested that people 'transport', in their minds, representations of parts of visual displays for the purpose of comparison. Such a comparison might make more apparent the relations holding between the parts and lead to aesthetic preference. Berlyne did not reject this possibility outright, saying that it "leans upon assumptions that remain to be experimentally verified" (page 229). He also rejected Arnheim's theory, noting that if it were strictly correct, the $\sqrt{2}:1$ rectangle would be the most preferred. Last, on the basis of his own research, and the observation that discussion of the golden section with reference to art has historically occurred only in "Mediterranean civilizations and their offshoots" (page 229), he tentatively endorsed his traditional position that there is a cultural basis to whatever preference for the golden rectangle there might actually be.

Having taken a cursory look at a number of possible explanations, Berlyne quickly moved on in his account to the work of the German psychologist H Frank (1959, 1964), who had suggested that the *Auffälligkeit* (commonly translated as the 'strikingness') of an artwork is what makes it aesthetically valued. The way, according to Frank, to make a work 'striking' is to maximize the information (in the technical sense developed by Shannon and Weaver 1949) it contains. The information function is given by the expression $-p\log_2 p$, and it is maximized at $1/e$, which is approximately equal to 0.368, a value close to $1-\phi'$ (0.382). Although Berlyne did not fully endorse this explanation of the apparent preference for the golden rectangle—how would it apply to preference for rectangles?—he was clearly intrigued by Frank's theory and results.

3.2 *The fall and rise of the golden rectangle: methodology in the 1970s and 1980s*

The early 1970s probably marked the low point of belief in any special psychological significance for the golden section. A student of Berlyne's, Michael Godkewitsch, argued that there is no need to explain the preference for the golden rectangle because it is nothing but "an artifact of stimulus range and measure of preference" (1974, page 269). Godkewitsch suggested the reason the golden rectangle traditionally received the best mean ranking was that it had often been placed at the center of the range of rectangles to be judged. He then reasoned that even if the first choices of rectangles are randomly distributed among subjects, but that their subsequent choices "are systematically and progressively further removed in size from their most preferred rectangle, then the stimulus with the lowest mean ... will lie in the middle of the range" (pages 271-272).

To test his hypothesis, Godkewitsch arranged twenty-seven rectangles, of fifteen distinct proportions, into three ranges of nine rectangles each—short, middle, and long. The golden rectangle was in each set. It was the second-longest of the short range, the middle of the middle range, and the second-shortest of the long range. As predicted, for each range, the middle rectangle achieved the best mean ranking. For both the short and the long range, however, the middle rectangle was actually the first choice *least* often. Both distributions were U-shaped. This was not so for the middle range, however, in which the golden rectangle was the middle rectangle. Though generally U-shaped, there was a substantial 'bump' at the bottom of the U, where the 1.41:1 ($\surd 2$), 1.51:1, and 1.62:1 (golden) rectangles were. Still, the results generally supported Godkewitsch's contention that the average-ranking procedure artifactually elevates the status of the middle figure.

Godkewitsch shied away from strong claims about the implications of his results for the psychology-of-the-golden-section enterprise as a whole, but he did say that "aesthetic theory has hardly any rationale left to regard the golden section as a decisive factor in formal visual beauty" (page 276). Contrary to common belief, however, Fechner neither put the golden rectangle at the center of his distribution, nor did he have his subjects rank their preferences. Thus, contrary to widespread opinion, Godkewitsch's results hold little significance for Fechner's original findings.

Piehl (1976) conducted an extended and slightly modified replication of the experiment of Godkewitsch. Unlike Godkewitsch's procedure, in which each subject took part in only one of the three ranges of rectangle size, each of Piehl's subjects took part in all three ranges, in random order. The subjects were also assigned to one of three conditions. One group was just involved in the standard rectangle-choosing procedure for each of the ranges. The two other groups took part in a mock psychophysics experiment involving rectangles before completing the rectangle-choosing procedure. One of these two groups saw the golden rectangle ten times more often than any other rectangle. The other saw the rectangles of varying proportions equally often. Piehl analysed the first choices of the subjects only. The subjects who did not experience the preliminary psychophysics procedure tended to choose rectangles at the extremes of each, like Godkewitsch's. The subjects who had taken part in the preliminary procedure with flat distribution of rectangles were said to have shown "similar" results. Subjects who saw more golden rectangles in the preliminary procedure, however, showed a great preference for the golden rectangle regardless of the range. Piehl argued the result "can only be accounted for by the demand characteristics of the experiment" (page 49). It is hard to see this as a case of classical demand characteristics. Instead, it seems that Piehl made a psychological discovery in line with Zajonc's (1968) famed "mere exposure effect". Subjects more familiar with the golden rectangle came to prefer it.

