Adaptive Kalman Filtering
Based on Posteriori Variance-Covariance Components Estimation

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ABSTRACT

There are different ways to construct adaptive Kalman filtering (AKF) algorithms. This paper proposes an innovative way to simultaneously estimate the variance matrix \( R \) of the measurement vector and the variance matrix \( Q \) of the process noise vector based on the variance-covariance component estimation by taking the advantages of the measurement residuals and the process noise residuals (Wang, 1997, 2009; Wang et al, 2009) and the measurement redundancy contribution (Ou, 1989). The core of the novel AKF algorithm lies in the projection of the system innovation vector into the three groups of residuals: the residuals of the measurement vector, the residuals of the process noise vector and the residuals of the predicted state vector exclusive of the effect of the process noise. The simulated and real GPS data in kinematic relative positioning mode were used to demonstrate the performance of the proposed adaptive Kalman filter. The results from the simulated datasets confirm to the simulated variance-covariance components well. The results from real kinematic GPS datasets are also provided and discussed.

Key words: Adaptive Kalman Filter, measurement residuals, process noise residuals, variance-covariance components, kinematic GPS, relative positioning, redundancy contribution.

1. INTRODUCTION

Kalman filter (KF) has become one of the most widely used recursive methods that estimate the states of a process. If the system and measurement model, the variance-covariance (VC) matrix \( Q \) of the process noise vector and the VC matrix \( R \) of the measurement vector are known, the KF produces an optimal solution. In practice, one cannot guarantee to have them all satisfied. In particular, the a-priori \( Q \) and \( R \) are unknown or approximated by applying the best available knowledge. This may produce unreliable results or cause the KF to diverge. The classical method that chooses the values for these two matrices is through empirical analysis on the system and measurement errors or manually in an ad hoc fashion. This requires an experienced developer. However, the results from empirical analysis may not quantitatively represent the real \( Q \) and \( R \). By performing the adaptive Kalman Filter (AKF), one can make the filter adaptively apply the realistic noise statistics \( Q \) and \( R \) that are based on the being processed samples.

There are different ways to construct AKF algorithms. The very pioneer work from Mehra (1970) used the autocorrelation functions of the innovation sequence to estimate both \( Q \) and \( R \) for the time invariant system. Another popular alternate method was to introduce the adaptive algorithms based on the system innovation sequence either to derive \( R \) and a scale factor for \( Q \) (Ding et al, 2007), or to indirectly estimate \( Q \) and \( R \) (Mohamed and Schwarz, 1999). Hu et al (2003) assumed \( R \) completely known in order to estimate \( Q \). Yang and Gao (2006) derived two optimal adaptive factors, based on either the estimated innovation VC matrix or the estimated VC of the predicted state vector, to balance the contribution of the dynamic system and the measurement model information.

In this work, the authors propose an alternate way to simultaneously estimate \( R \) and \( Q \) by performing the variance-covariance component estimation using the measurement residual vector and the process noise residual vector that were derived in (Wang, 2009) and the measurement redundancy index formulated by (Ou, 1989). After the overview of the Kalman filtering and adaptive Kalman filtering in Section 2, the proposed AKF algorithm is presented in Section 3. To illustrate the proposed adaptive Kalman filter, the kinematic relative GPS positioning is taken as an example (Section 3.4). Both of the simulated and real GPS data were processed. Our numerical analysis of the proposed algorithm limits \( Q \) to be diagonal and \( R \) to be correlated. Section 4 provides the corresponding results along with specific analysis. Remarks and conclusions are provided in Section 5.

2. ADAPTIVE KALMAN FILTERING

2.1. OVERVIEW OF KALMAN FILTER

With provision for algorithm formulation, a concise summary of Kalman filter is given in this section. A linear
or linearized multivariable discrete system is considered over a discrete time series \( \{t_0, t_1, \ldots, t_k, \ldots, t_N \} \), often simplified to \( \{0, 1, \ldots, k, \ldots, N \} \). Without loss of generality, the deterministic system input is intentionally omitted here. Straightforward, the system can be described at instant \( k \) as follows:

\[
x(k) = A(k-1)x(k-1) + B(k-1)w(k-1) \quad (1)
\]

\[
z(k) = C(k)x(k) + \Delta(k) \quad (2)
\]

