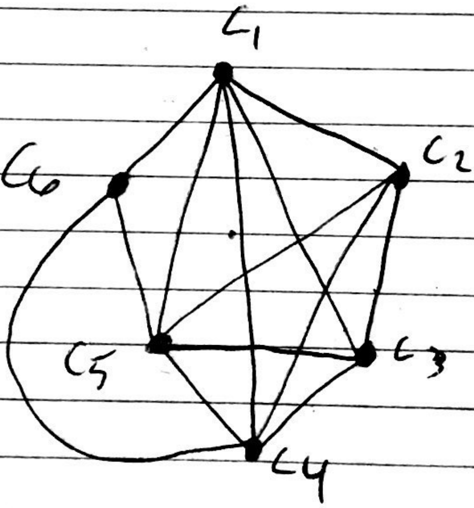
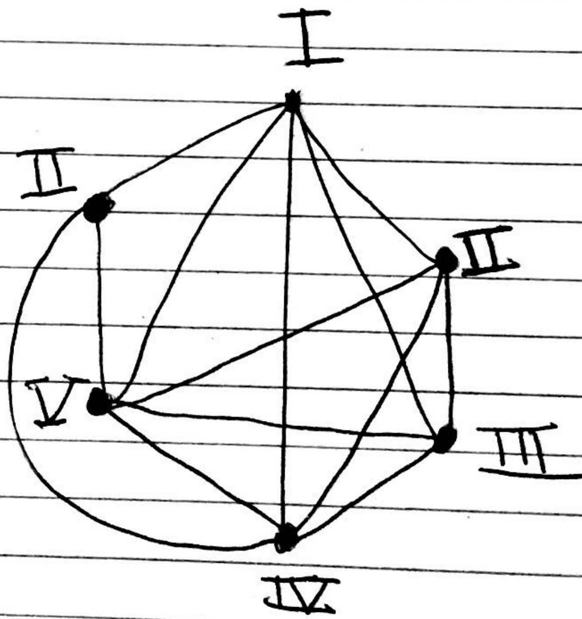
Graph Coloring

A company has n comittees $C_1, C_2, \ldots, C_n$ how can the company schedule meeting times for each comittee so that no employee has to be in two (or more) meetings at the same time?

Committee	Employees
C_1	Alice, Bob, Chuck
C_2	Bob, Don, Evan
C_3	Alice, Bob, Don
C_4	Chuck, Frank, Gina, Don
C_5	Alice, Gina, Evan
C_6	Chuck, Gina, Mike





A proper coloring of a graph is an assignment of color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

The chromatic number of a graph is the least number of colors needed for a proper coloring. Denoted $\chi(G)$.

Some graphs

$K_n = (V, E)$ where $n \geq 1$ is an integer

$$V = \{1, 2, \ldots, n\}$$

$$E = \{\{i, j\} : 1 \leq i < j \leq n\}$$

Find the chromatic number of K_n.

$K_{n,m} = (V, E)$ where $n, m \geq 1$ are integers

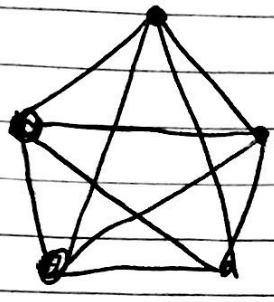
$$V = \{a_1, a_2, \ldots, a_n\} \cup \{b_1, b_2, \ldots, b_m\}$$

$$E = \{\{a_i, b_j\} : 1 \leq i \leq n, 1 \leq j \leq m$$

Find the chromatic number of $K_{n,m}$.