Benjafield (1976) attempted another replication of Godkewitsch's procedure, though he modified two variables he thought to be of crucial importance. First, he controlled for the areas of the rectangles by running two series of figures, one in which the proportion was changed simply by extending the length of the longer side (à la Godkewitsch and Piehl) and another in which both dimensions were varied in order to keep the total areas of the rectangles the same. Recall that this had already been established as an important factor by Shipley et al (1947). Second, instead of having subjects simply pick the rectangle they liked best from the set 'cold', so to speak, he adapted Kelly's (1955) 'repertory-grid' technique to his needs. He had his subjects (1) separate the ones they "liked" from those they "disliked"; then (2) separate, from those that they "liked", the ones they "liked most"; then (3) finally pick, from among those they "liked most", the single rectangle they "liked best". The "liked-best" rectangle was given a score of 3, those "liked most" were given scores of 2, those "liked" given 1, and those "disliked" given 0. For the increasing-size series of rectangles, people generally liked larger rectangles better. With the equal-area series of rectangles, however, people preferred the golden rectangle, regardless of the range in which it was presented. Thus the plausibility of the idea that the golden rectangle has some special aesthetic significance was revived.

Piehl (1978) used the method of paired comparisons to check Benjafield's findings. His subjects were shown all possible pairs of seven equal-area rectangles, ranging from the square to the 2.31:1 rectangle, and asked to select the preferred member of each pair. Under these conditions, the subjects showed a marked preference for the golden rectangle. Piehl reversed himself, concluding that "when a control for size is introduced and subjects are allowed to gradually articulate their judgments, preference for golden rectangles or rectangles near the golden section emerges" (page 834).

McManus (1980) conducted three overlapping experiments into people's preferences for rectangles and triangles. In the first study, twenty-three subjects were presented with all possible pairs of fifteen horizontal rectangles, ranging in side-to-side ratio from 1:1 to 4:1. Fifteen of these same subjects also saw all possible pairs of fifteen vertical rectangles with the same ratio range. In the second study, twenty-seven subjects saw all possible pairs of fifteen mixed vertical and horizontal rectangles, as well as all pairs of twelve upright isosceles triangles, and all pairs of twelve sideways isosceles triangles. The triangles had height-to-base-width ratios covering about the same range as the rectangles. In the third study, forty subjects saw all pairs of the same shapes as in experiment 2 (but only ten triangles of each orientation this time), as well as all pairs of eleven right-angled triangles of mixed orientations with leg-to-leg ratios from about 1:1 to 3:1. No exactly golden triangles or rectangles were used in this study. In all studies, subjects were asked to pick the figure they preferred, as well as to rate how much they preferred it to the other.

As a group, McManus's subjects preferred rectangles near the golden but, as usual, the peaks of the curve were rather broad, spanning over the ratio range from about 1.5:1 to 2:1, and with sizable dips around 1:1, and at the extremes. Detailed analyses of the data of individual subjects showed, however, that beneath the group preference lay a heterogeneous array of individual preference patterns. Unfortunately, no attempt was made to see how many, if any, individuals preferred golden rectangles. The group curves for triangle preference also showed broad peaks at height-to-base-width ratios just short of the golden section. Although McManus does not make mention of it, the curves peaked at almost exactly the golden triangle (1.53:1). A factor analysis of the data revealed two main dimensions to people's preferences: one that reflected a preference for squares and triangles inscribable inside squares and another "showing a strong suggestion of being related to the golden section" (page 520). McManus concluded, to his admitted surprise, "that there is moderately good evidence for the phenomenon which Fechner championed" (page 522), though the broadness of the peaks, as usual, made precise determinations difficult.