wherein \( x(k) \) is the \( n \times 1 \) state-vector; \( z(k) \) is the \( p \times 1 \) observation vector, \( w(k-1) \) is the \( m \times 1 \) process noise vector, \( \Delta(k) \) is the \( p \times 1 \) measurement noise vector; \( A(k-1) \) is the \( n \times n \) transition matrix; \( B(k-1) \) is the \( n \times m \) coefficient matrix of \( w(k-1) \); \( C(k) \) is the \( p \times n \) output matrix. The random vectors \( w(k-1) \) and \( \Delta(k) \) are generally assumed to be: \( w(k-1) \sim N(0, Q(k-1)) \) and \( \Delta(k) \sim N(0, R(k)) \) with zero-means \( o \) and the variance matrices as \( Q(k-1) \) and \( R(k) \) positive definite, respectively. Further assumptions about \( w(\cdot) \) and \( \Delta(\cdot) \) are considered: \( \text{Cov}(w(i), w(j)) = O \), \( \text{Cov}(\Delta(i), \Delta(j)) = O \) and \( \text{Cov}(w(i), \Delta(j)) = O \) for \( i \neq j \). Commonly, one also assumes to have the initial state vector \( x(0) \) and its variance-covariance matrix \( D_{xx}(0) \) available and independent of arbitrary \( w(\cdot) \) and \( \Delta(\cdot) \). Under the given assumptions, the unbiased optimal estimation of \( x(k) \) can be derived in the sense of minimum variance an in table 1.

Table 1: Solution of Kalman filter

<table>
<thead>
<tr>
<th>The predicted state vector and its variance matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{x}(k / k-1) = A(k-1)\hat{x}(k-1) )</td>
</tr>
<tr>
<td>( D_{xx}(k / k-1) = A(k-1)D_{xx}(k-1)A^T(k-1) )</td>
</tr>
<tr>
<td>( + B(k-1)Q(k-1)B^T(k-1) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The optimal estimated state vector and its variance matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{x}(k) = \hat{x}(k / k-1) + G(k)d(k) )</td>
</tr>
<tr>
<td>( D_{xx}(k) = G(k)R(k)G^T(k) )</td>
</tr>
<tr>
<td>( + [E - G(k)C(k)]D_{xx}(k / k-1)[E - G(k)C(k)] )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The optimal estimated state vector and its variance matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d(k) = z(k) - C(k)\hat{x}(k / k-1) )</td>
</tr>
<tr>
<td>( D_{zz}(k) = C(k)D_{xx}(k / k-1)C^T(k) + R(k) )</td>
</tr>
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<table>
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<tr>
<th>The gain matrix</th>
</tr>
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<tbody>
<tr>
<td>( K(k) = D_{xx}(k / k-1)C^T(k)D_{zz}^{-1}(k) )</td>
</tr>
</tbody>
</table>

2.2. ADAPTIVE KALMAN FILTERING

As a fact, the effect of the initial state vector along with its variance will be forgotten with the time being in Kalman filtering. However, a major obstacle in applying Kalman filter is specifying the variance matrices \( Q(\cdot) \) and \( R(\cdot) \) (Louv, 1984 etc.). Their true values are not known. Users make their good effort to approximate \( Q(\cdot) \) and \( R(\cdot) \) using the best available information about their applications. In general, the specified values for the variance matrices \( Q(\cdot) \) and \( R(\cdot) \) are experimental.

Primarily, two different strategies for running adaptive Kalman filtering have been developed. The majority of the adaptive algorithms focused on how to sequentially improve \( Q(\cdot) \) or \( R(\cdot) \), or both of \( Q(\cdot) \) and \( R(\cdot) \) (Jazwinski, 1969; Mehra, 1970; Louv, 1984; Mohamed and Schwarz, 1999; Hu and Liu, 2002; etc.). Another strategy aims to find a balance between the time update and the measurement update (Wang, 1997; Yang and Gao, 2006;), i.e.

\[
v_{i+1}^T(k)R(k)v_i(k) + \alpha_k |\Delta v(k)D^{-1}(k / k-1)\Delta v(k)| = \min
\]

with \( \Delta v(k) = x(k) - x(k / k-1) \).

Further derivative algorithms similar to (3) were also constructed (Ding, et al, 2007; Yang, et al, 2001; Oussalah and Schutter, 2000 etc.). This is not more adaptive than robust because the formulation is modified in addition to providing the adaptive \( Q(\cdot) \) or \( R(\cdot) \).

In this manuscript, the proposed adaptive algorithm only deals with improving of \( Q(\cdot) \) and \( R(\cdot) \) in Kalman filtering.

