$\{X_i\}$ partition a set X if

(1.) $\bigcup_i X_i = X$ (2.) $X_i \cap X_j = \emptyset$ for $i \neq j$

$\boxed{\text{Theorem}}$ If R is an equivalence relation on A, then the equivalence classes partition A.

Pf) For any $a \in A$ we know that $a \in [a]$ because $(a,a) \in R$ since R is reflexive. This means A is equal to the union of the equivalence classes

By the previous theorem for $a,b \in A$ either $[a]=[b]$ or else $[a] \cap [b] = \emptyset$. That means distinct equivalence classes are disjoint. Therefore the equivalence class form a partition.

$(\Sigma = \{0,1\})$

Let Σ be a finite set and consider the following relations on Σ^*

$R_1 = \{(x,y) \in \Sigma^* \times \Sigma^* : \ell(x) = \ell(y)\}$

$R_2 = \{(x,y) \in \Sigma^* \times \Sigma^* : y = xz \text{ for some } z \in \Sigma^*\}$

$R_3 = \{(x,y) \in \Sigma^* \times \Sigma^* : y = xa \text{ for some } a \in \Sigma\}$

Determine whether or not each relation is

(i) reflexive

(ii) symmetric

(iii) transitive

R_1 is reflexive, symmetric, transitive

$01 \in \Sigma^*$ $[01] = \{01, 10, 00, 11\}$ $(01, 11) \in R_1$

R_2 reflexive , NOT symmetric

$(x,x) \in R_2$ $x = x\lambda$

$(0,01) \in R_2$

$(01,0) \notin R_2$

transitive

R_3 NOT reflexive, NOT symmetric, transitive

R_2 is transitive

$(u,v) \in R_2 \quad (v,w) \in R_2$

$v = uz \qquad w = v\tilde{z} \qquad z, \tilde{z} \in \Sigma^*$

$w = u z \tilde{z}$

$(00, 0011) \in R_2 \qquad (0011, 001101) \in R_2$

$(00, 001101) \in R_2$

$u = 00$

$v = 0011 \qquad z = 11$

$w = 001101 \qquad \tilde{z} = 01$

Find all possible partitions of the sets
$A = \{1,2,3\}$ and $B = \{1,2,3,4\}$

Ex) $\{\{1,3\},\{2\}\}$

$\{\{1\},\{2\},\{3\}\}$

$\{\{1,2\},\{3\}\}$

$\{\{1,3\},\{2\}\}$

$\{\{2,3\},\{1\}\}$

$\{\{1,2,3\}\}$

$\{\{1\},\{2\},\{3\},\{4\}\}$

$\{\{1,2\},\{3\},\{4\}\}$ $\{\{1\}\{2\},\{3,4\}\}$

$\{\{1,2\},\{3,4\}\}$

$\{\{1,3\},\{2\},\{4\}\}$ $\{\{1\},\{3\},\{2,4\}\}$

$\{\{1,3\},\{2,4\}\}$

$\{\{1,4\},\{2\},\{3\}\}$ $\{\{1\},\{4\},\{2,3\}\}$

$\{\{1,4\},\{2,3\}\}$

$\{\{1,2,3\},\{4\}\}$

$\{\{1,2,4\},\{3\}\}$

$\{\{1,3,4\},\{2\}\}$

$\{\{2,3,4\},\{1\}\}$

$\{\{1,2,3,4\}\}$