3.3 *Recent research on line lengths*

Although investigation into rectangle preferences has been the most popular form of golden-section research, it has not been the only form. Svensson (1977) had forty-eight subjects (twenty-four students of psychology and twenty-four of art) divide both horizontal and vertical lines "at the point where the resulting line segments formed the most pleasing ratio" (page 79). He reported that a greater proportion of each group (37.5% for both) selected ratios in the range between 1.5 and 1.7 than any other range of 0.2 width. The mean ratio of long to short segments was 1.60 for the psychologists and 1.55 for the art students. Both distributions were slightly positively skewed. The ratios for vertical lines were slightly lower than for horizontal lines in both groups. Svensson concluded that his findings "point toward the possibility that the golden section is a 'true' ratio when subjects are instructed to partition a line so that they find the partition harmonious and pleasing" (page 80).

Schiffman and Bobko (1978) criticized Svensson, arguing that the "procedure [Svensson] reported suggests that the subjects were instructed to partition the experimental lines into *unequal* segments. This would tend to produce a skewed distribution" (page 102). Nothing in Svensson's report substantiates their claim in this. The skewness is clearly due to the closeness of his subjects' preferences in equality, which serves as a floor to possible responses in his procedure. Moreover, a stronger preference for equality would have only served to skew the distribution more strongly. Schiffman and Bobko showed twenty-two subjects eight lines; two each in the horizontal, vertical, and both diagonal (45°, 135°) orientations. Their exact instructions were, "your task is to divide a line with a pencil mark so that the resulting two line segments form the most pleasing proportion" (page 102). Overall, their subjects chose, on average, a 0.59 short-to-long segment ratio (1.69, in Svensson's long-to-short terms). Of the four orientations, the horizontal orientation gave rise to the ratio closest to equality (0.68). This was followed by the 45° (0.60), the 135° (0.57), and the vertical (0.51).

Although these numbers are fairly close to Svensson's, Schiffman and Bobko considered their main finding to be the high degree of variability in their subjects' preferences (mean $s = 0.318$). Notably, Schiffman and Bobko's distribution for horizontal lines was even more highly skewed than Svensson's had been ($g_1 = 0.677$ and 0.517, respectively). Last, they wrote of their data, "there is little agreement between subjects .... Moreover, there is little consistency or reliability in intrasubject judgments" (pages 102–103). The point had been made long prior to their study. In short, nothing particularly contrary to Svensson's findings was discovered.

As in the rectangle debate, the intervention of Benjafield (with Pomeroy and Saunders 1980) in the line-division debate served to cast things in a new light. Benjafield et al employed thirty-two subjects in a procedure in which they were asked merely to copy by hand line divisions presented to them, both horizontally and vertically, on cards. The divisions, presented on lines of 7, 6, 8, 9, and 10 cm, broke them at proportions of 0.5, ϕ', 0.67, and 0.75. A series of analyses showed that subjects made fewer mistakes and, when they made them, significantly smaller mistakes in copying divisions of 0.5 and ϕ' than they did when copying divisions of 0.67 and 0.75.[4]

It has long been a strategy of opponents of the golden section to pit it against equality (eg Angier 1903; Berlyne 1969; A H Pierce 1894). As noted above, however, this tactic misses the point of the 'Pythagorean' psychologist. As Benjafield et al concluded,

> "people tend to use two proportions more frequently than any others. When circumstances require a division into two equal parts, it seems obvious that people should be able to do so fairly accurately. However, when equal division is not required, we believe people tend to use the G[olden] S[ection]" (1980, page 253).

An alternative interpretation might be that the golden section has nothing to do with the matter, but people simply do better on the task when the division is closer to equality. Since subjects had to copy no divisions intermediate between equality and ϕ', there is nothing to decide between these two interpretations. Benjafield et al countered this critique by pointing out that often their subjects actually performed better on the golden section than on equal division. Still, it would be more convincing if they could show a substantial dip in performance between 0.5 and ϕ'.