3. VARIANCE-COVARIANCE ESTIMATION-BASED ADAPTIVE KALMAN FILTERING

This section furnishes the novel adaptive algorithm in Kalman filtering in details. First, the residual vectors for the measurement vector and the process noise vector are derived. Then, the redundancy contribution in Kalman filtering is discussed. At the end of this section, the posteriori estimation of the variance and covariance factors is formulated on the ground of the measurement or pseudo-measurement residuals, not the system innovation residuals instead as usual.

3.1. MEASUREMENT AND PROCESS NOISE RESIDUALS
In order to derive the residuals of measurement vector and process noise vector, an alternate prospect was given about Kalman filter in (Wang, 1997, 2009; Caspary and Wang, 1998; Wang et al, 2009).

There exist three groups of stochastic information that is associated with the estimation of the state vector \( \mathbf{x}(k) \) at the instant \( k \):

1. the observation noise vector \( \Delta(k) \),
2. the system process noise vector \( \mathbf{w}(k-1) \) and
3. the noise on the predicted state vector \( \mathbf{z}(k) \) brought by \( \mathbf{z}(k-1) \) through the propagation of \( \{\Delta(1), \ldots, \Delta(k-1)\} \) and \( \{\mathbf{w}(0), \ldots, \mathbf{w}(k-2)\} \).

Customarily, “(2)” and “(3)” are considered together in the one step predicted state vector \( \mathbf{x}^*(k) \) from time \( k-1 \) to \( k \). As a matter of fact, these three groups of stochastic information should be studied separately. By defining the independent (pseudo-)observation groups

\[
I_x(k) = A(k-1)\hat{\mathbf{x}}(k-1) \quad (4) \\
I_w(k) = \mathbf{w}(k-1) \quad (5) \\
I_z(k) = \mathbf{z}(k) \quad (6)
\]

with their variance-covariance matrices

\[
D_{I_xI_x}(k) = A(k-1)D_{xx}(k-1)A^T(k-1) \quad (7) \\
D_{I_wI_w}(k) = \mathbf{Q}(k-1) \quad (8) \\
D_{I_zI_z}(k) = \mathbf{R}(k) \quad (9)
\]

The system model with (1) and (2) can be reformulated through the residual vectors of these three groups of the error sources[Wang, 1997; Caspary & Wang, 1998; Wang, 2008]:

\[
v_{x}(k) = \hat{\mathbf{x}}(k) - \mathbf{B}(k-1)\hat{\mathbf{w}}(k-1) - I_x(k) \quad (10) \\
v_{w}(k) = \hat{\mathbf{w}}(k) - I_w(k) \quad (11) \\
v_{z}(k) = \mathbf{C}(k)\hat{\mathbf{x}}(k) - I_z(k) \quad (12)
\]

Here, \( I_x(k) \), \( I_w(k) \) and \( I_z(k) \) are the \( n \), \( m \) and \( p \)-dimensional measurement or pseudo-measurement vectors, respectively. Usually, one has \( \mathbf{w}(0) = \mathbf{0} \).

By applying the principle of least squares to \( I_x(k) \), \( I_w(k) \) and \( I_z(k) \), the identical solution can be obtained as in table 1 [Wang, 1997, 2009]. Moreover, this alternate prospect directly makes the measurement residual vectors available at each epoch for further error analysis in Kalman filter:

\[
v_{x}(k) = D_{I_xI_x}(k)D_{xx}^{-1}(k)\mathbf{K}(k)d(k) \quad (13) \\
v_{w}(k) = \mathbf{Q}(k-1)\mathbf{B}^T(k-1)D_{xx}^{-1}(k)\mathbf{K}(k)d(k) \quad (14) \\
v_{z}(k) = \{\mathbf{C}(k)\mathbf{K}(k) - E\}d(k) \quad (15)
\]

with their variance matrices

\[
D_{v_{x}v_{x}}(k) = A(k-1)D_{xx}(k-1)A^T(k-1)C^T(k) + D_{w} \quad (16) \\
D_{v_{w}v_{w}}(k) = \mathbf{Q}(k-1)\mathbf{B}^T(k-1)C^T(k)D_{w} \quad (17) \\
D_{v_{x}v_{z}}(k) = \{\mathbf{E} - \mathbf{C}(k)\mathbf{K}(k)\}R(k) \quad (18)
\]

Obviously, all of the three residual vectors are the functions of the system innovation vector \( d(k) \). The measurement residuals are characterized as an uncorrelated series epochwise

\[
\text{Cov}(v(i), v(j)) = \mathbf{O} \quad (i \neq j)
\]

as how the system innovation series are characterized.