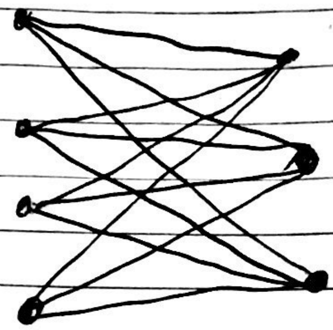
$\chi(K_n) = n$

$\chi(K_{n,m}) = 2$



K_5

Complete graph



$K_{4,3}$

Complete bipartite graph

Scanned with CamScanner

6 people are at a party. For any pair of 2 people they are either

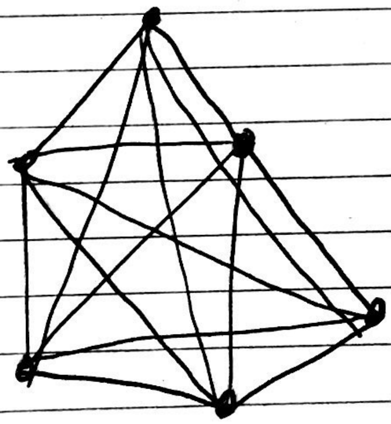
- strangers, or
- acquaintances

Show at least 3 people must be mutual strangers or mutual acquaintances

That is there is a group of 3 out of the 6 ~~[illegible]~~ such that either

- None of them know eachother, or
- They all know each other.

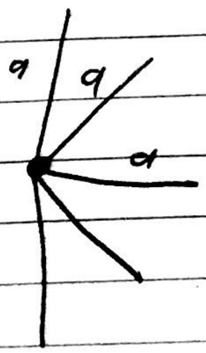
Proof)

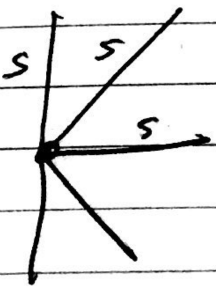


K_6

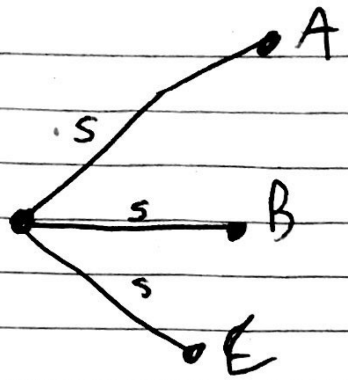
each edge represents a pair.

Color each edge 'a' for acquaintance or 's' for stranger.

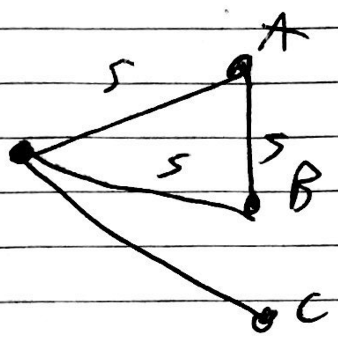




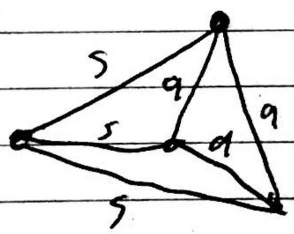
any person has at least 3 s's or at least 3 a's



If any one of A, B, C is a stranger with another one of A, B, C we have 3 mutual strangers



Otherwise A, B, and C are 3 mutual acquaintances



A **edge coloring** is an assignment of colors to each edge of the graph.

we just showed any edge coloring of K_6 with 2 colors will have a triangle all the same color

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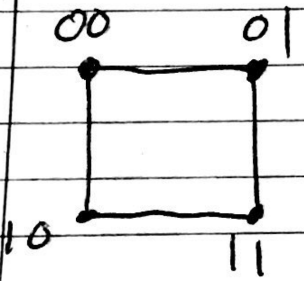
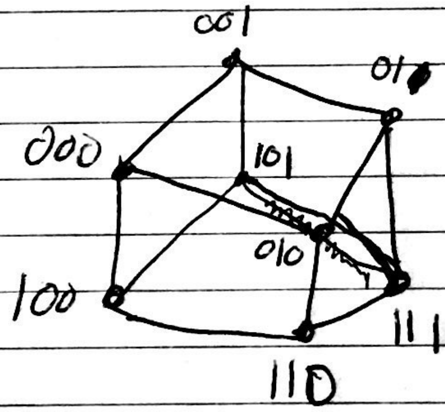
$Q_n = (V, E)$

$V = \{ w \in \{0,1\}^* : l(w) = n \}$ length n bit strings

$E = \{ \{w_1, w_2\} : w_1, w_2 \in \{0,1\}^n$ such that differ is one place $\}$



Q_2



Q_3

Find $\chi(Q_n)$.