3.4 *Recent research on other figures*

There is a smattering of research on figures other than rectangles and lines as well. Boselie (1984a, 1984b) has hypothesized that complex ratios—such as ϕ, $\sqrt{2}$, and $\sqrt{3}$—enhance the aesthetic character of a figure *only* if other parts of the figure contain simple ratios—such as 1:1 or 2:1. In one experiment (Boselie 1984a), he showed fifty subjects twelve pairs of irregular polygons, and asked which of the two they preferred. Seven of the pairs were matched for relative line lengths, but their respective internal angles differed. Five pairs were matched for angles, but differed in relative line lengths. One member of each pair contained both complex and simple ratios among their line lengths or angles. The other member of each pair contained the complex ratio of the first, but not the simple one. Boselie found that a majority of subjects favored the member of each pair containing both complex and simple ratios for eleven of the twelve pairs. These preferences achieved significance, however, for only seven of the twelve pairs.

In a second experiment (Boselie 1984b) one group of fifty subjects was shown ten pairs of irregular polygons (some with internal lines) in which one member contained a golden-section ratio between lines or angles, but no equal relations, and the other member contained neither golden nor equal ratios among its lines or angles. It was hypothesized that the polygons containing the golden-section ratios would not be preferred over those not containing them because, according to Boselie, there were no simple ratios present. As predicted, in only one of the ten pairs was the golden member preferred by a significant majority of subjects. There was no overall significant difference in preference between the pair members. A second group of fifty subjects were shown pairs of irregular polygons both of which contained golden ratios among the lines, but only one of which also contained equal lines in virtue of the particular golden relation that was present (eg a line that divided another in the golden section was also the same length as the shortest of the two golden segments).

[4] Note that Benjafield's ratios are long-segment-to-whole-line, not short-segment-to-long-segment, as used by Schiffman and Bobko. Recall that these two ratios are the same for the golden section, but not for other divisions of the line.

As Boselie predicted, the subjects significantly preferred the member containing equal relations *and* the golden relation for seven of the ten pairs. There was a significant overall preference for these figures as well.

With respect to rectangles in particular, Boselie noted that three rectangles, the golden, the 1.5:1, and the $\sqrt{2}:1$, all exhibit the sorts of ratios preferred by his subjects—complex ratios that give rise to simple ones within the figure. He suggested, but did not explicitly test, the hypothesis that, because these rectangles have such similar side-to-side ratios, the 'broad peak' near the golden rectangle typically seen in the literature may be a combined preference for all three that the methods typically used are not sensitive enough to tease apart. This would explain why it is that one regularly sees a 'spike' in preference curves near equality, but typically not near the golden section. According to Boselie, three 'spikes' corresponding to the golden, 1.5:1, and $\sqrt{2}:1$ rectangle are being 'blurred' to produce a blunt curve.

Boselie (1992) later ran three other experiments on the golden rectangle per se. The first of these was an empirical test of Bouleau's (1963/1980) analysis of a painting by Mondrian, and need not concern us further here because, as noted by researchers from Fechner to Arnheim, real artworks have many perceptual dynamics operating in them that can swamp whatever golden-section effects might be present. Specifically, the results of experiments with real artworks, no matter which way they come out, do not offer clear support for either side.

In the second experiment, he had twenty-five subjects compare eight golden rectangles each containing one interior line, and eight similar 1.5:1 rectangles, all presented in the horizontal orientation. The interior lines were constructed by dividing each side of each golden rectangle into the golden section, and each side of each 1.5:1 rectangle in a 1.5:1 ratio. By measuring once from each end of each side, two dividing points per side were obtained. Each of the resulting eight points of division (two on each of the four sides) served as the termination points for interior lines. Eight interior lines of the logically possible twenty-eight interior lines were chosen for use in the experiment (the others are all simple rotations of the chosen eight). The sixteen resulting figures (eight golden and eight 1.5:1) were presented to the subjects *en masse*, who were asked to order them from most preferred to least preferred. The mean rankings for the golden and 1.5:1 rectangles were found to be very similar (8.7 and 8.3, respectively), though Boselie argues that it is a clear refutation of the importance of goldenness.

