5.3 #40

Recursively define the set of bit strings that are palindromes.

A palindrome is a bit string which is the same forwards and backwards

Examples: 000, 01010, 01110

Non-examples: 01, 0011, 010101

Let P be the set of palindromes

(1) $\lambda \in P$, $0 \in P$, $1 \in P$

(2) If $w \in P$, then $0w0 \in P$ and $1w1 \in P$

8.1 #16

Find a recurrence relation for the number of ternary strings of length n that contain either two consecutive 0's or two consecutive 1s

$\Sigma = \{0, 1, 2\}$

$\boxed{\text{length } n-2}\,00 \quad 3^{n-2}$

$\boxed{\text{length } n-2}\,11 \quad 3^{n-2}$

8.2 #4 b)

Solve $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2$, $a_0 = 2, a_1 = 1$

$r^2 - 7r + 10 = (r-5)(r-2)$

$a_n = A5^n + B2^n$

$a_0 = A + B = 2$ $\quad$ $3A = -3$ $\quad$ $A = -1$

$a_1 = 5A + 2B = 1$ $\quad$ $B = 3$

$a_n = (-1) \cdot 5^n + 3 \cdot 2^n$

8.3 #6

The question translates to

$\rightsquigarrow$ what is $f(32)$ if $f(n) = 7f(\frac{n}{2}) + \frac{15n^2}{4}$

when $f(1) = 1$

$f(32) = 7f(16) + \frac{15 \cdot 32^2}{4} = 95722$

$f(16) = 7f(8) + \frac{15 \cdot 16^2}{4} = 13126$

$f(8) = 7f(4) + 15 \cdot 8^2/4 = 1738$

$f(4) = 7f(2) + 15 \cdot 4^2/4 = 214$

$f(2) = 7f(1) + 15 \cdot 2^2/4 = 22$

9.1 #6 b)

Determine if the relation R on the set of real numbers is reflexive, symmetric, and/or transitive where $(x,y) \in R$ iff $x = \pm y$

Since $x = x$ for every $x \in \mathbb{R}$ we have that $(x,x) \in R$ and R is reflexive.

If $(x,y) \in R$, then $x = \pm y$. When $x = y$ then also $y = x$ so $(y,x) \in R$. When $x = -y$ then $y = -x$ so $(y,x) \in R$. Thus R is symmetric.

If $(x,y) \in R$ and $(y,z) \in R$, then $x = \pm y$ and $y = \pm z$.

If $x = y$ and $y = z$, then $x = z$ and $(x,z) \in R$
If $x = y$ and $y = -z$, then $x = -z$ and $(x,z) \in R$
If $x = -y$ and $y = z$, then $x = -z$ and $(x,z) \in R$
If $x = -y$ and $y = -z$, then $x = z$ and $(x,z) \in R$
Therefor R is transitive.

9.3 #46 c), d), and e)
Which are partitions of $\mathbb{R}$.

The set of intervals $[k, k+1]$, $k = \ldots, -2, -1, 0, 1, 2, \ldots$

The set of intervals $(k, k+1)$, $k = \ldots, -2, -1, 0, 1, 2, \ldots$

The set of intervals $(k, k+1]$, $k = \ldots, -2, -1, 0, 1, 2$

Only the last gives a partition.

The intervals $[k, k+1]$ are not disjoint

$$[1,2] \cap [2,3] = \{2\} \neq \emptyset$$

The intervals $(k, k+1)$ do not ever contain any integers

$$\bigcup_{k \in \mathbb{Z}} (k, k+1) \neq \mathbb{R}$$

In the last case

$$(k, k+1] \cap (j, j+1] = \emptyset \quad \text{when } k \neq j$$

$$\bigcup_{k \in \mathbb{Z}} (k, k+1] = \mathbb{R}$$

Define $G_n = (V, E)$ where $V = \{1, 2, \ldots, n\}$ and

$$E = \{\{i, j\} : i, j \in V \text{ and } i+j \text{ is odd}\}$$

Draw G_8 and show G_n is bipartite