### 3.2. THE REDUNDANCY CONTRIBUTION IN KALMAN FILTERING

According to the reliability theory in least squares method, one can judge of the redundancy contribution of the individual measurements through the matrix \( D_{v_{x}v_{x}}(k)P(k) \) or \( D_{v_{w}v_{w}}(k)D_{w}^{-1}(k) \), wherein \( P(k) \) is the corresponding weight matrix of the measurement vector. In case that the measurements are not correlated to each other, the detailed analysis can be found in (Wang, 1997, 2009).

For three independent measurement groups as from (3) to (5), the individual group redundant indices are generally equal to

\[
r_x(k) = \text{tr}[A(k-1)D_{xx}(k)A^T(k-1)C^T(k)D_{w}^{-1}(k)\mathbf{C}(k)] \quad (20) \\
r_w(k) = \text{tr}[\mathbf{Q}(k)\mathbf{B}^T(k)C^T(k)D_{w}^{-1}(k)\mathbf{C}(k)B(k-1)] \quad (21) \\
r_z(k) = \text{tr}[\mathbf{I} - \mathbf{C}(k)\mathbf{K}(k)] \quad (22)
\]

wherein “tr” stands for trace. The number of the total redundancy at \( k \) is given by

\[
r(k) = r_x(k) + r_w(k) + r_z(k) = p(k) \quad (23)
\]
which is exactly equal to the number of the measurements in $\mathbf{z}(k)$

### 3.3. VARIANCE-COVARIANCE ESTIMATION

The most widely known method for the variance and covariance component estimation (VCCE) is the Helmert's method. More on development to simplify, unify or extend the various VCCE algorithms can be found in (Förstner, 1979; Koch, 1986; Ou, 1989; Yu, 1996). The research in this paper adapts the derivation of VCCE algorithm from (Qu, 1989).

Let $\mathbf{I}$ be a $n$-dimensional observation vector as the function of the $u$-dimensional parameter vector $\mathbf{y}$

$$\mathbf{I} = \mathbf{Fy} + \mathbf{e}$$

(24)

wherein $\mathbf{e}$ is the observation error vector assumed to be Gaussian white noise with the observation variance matrix

$$\mathbf{\Sigma}_e = \begin{pmatrix}
\sigma_{11}^2 & \cdots & \sigma_{1u} & \cdots & \sigma_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\sigma_{nu} & \cdots & \sigma_{uu} & \cdots & \sigma_{nn}
\end{pmatrix}$$

and

$$\mathbf{\Sigma}_j = \begin{pmatrix}
\sigma_{(0)1}^2 & \cdots & \sigma_{(0)u} & \cdots & \sigma_{(0)n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\sigma_{(nu)} & \cdots & \sigma_{(nu)} & \cdots & \sigma_{(nn)}
\end{pmatrix}$$

(25)

wherein $\sigma_{(0)u}$, $\sigma_{(0)v}$, $\sigma_{uv}$ and $\sigma_{uv}$ stand for the corresponding variance or covariance of unit weight and the cofactors, respectively. In general, it maintains $\sigma_{(0)u}q_{uk} = \sigma_{(0)v}q_{vk}$. In practice, one seeks to estimate either $\sigma_{uv}^2$ or $\sigma_{uv}$ by taking the advantages of the available measurement residuals.

The least squares solution delivers the residual vector $\mathbf{r}$ of $\mathbf{I}$ with its variance matrix $\mathbf{D}_r$. Statistically, the following equation exists for the expectation of weighted sum of the residuals squared

$$\mathbf{E}(\mathbf{r}^T \mathbf{P} \mathbf{r}) = \text{tr}((\mathbf{Q}_r \mathbf{P})^T \mathbf{P} \mathbf{\Sigma}_e)$$

(26)

whereas $\mathbf{r}$ and $\mathbf{P}$ are the residual vector and weight matrix of $\mathbf{I}$, respectively, and $\mathbf{Q}_r$ is the cofactor matrix of $\mathbf{r}$. Through appropriate partition, the following normal equation for the individual variance-covariance factors is given by (Ou, 1989; etc.):

$$\mathbf{w} = \mathbf{M} \mathbf{\theta}$$

(27)

with

$$\mathbf{\theta} = \begin{pmatrix}
\sigma_{(0)1}^2 & \cdots & \sigma_{(0)u} & \cdots & \sigma_{(0)n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\sigma_{(nu)} & \cdots & \sigma_{(nu)} & \cdots & \sigma_{(nn)}
\end{pmatrix}$$