In the third experiment, Boselie had thirty subjects compare six pairs of plain rectangles. One of each pair was golden, the other 1.5:1. In half of the pairs both rectangles had equal-length vertical sides. In the other half both had equal areas. The orientation was horizontal. In 74 of the 180 comparisons, the golden rectangle was preferred. In the 106 others, the 1.5:1 rectangle was preferred. No breakdown between equal-side and equal-area rectangles was reported even though this has often been shown to be a crucial factor in past research. Boselie concludes that his results "allow only one conclusion: the golden section has no special perceptual aesthetic attractivity" (1992, page 16). This appears to be a clear refutation of the strong golden-section hypothesis (viz that the golden rectangle is unique in its aesthetic appeal). As with many opponents of the golden section, however, Boselie has pitted the golden rectangle against another of interesting proportions—the 1.5:1—and most proponents of the golden-section hypothesis are more broadly 'Pythagorean' in their outlook; they believe that several rectangles of simple or otherwise interesting ratios will show an aesthetic advantage over those that do not have such ratios. Boselie's result is still consistent with this view.

Nakajima and Ohta (1989) employed concentric circles in which the inner figure of each had a diameter that was set to one of nine proportions of the diameter of the outer circle. These proportions employed ranged from 0.16 to 0.86. Four of these nine proportions implied golden proportions between various parts of the 'doughnut', as they were called. The one hundred and twenty-four subjects that Nakajima and Ohta tested were shown all thirty-six possible pairs of the nine doughnuts and asked which they preferred. One of the four containing a golden ratio was most preferred, but in general, the extremes of the range were rejected in favor of the moderate relations. No clear effect was found.

Davis and Jahnke (1991) looked at preferences for divided squares, rectangles, circles, ellipses, trapezoids, and parallelograms. The figures were divided in such a way that the *areas* on either side of the dividing line were in a ratio of equality, 1.268, ϕ, 2.791, or 3.885. In the first experiment, forty-nine subjects were shown sets of six identical figures (eg all squares) divided in the six different proportions. The various possible combinations of orientation, direction of division, and location of division (relative to the center of the figure) produces forty-two such displays. Subjects overwhelmingly preferred equal division over all others. In a second experiment, forty subjects were allowed to draw their preferred dividing line themselves. These, too, strongly preferred equality. Finally, in a third experiment forty subjects were asked to select the preferred member of 114 pairs of figures, each consisting of a divided square and a divided rectangle. These subjects also showed a strong preference for equality of division. When asked to compare undivided squares with undivided rectangles, of various dimensions, however, no clear preference emerged. Davis and Jahnke concluded that their results "provide no basis for the golden section as an aesthetic ideal, nor do we wish to champion some other rule, such as that of unity" (page 275). Still, their procedure was geared to pit equality against the golden section, a standard ploy of golden-section critics.

There were some anomalous aspects to Davis and Jahnke's procedure. First, their figures were divided so as to produce specific ratios of area, rather than ratios of length. Areas are notoriously difficult for people to estimate. Moreover, dividing a figure such as a circle or an ellipse so that the areas on either side of the divide stand in a certain relation will not result in the axis perpendicular to the line of division being divided in the same ratio. Second, in the only experiment in which rectangles were used, they were *all* golden rectangles. Thus any preference for 'goldenness' might well have been satisfied by the base figure itself, entirely other considerations leading to the preference for a particular division of the figure.

4 Assessment of the research on aesthetics and ϕ

This review began with the question of whether there really is a set of aesthetic phenomena associated with the golden section, or whether it is nothing more than an ancient superstition that dispassionate empirical research could show to be false. Having now examined the major empirical studies of the last 130 years, what are we to make of the resulting mass of conflicting data and conclusions? The studies that have been discussed in this paper (barring those in which there was no empirical work, or quite eccentric work) are summarized in table 1.

