SC/MATH 1019B - HOMEWORK 3
DUE NOVEMBER 20, 2018

Solutions to the problems below must be brought to class on November 20, 2018. Solutions may by typed or neatly hand written. You must clearly indicate which problem you are solving. All solutions must be fully justified.

 # 1. Let $\Sigma = \{a, b, c\}$. Find a recurrence for the number of length n strings in Σ^* that do not contain any two consecutive a's, b's, nor c's (i.e. none of aa, bb, nor cc are in the string).
Let s_n denote the number of string which do not contain any two consecutive a's, b's, nor c's. For any such string of length $n-1$

- if it ends in an a, I can add either b or c;
- if it ends in a b, I can add either a or c;
- if it ends in a c, I can add either a or c;

Thus any string of length $n-1$ can be extended to a string of length n in two possible ways and so $a_n = 2a_{n-1}$.

 # 2. Solve the recurrence $a_n = 5a_{n-1} - 6a_{n-2}$ where $a_0 = 2$ and $a_1 = 5$.
The characteristic equation is $r^2 - 5r + 6 = (r-2)(r-3)$. So, the solution of the recurrence is

$$A2^n + B3^n$$

for some constants A and B. Using the initial conditions we find that

$$A + B = 2$$
$$2A + 3B = 5$$

and so $A = 1$ and $B = 1$. Therefore $a_n = 2^n + 3^n$.

 # 3. Consider the relation R on $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ defined by

$$R = \{((a, b), (c, d)) : ad = bc\}.$$

Prove that R is an equivalence relation. Does the relation R have any meaning to you?
Since $ab = ba$ for any $(a, b) \in \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ we find that $((a, b), (a, b)) \in R$ and R is reflexive.
Assuming that $((a, b), (c, d)) \in R$ we have that $ad = bc$. This also means $cb = da$ and so $((c, d), (a, b)) \in R$ and R is symmetric.
Assuming that $((a, b), (c, d)) \in R$ and $((c, d), (e, f)) \in R$ we have that

$$ad = bc \qquad\qquad cf = de.$$

Since $b \neq 0$, $d \neq 0$, and $f \neq 0$ we see that

$$\frac{a}{b} = \frac{c}{d} \qquad\qquad \frac{c}{d} = \frac{e}{f}.$$

so then $a/b = e/f$. This means the $af = be$ and so $((a,b),(e,f)) \in R$. Therefore R and transitive and we have shown its an equivalence relation.