SC/MATH 1019B - HOMEWORK 1
DUE SEPTEMBER 20, 2018

Solutions to the problems below must be brought to class on September 20, 2018. Solutions may by typed or neatly hand written. You must clearly indicate which problem you are solving. All solutions must be fully justified.

 # 1. Show that the conditional statement $(p \wedge q) \to (p \vee q)$ is a tautology both

(a) by using truth tables,
(b) and by applying a chain of logic identities.

In the truth table

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

we see that the last column which corresponds to $(p \wedge q) \to (p \vee q)$ contains only T. Therefore $(p \wedge q) \to (p \vee q)$ is a tautology.

Alternatively, the chain of logical equivalences

$$\begin{aligned}(p \wedge q) \to (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q)\\ &\equiv (\neg p \vee \neg q) \vee (p \vee q)\\ &\equiv \neg p \vee \neg q \vee p \vee q\\ &\equiv (\neg p \vee p) \vee (\neg q \vee q)\\ &\equiv T \vee T\\ &\equiv T\end{aligned}$$

also demonstrates that $(p \wedge q) \to (p \vee q)$ is a tautology.

 # 2. Find the negation of the quantified statement

$$\exists x, \forall y \Big(\big((x < y) \vee (x > 2)\big) \to (xy = 2) \Big)$$

where the symbol $\neg$ does not appear in the final answer.

$$\begin{aligned}\neg\left(\exists x, \forall y \Big(\big((x < y) \vee (x > 2)\big) \to (xy = 2) \Big)\right) &\equiv \forall x, \exists y \neg \Big(\big((x < y) \vee (x > 2)\big) \to (xy = 2) \Big)\\ &\equiv \forall x, \exists y \neg \Big(\neg\big((x < y) \vee (x > 2)\big) \vee (xy = 2) \Big)\\ &\equiv \forall x, \exists y \Big(\big((x < y) \vee (x > 2)\big) \wedge (xy \neq 2) \Big)\end{aligned}$$

 # 3. Give an example of a statement $P(x, y, z)$, where the domain for each of the variables x, y, and z consists of all real numbers, so that

(i) $\forall x, \forall y, \exists z P(x, y, z)$ is true,

(ii) but $\forall x, \exists y, \forall z P(x, y, z)$ is false.

You must prove the example you give satisfies (i) and (ii).

We can take $P(x, y, z)$ to be the statement that $x + y + z = 0$. To prove that $\forall x, \forall y, \exists z P(x, y, z)$ we take arbitrary real numbers x and y, then let $z = -x - y$. We see that $x+y+z = 0$ and we have demonstrated that $\forall x, \forall y, \exists z P(x, y, z)$ is true.

To show that $\forall x, \exists y, \forall z P(x, y, z)$ is false we can show that negation is true. That is, we show that $\exists x, \forall y, \exists z \neg P(x, y, z)$ is true. To do this we can take $x = 0$, then for any arbitrary real number y we must find a z such that $y + z \neq 0$. It works to take $z = -y + 1$.