One might argue that the effects, if there be any at all, are so fragile as not to be of much interest; that a substantial underlying psychological factor related to ϕ, if there were one, would prove more robust. On the other hand, one might argue that there has been something of a concerted effort among some psychologists to show that there is nothing to the alleged effects of the golden section; that the unreliability of the effects is due to research practices geared to show it to be a fraud, rather than to an inherent weakness in the effect. Both of these positions are, I think, too crude to do justice to the data, but they are reasonable places to begin a discussion.

Table 1. Summary of golden-section research.

Researcher(s)	Stimulus shape	Orientation of stimulus	Range of stimulus ratios	ϕ stimulus in center of range?	Area of stimuli controlled?	Method	Analysis	Result
Fechner (1876)	rectangle	"kreuz und quer"	1.0–2.5	no (7/10)	yes	choice (serial)	% 1st choice	35% golden, 76% near
	ellipse	"kreuz und quer"	1.0–2.5	no (6/9)	yes	choice (mass)	% 1st choice	16% golden; 72% near
Witmer (1894)	rectangle	?			?	choice (serial)	% 1st choice	1.651 most preferred
	triangle	horizontal			?	choice (mass)	% 1st choice	0.41 most preferred
E Pierce (1894)	lines	vertical	NA	NA	NA	production	qualitative	"everyone chose position roughly corresponding to golden section"
Angier (1903)	line	horizontal	NA	NA	NA	production	means, raw frequencies	mean near ϕ but only 2/11 chose ϕ
Haines and Davies (1904)	rectangle	?	1.0–4.8	no	no	choice (serial)	raw frequencies	no sizeable effect
	rectangle	NA	NA	NA	NA	production	raw frequencies	no trend
Lalo (1908)	rectangle	horizontal	1.0–2.5?	no (7/10)	yes	choice (mass)	% 1st choice	30% gold, 71% near
Thorndike (1917)	rectangle	horizontal	1.3–3.75	no (9/12)	no	choice (mass)	% 1st choice	16% gold, 41% near
	triangle	vertical	1.1–3.3	no (4/12)	no	choice (mass)	% 1st choice	14% gold, 42% near
Weber (1931)	rectangle	vertical	1.0–2.2	yes (4/9)	yes	choice (PC)	most-often preferred	14% gold, 40% near
Farnsworth (1932)	rectangle	both	0.4–2.5	no (3/17) (16/17)	yes	choice (PC)	sigma units[a]	vertical gold ranked 1st
Davis (1933)	rectangle	NA	NA	NA	NA	production	raw frequencies	modes at $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$
Thompson (1946)	rectangle	horizontal	0.25–0.75	no (8/12)	no	choice (serial)	median rank	max medians at 0.55–0.65
Shipley et al (1974)	rectangle	horizontal	0.27–0.75	no (4/6)	yes	choice (PC)	median rank	max median at 0.65
Nienstedt and Ross (1951)	rectangle	horizontal	0.25–0.75	yes (4/6)	yes	choice (mass)	median rank	max medians at 0.55–0.75
Austin and Sleight (1951)	triangle	horizontal	0.25–3.0	yes (6/12)	no	choice (mass)	% 1st choice	broad peak at 1.0–1.75
Schiffman (1966)	rectangle	NA	NA	NA	NA	production	mean, median h–w ratios	mean = 0.525, median = 0.500