(28)

and

$$\mathbf{w} = \begin{pmatrix}
p_{11}v_1^2 & \cdots & p_{1u}v_1v_u & \cdots & 2p_{uu}v_u^2 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
2p_{2u}v_u^2 & \cdots & p_{(n-1)u}v_{u-1}^2 & \cdots & 2p_{(n-1)u}v_{u-1}v_u
\end{pmatrix}^T$$

(29)

$$\mathbf{M}_{jk} = \text{tr}((\mathbf{Q}_r \mathbf{P})^T \mathbf{P} \mathbf{J}_k)$$

(30)

wherein $p_{ij}$ is the elements intersected between the $i$th row and the $j$th column in $\mathbf{P} = \mathbf{\Sigma}_e^{-1}$, and

$$\mathbf{P}_j = \begin{pmatrix}
0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0
\end{pmatrix}$$

or

$$= \begin{pmatrix}
0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0
\end{pmatrix}$$

(31)

$$\mathbf{T}_k = \begin{pmatrix}
0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0
\end{pmatrix}$$

or

$$= \begin{pmatrix}
0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0
\end{pmatrix}$$

(32)

If multiple variance or covariance factors are assumed to be equal, $\mathbf{P}_j$ and $\mathbf{T}_k$ will be changed accordingly so that it may have more non-zero elements in them, and the dimension of $\mathbf{\theta}$ may be reduced as well.

Furthermore, the matrix $\mathbf{Q}_r \mathbf{P}$ has the total redundancy number $r_j$ that $\mathbf{I}$ possesses. At most $r_j(r_j+1)/2$ independent variance and covariance components can be uniquely determined [Xu et al., 2007].

In general, the non-diagonal elements of the coefficient matrix $\mathbf{M}$ in (24) are much smaller than its diagonal ones so that the successive approximation has practically been made to ignore the non-diagonal elements of $\mathbf{M}$ (Förstner, 1979; Ou, 1989). Therefore, the following
solution of the variance and covariance factors is given as follows
\[
\sigma_{ii}^2 = \frac{p_{ii} v_i^2}{M_{ii}} \tag{33}
\]
\[
\sigma_{jk} = \frac{2p_{jk} v_j v_k}{r_{jk}} \tag{34}
\]
with
\[
r_{jk} = tr((Q_{vv}P)^T P_j T_k) + tr((Q_{vv}P)^T P_k T_j)) \tag{35}
\]
The result given above in this subsection is only corresponding to the variance and covariance components estimation at a single epoch, namely \( k \) here. In Kalman filtering, the residuals from previous multiple epochs, even all of the past epochs can be used to accumulatively estimate the variance and covariance components of \( R(k) \) and \( Q(k) \) so that the estimation quality can be improved. Accordingly, (33) and (34) become
\[
\sigma_{ii}^2(k) = \frac{\sum_{k=k_0}^{k} p_{ii}(k)v_i^2(k)}{\sum_{k=k_0}^{k} M_{ii}(k)} \tag{36}
\]
\[
\sigma_{jk}(k) = \frac{2\sum_{k=k_0}^{k} p_{jk}(k)v_j v_k}{\sum_{k=k_0}^{k} r_{jk}(k)} \tag{37}
\]
where \( k_0 \) is the number of the used previous epochs. In practice, the equations (33), (34) and (36), (37) can further be reformulated to fit to other reasonable assumptions, for example, the measurement variances should be identical in GPS positioning if the satellites have the same elevation angles or if their elevation differences are within a given range (e.g. 5 degrees).

By the way, readers are referred to (Wang, 1997; Wang et al, 2009) for the case that \( R(k) \) and \( Q(k) \) are diagonal, i.e. the measurements and process noise factors are uncorrelated.

### 3.4 Kinematic Relative GPS Positioning

The objective of kinematic relative position is to determine the coordinates of moving object \( A \) with respect to a stationary known point \( B \). This type of positioning requires simultaneous observations at two points and can be performed with code ranges or/and carrier phases (Hofmann-Wellenhof et al., 2008).

