SC/MATH 1019B - HOMEWORK 2
DUE OCTOBER 23, 2018

Solutions to the problems below must be brought to class on October,23 2018. Solutions may by typed or neatly hand written. You must clearly indicate which problem you are solving. All solutions must be fully justified.

 # 1.

(a) Given an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is injective but not surjective.

(a) Given an example of a function $g : \mathbb{N} \to \mathbb{N}$ which is surjective but not injective.

An example of an injective but not surjective function is $f : \mathbb{N} \to \mathbb{N}$ by $f(n) = n+5$. If $f(n_1) = f(n_2)$, then $n_1 + 5 = n_2 + 5$ and so $n_1 = n_2$. Therefore f is injective. To see that f is not surjective consider $1 \in \mathbb{N}$ (the codomain). If $f(n) = 1$, then $n + 5 = 1$ and $n = -4$. However, $-4 \notin \mathbb{N}$ (the domain) so f is not surjective.

An example of a surjective but not injective function is $g : \mathbb{N} \to \mathbb{N}$ by $g(n) = \lfloor n/2 \rfloor$. Indeed for any $n \in \mathbb{N}$ (the codomain) take $2n \in \mathbb{N}$ (the domain) and $g(2n) = n$. The function g is not injective since $g(2) = 1 = g(3)$ but $2 \neq 3$.

 # 2. Let A, B, and C be nonempty sets. Let $f : A \to B$ and $g : B \to C$ be functions. Show that if $g \circ f$ is one-to-one, then f must be one-to-one. Is it true that g must also be one-to-one?

Let us use proof by contrapositive. Assume that f is not one-to-one. Then there exists $a_1, a_2 \in A$ with $a_1 \neq a_2$ such that $f(a_1) = f(a_2)$. This means

$$(g \circ f)(a_1) = g(f(a_1)) = g(f(a_2)) = (g \circ f)(a_2)$$

and $g \circ f$ is not one-to-one.

To see an example where $g \circ f$ is one-to-one, but that g is not one-to-one tkae $A = \{0\}, B = \mathbb{Z}$, and $C = \{0\}$ with $f(0) = 0$ and $g(n) = 0$ for $n \in \mathbb{Z}$. Then g is not one-to-one since $g(0) = 0 = g(1)$, but $g \circ f : \{0\} \to \{0\}$ is one-to-one.

 # 3. Solve the recurrence relation given by $a_1 = 2$ and $a_n = 2na_{n-1}$ for $n > 1$.

We claim that $a_n = 2^n n!$. Indeed $a_1 = 2 = 2^1 1!$. Assume that $a_n = 2^n n!$ for some $n \geq 1$, then

$$a_{n+1} = 2(n+1)a_n = 2n(2^n n!) = 2^{n+1}(n+1)!.$$