Schiffman (1969)	rectangle	NA	NA	NA	NA	production	mean, median h–w ratios	mean = 0.489, median = 0.500
	rectangle	both	0.318–0.818	yes (4/6)	no	choice (PC)	most often preferred	no significant preference
	rectangle	both	0.318–0.718	no (4/5)	no	choice (PC)	most often preferred	no significant preference
Hintz and Nelson (1970)	rectangle	horizontal	0.10–1.0	yes (8/14)	yes	choice (PC)	median, modal preference	median = 0.558, mode = 0.600
	rectangle	NA	NA	NA	NA	production	median, modal h–w ratio	median = 0.545, mode = 0.57
Plug (1976)	rectangle diamond pear	both both both	1.0–8.5 1.0–8.5 1.19–2.88	no (6/18) no (6/18) yes (4/9)	? ?	choice (mass)	mean preference	"somewhat less than 2.0"
Hintz and Nelson (1971)[b]	rectangle	vertical?	0.10–1.0	yes (8/14)	yes	choice (see description)	median, modal preference	median = 0.558, mode = 0.600
Eysenck and Tunstall (1968)	rectangle	vertical	0.25–1.0	yes (8/14)	no	choice (mass)	mean rank	0.69 (introverts), 0.75 (extroverts)
Berlyne (1970)	rectangle	vertical	1.0–2.5	no (7/10)	no	choice (mass)	mean rank, % 1st choice	golden rectangle best for Canadian middling for Japanese square preferred
Godkewitsch (1974)	rectangle	vertical	1.0–2.5	(see text)	no	choice (mass)	mean rank, % 1st choice	golden rectangle only preferred on average, and only when in center c distribution
Piehl (1976)	rectangle	vertical	1.0–2.5	(pace Godkewitsch)	no	choice (mass)	% 1st choice	controls preferred extremes; those exposed to golden rectangle preferre
Benjafield (1976)	rectangle	vertical	1.0–2.5	(pace Godkewitsch)	yes	repertory grid[c]	% 1st choice	golden rectangle preferred regardle of position in range
Piehl (1978)	rectangle	vertical	1.0–2.3	(pace Godkewitsch)	yes	choice (PC)	most-often preferred	golden rectangle preferred
McManus (1980)	rectangle and triangle	both both	0.25–4.0 0.33–3.0	no no	? ?	choice (PC) choice (PC)	weighted preferences	near-golden preferred by groups, b individuals varied

Table 1 (continued)

Researcher(s)	Stimulus shape	Orientation of stimulus	Range of stimulus ratios	ϕ stimulus in center of range?	Area of stimuli controlled?	Method	Analysis	Result
Svensson (1977)	line	both	NA	NA	NA	production	raw frequencies	37.5% of subjects between 1.5 and 1.7
							mean ratio	means = 1.60 and 1.55
Schiffman and Bobko (1978)	line	both and 2 diagonals	NA	NA	NA	production	mean ratio	mean = 1.69
Benjafield et al (1980)	line	both	0.5 – 0.75	yes (2/4)	NA	production (copy given division)	mean errors in copying	errors significantly smaller for equality and golden section than for 0.67 and 0.75
Boselie (1984a)	(see text)				no, control for line and angle size	choice (PC)	most-often preferred	combinations of simple and complex ratios preferred to complex ratios alone
Boselie (1984b)	(see text)				pace Boselie (1984a)	choice (PC)	most-often preferred	polygons bearing golden section
Boselie (1992)	(see text)				yes	choice (mass)	mean rank	golden and 1.5:1 rectangles equal in preference
	pairs of rectangles		ϕ and 1.5	NA	yes (but see text)	choice (PC)	most-often preferred	1.5:1 preferred to golden rectangle
Nakajima and Ohta (1989)	doughnuts	NA	0.16 – 0.86	(see text)	no	choice (PC)	most-often preferred	golden section not preferred over others
Davis and Jahnke (1991)	divided square	NA	?	?	no	choice (?)	?	strong preference for equal division
	divided square	NA	NA	NA	NA	production	?	strong preference for equal division
	divided square and rectangle	horizontal	?	?	no	choice (?)	?	strong preference for equal division

Note: [a] See Guilford 1928. [b] Sighted group. [c] See Kelly 1955. NA, not applicable; h – w, height – width; PC, paired comparisons.

I do not think it unreasonable to suggest that there has been a tendency among many psychologists to discount the golden section a priori as a 'numerological fantasy'. I also think that it is clear, particularly in the tone of their writing, that doing away with this 'fantasy' has been the guiding intent of many of them. Consequently, many of the studies have been carried out crudely, some even sloppily, rather than with a desire to 'tease out' what might be a somewhat fragile, but nonetheless consistent, effect. Still, the presence of bad attitudes is not a sufficient ground for claiming that an apparent noneffect is, in reality, an effect; there must be some evidence.