The system equation is given as follows
\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} + \begin{bmatrix}
\frac{1}{2} \Delta t_x^2 \\
\frac{1}{2} \Delta t_y^2 \\
\frac{1}{2} \Delta t_z^2
\end{bmatrix} \tag{38}
\]
where \( x \) is the position vector, \( \dot{x} \) is the velocity vector, \( \ddot{x} \) is the acceleration vector all in ECEF, \( \Delta \text{VN} \) is the double differenced ambiguity vector and \( \Delta t = t_k - t_{k-1} \).

\( \ddot{x} \) is modeled as process noise.
\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} \sim N\left( \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}, \begin{bmatrix}
\sigma^2_x & 0 & 0 \\
0 & \sigma^2_y & 0 \\
0 & 0 & \sigma^2_z
\end{bmatrix} \right) \tag{39}
\]

Double-differenced measurements are usually used and the measurement equation for a single frequency receiver is given by:
\[
\Delta \text{VN}_{AB} = \Delta \text{VN}_{AB}^j + \Delta \text{VN}_{jAB} \tag{40}
\]
\[
\lambda \Delta \text{VN}_{jAB} = \lambda \Delta \text{VN}_{jAB}^j + \lambda \Delta \text{VN}_{AB} + \Delta \text{VN}_{L1,AB} \tag{41}
\]
where \( \Delta \text{VN}_{AB}^j \) is the double differenced L1 code range measurement between satellite \( j \) and satellite \( k \) and receiver \( A \) and receiver \( B \) is meters, \( \Delta \text{VN}_{jAB} \) is the corresponding double differenced carrier phase measurement in cycles, \( \Delta \text{VN}_{AB} \) is the geometric range, \( \lambda \) is the wave length of L1 carrier phase, \( \Delta \text{VN}_{AB} \) is the double differenced ambiguity unknown of L1 carrier phase, \( \Delta \text{VN}_{L1,AB} \) is the code measurement noise and \( \Delta \text{VN}_{L1,AB} \) is the phase measurement noise, respectively.

For short baselines, the residual tropospheric and ionospheric effects in double differenced measurements are assumed to be negligibly small and are ignored. The double difference linear combination in (29) and (30) introduces measurement correlations and the corresponding variance-covariance matrix is as follows:
\[
R_{\text{SV}} = \begin{bmatrix}
\sigma_{i,j}^2 + \sigma_{i,j}^2 + \sigma_{i,j}^2 & \sigma_{i,j}^2 + \sigma_{i,j}^2 \\
\sigma_{i,j}^2 + \sigma_{i,j}^2 & \sigma_{i,j}^2 + \sigma_{i,j}^2 + \sigma_{i,j}^2 \\
\vdots & \vdots \\
\sigma_{i,j}^2 + \sigma_{i,j}^2 & \sigma_{i,j}^2 + \sigma_{i,j}^2
\end{bmatrix}
\]
For each epoch there are a maximum of 1 elevation angles. Measurements can accordingly be grouped based on the satellites decreases, the signal-to-noise ratio (Hofmann-Wellenhof et al., 2008). As the elevation of path length and the strength of the received signal factors that can be estimated. The elevation angle of the satellite influences the signal variances cannot be separated. Thus, (42) is written as each entry in the matrix. Therefore, the individual can only estimate the variance and covariance factors for components were grouped against satellite elevation and angles the variance does not change much. The ranges of it is assumed that for short intervals in satellite elevation components are estimated. The floated ambiguity process noise is assumed to be zero. The variance-covariance at least 120 epochs and 171 to third group starts at 583 seconds. The results show that one with the highest, average and lowest elevation angles. The estimation of the first group (Elevation 65 to 72) starts at 3806 seconds because it the time when one or more satellites enter this elevation range. Similarly the angles. The undifferenced code and phase kinematic measurements were simulated at 1Hz for 4800 seconds with an elevation mask of 10 degrees. The simulation algorithm is as follows:

The ECEF acceleration process noise variance is set at a constant value of \[ \sigma^2_E = \sigma^2_0 + 0.5 \exp(17.5/E) \] (45)
where \( \sigma^2_0 \) is the variance at the zenith and \( E \) is the satellite elevation angle in degrees. The zenith variances used for code and phase measurements are \((0.300m^2/s^2)\) and \((0.003m^2/s^2)\), respectively. In this test the initial Q for the process noise vector is diag\([0.35^2 0.35^2 0.35^2][m^2/s]\). The initial double differenced code and phase variances are \((1.200m^2)\) and \((0.012m^2)\).

