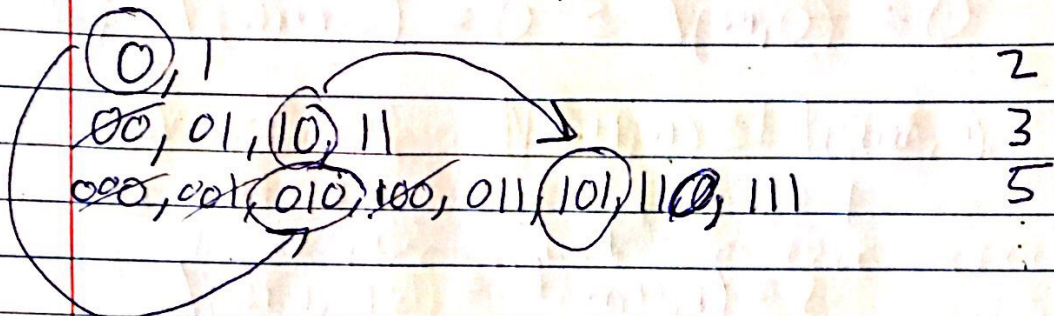


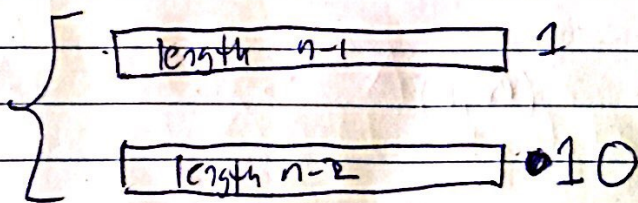
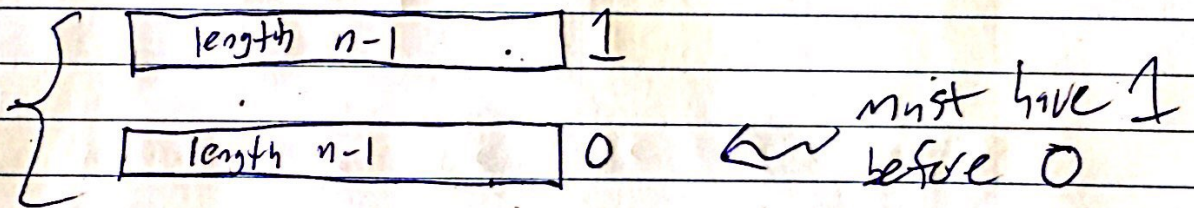
# 8.1 Applications of Recurrence Relations

Counting bit strings of length  $n$  with no two consecutive zeros



Consider length  $n$  bit string with no two consecutive zero

Either ends in 1 or ends in 0



$a_n = \#(\text{length } n \text{ bit string with no two consecutive zeros})$

$$a_n = a_{n-1} + a_{n-2} \quad \text{for } n \geq 3$$

Find a recurrence relation for the number of length  $n$  bit strings that do contain two consecutive zeros

Any length  $n-2$   $00$   $2^{n-2}$

Length  $n-2$  w/  $00$  ~~00~~  $10$   $a_{n-2}$

Length  $n-1$  w/  $00$   $1$   $a_{n-1}$

$$a_n = a_{n-1} + a_{n-2} + 2^{n-2}$$

$$a_0 = 0, a_1 = 0$$

$$a_2 = 0 + 0 + 1 = 1$$

$$a_3 = 1 + 0 + 2 = 3$$

$$a_4 = 3 + 1 + 4 = 8$$

00

000 001 100

0000 0001 0010 0011

0100 1000 1001 1100