What has been found? Apart from the Fechner and Witmer studies—the ones that are consistently put forward by advocates of the golden section—the early study by E Pierce (1894) revealed a popular preference for the golden section (despite Pierce's own efforts to minimize the finding). Angier (1903) found a preference for line divisions near the golden section *on average*, but did not find that it was preferred by individuals. Haines and Davies (1904) found no sizable effect, but Lalo (1908), replicating Fechner's procedure, found an effect nearly as strong as had Fechner himself. Studies by Thorndike (1917) and Weber (1931) revealed general trends in favor of figures with proportions in the range of the golden section, but nothing specific. Farnsworth (1932), on the other hand, found fairly strong support for a preference for the golden rectangle. Davis (1933) found modal preferences at $\surd 3$, $\surd 4$, and $\surd 5$, but not at ϕ. Importantly, however, he was the first to suggest that the proximity of ϕ to other 'basic' proportions, such as $\surd 2$ and $\surd 3$, might be masking an otherwise reliable effect. Interestingly, although this kind of 'Pythagorean' attitude has not been popular in mainstream psychology, it was the metaphysical backbone of the psychophysics developed by Weber and Fechner in the 19th century. Thompson (1946), Shipley et al (1947), and Nienstedt and Ross (1951) all showed trends in favor of golden rectangles, but their use of median rankings, rather than modes or raw frequencies, make their results suspect in the eyes of many critics.

Schiffman (1966, 1969) failed to find any effect for the golden rectangle, confirming the growing suspicion that golden-section research was a wild-goose chase. Eysenck and Tunstall (1968) found golden-rectangle effects, especially for introverts, but used the dubious tool of mean rankings. Berlyne (1970) found similar effects among Canadian subjects, using mean rankings as well, but showed, as had Angier in the early part of the century, that this does not accurately reflect individuals' preferences. The research of Hintz and Nelson (1970, 1971), however, revealed *modal* preferences quite close to the golden rectangle. Significantly, modes are not subject to the criticisms that have historically been made of means in this area of research.

Godkewitsch (1974) claimed to show that historically established preferences for the golden rectangle were nothing but artifacts of poorly conceived experimental procedures, but he did not treat those studies in which other procedures and methods of analysis had been used. Still, the replication of Godkewitsch's finding by Piehl (1976) boded ill for golden-section research. Reversing this apparent fate, Benjafield (1976) showed that a more carefully conceived experiment would give rise to the traditional effect for the golden rectangle, even when Godkewitsch's criticisms were taken into account. The results of Piehl (1978) supported this conclusion, and golden-section research was restored to the psychological agenda. An interestingly parallel case occurred in the case of research on divided lines. McManus's (1980) result, too, lent some additional, though inconclusive, credence to at least group preferences for the golden rectangle. Schiffman and Bobko (1978) claimed to refute the positive findings of Svensson (1977), but again, an experiment carefully conceived and conducted by Benjafield et al (1980) restored an effect that had been lost in less-exacting work. Boselie (1984a, 1984b) has argued that apparent preferences for complex proportions, such as square roots and ϕ are, in fact, the result of subordinate simple proportions, such as equality. Boselie (1992) showed that the 1.5:1 rectangle may also be preferred to ϕ. The failures of Nakajima and Ohta (1989) and of Davis and Jahnke (1991) to find positive results are both beset by methodological problems.

I am led to the judgment that the traditional aesthetic effects of the golden section may well be real, but that if they are, they are fragile as well. Repeated efforts to show them to be illusory have, in many instances, been followed up by efforts that have restored them, even when taking the latest round of criticism into account. Whether the effects, if they are in fact real, are grounded in learned or innate structures is difficult to discern. As Berlyne has pointed out, few other cultures have made mention of the golden section but, equally, effects have been found among people who are not aware of the golden section. In the final analysis, it may simply be that the psychological instruments we are forced to use in studying the effects of the golden section are just too crude ever to satisfy the skeptic (or the advocate, for that matter) that there really is something there.

Acknowledgements. This research was supported by the financial assistance of the University of Toronto Department of Psychology, the York University Faculty of Arts, and the Natural Sciences and Engineering Research Council of Canada. I would like to express my personal appreciation to Dr Rolf Kroger for his assistance in the translation of passages of Fechner's and Witmer's books and articles.

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