Fig. 1 shows the process noise standard deviation estimation as a function of time with its true value. The results show that the process noise can be estimated with 400 epochs or less.

Fig. 2 shows the covariance estimation as a function of time together with its true value. The results show that the covariance component for the code and phase measurement can be estimated with at least 250 and 200 epochs respectively.

Fig. 3 and 4 show the standard deviation (SD) estimation of the code and phase measurements as a function of time together with its true value. Three groups are presented, one with the highest, average and lowest elevation angles. The estimation of the first group (Elevation 65 to 72) starts at 3806 seconds because it the time when one or more satellites enter this elevation range. Similarly the third group starts at 583 seconds. The results show that the algorithm takes at least 120 epochs and 171 to estimate the code and phase standard deviations.

Fig. 5 shows the estimated standard deviation for all the groups as a function of satellite elevation angles together with its corresponding initial and true value plots. The estimated components fit closely to the true curve.
Fig. 1 Process noise vs. time

Fig. 2 Covariance vs. time

Fig. 3 Selected code SD vs. time

Fig. 4 Selected phase SD vs. time

Fig. 5 Code and phase SD vs. Satellite Elevation

Fig. 6 and 7 show the position error together the estimated standard deviation for two cases; first without AKF and second with AKF.

The estimated standard deviation is more realistic with AKF. Fig. 8 and 9 show the probability density functions (PDFs) of standardized position error without and with the application of AKF together with a standardized normal distribution curve. The PDFs with the AKF algorithm fits the standardized normal distribution curve which suggests that the estimated variances for the states represent the true distribution of the states.
4.2 RESULTS FROM REAL GPS DATA

This section presents the processing results using the VC estimation algorithm on real GPS data. The data was collected on 24 April 2010 using two Leica 1200 receivers for the duration of 80 minutes. The base station was located at N43°46'26.34512", W79°30'43.23784", 158.697[m] on the Keele Campus of York University, Toronto, Ontario, Canada. The rover trajectory is shown in fig. 10 with its velocity profile as in fig. 11. Both receivers were set at sampling rate of 1.0 Hz and with an elevation mask of 10 degrees. The maximum baseline length and velocity are 3.5 km and 23 m/s respectively. POSGNSS (Waypoint® Software) was used to generate a reference solution (the ‘true’ rover position).

In this test the initial Q for the process noise vector is diag\([0.75^2 \ 0.75^2 \ 0.75^2 \ 2] \text{[m}^2\text{s}^{-4}]\). The initial double differenced code and phase variances are \((0.800 \text{m})^2\) and \((0.016 \text{m})^2\).

Figure 8 PDFs of standardized position error using initial variances (i.e. without AKF)

Fig. 9 PDFs of standardized position error with AKF

Fig. 10 Kinematic dataset trajectory

Fig. 11 Speed vs. time
stabilizes after 210 epochs while the one for the phase measurement takes significantly longer to be convergent.

Fig. 12 Process noise vs. time

Fig. 13 Covariance vs. time

Fig. 14 Selected code SD vs. time

Fig. 15 Selected phase SD vs. time

Fig. 16 Code and phase SD vs. Satellite Elevation

Fig. 17 and 18 show the estimated standard deviation of the code and phase measurements as a function of time together. Three measurement groups are presented, one with the highest, average and lowest elevation angles.

Fig. 16 shows the estimated standard deviation for all the groups as a function of satellite elevation angles together with its initial standard deviations. Fig. 17 and 18 show the position errors together with the corresponding standard deviation before and after the application of the AKF algorithm. The reference solution contains errors as it comes from POSGNSS commercial software.

Fig. 19 and 20 show the probability density functions (PDFs) of standardized position error without and with the application of AKF together with a standardized normal distribution curve.
5. REMARKS AND CONCLUSIONS

The paper presents an adaptive Kalman filtering algorithm based on variance and covariance component estimation. The distinction of the proposed AKF comparing with others in the literature lies in its capability of the simultaneous estimation of the variance and covariance factors for both of the measurement vector and the process noise vector. The results from simulated and real datasets showed the feasibility, efficiency and practicality of the algorithm. Furthermore the estimated variance and covariance factors possess good convergence. The authors will further apply this AKF to the integrated navigation and study how the system quality control can benefit from it.

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7. REFERENCE:


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