2.2 Set Operations

Let A and B be two sets inside a universal set U. We have the following operations.

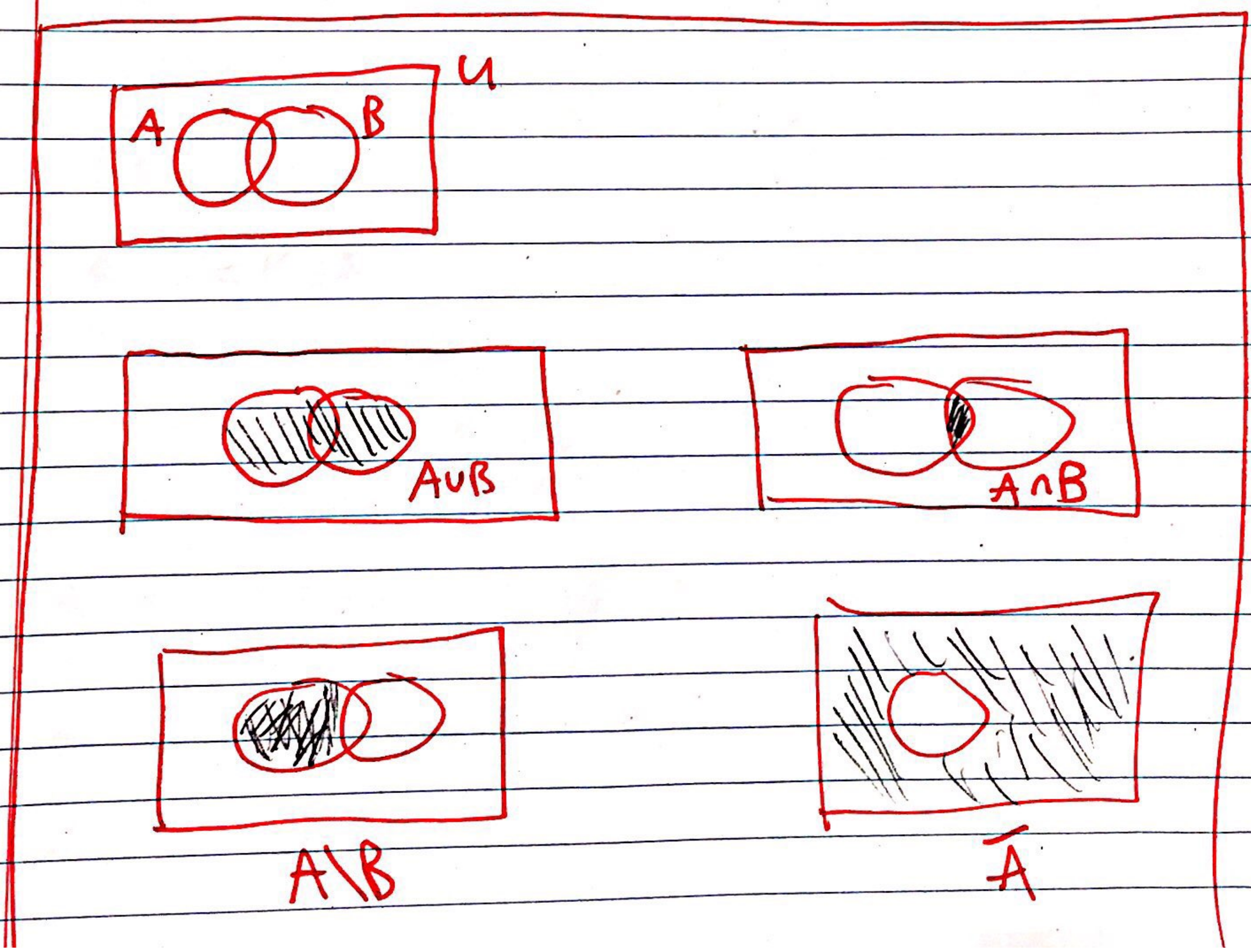
<u>Union</u> $A \cup B = \{x : x \in A \vee x \in B\}$

<u>intersection</u> $A \cap B = \{x : x \in A \wedge x \in B\}$

<u>difference</u> $A \setminus B = \{x : x \in A \wedge x \notin B\}$ $A - B$

<u>complement</u> $\bar{A} = \{x \in U : x \notin A\}$

Try drawing Venn diagrams for these operations



Two sets A and B are called disjoint if $A \cap B = \phi$.

Notice analogy between $\wedge, \vee$ and $\cap, \cup$.

Try to come up with rules for set operations mimicking our logic rules

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$A \cap U = A$ $\quad\quad$ $A \cup (A \cap B) = A$

$A \cup \phi = A$ $\quad\quad$ $A \cap (A \cup B) = A$

$A \cup U = U$ $\quad\quad$ $A \cup \overline{A} = U$

$A \cap \phi = \phi$ $\quad\quad$ $A \cap \overline{A} = \phi$

$A \cup A = A$

$A \cap A = A$

$\overline{(\overline{A})} = A$

$A \cup B = B \cup A$

$A \cap B = B \cap A$

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = A \cap (B \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

Let's prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cup (B \cap C) = \{x : x \in A \vee x \in (B \cap C)\}$$
$$= \{x : x \in A \vee (x \in B \wedge x \in C\}$$
$$= \{x : (x \in A \vee x \in B) \wedge (x \in A \vee x \in C)\}$$
$$= \{x : x \in A \cup B \wedge x \in A \cup C\}$$
$$= (A \cup B) \cap (A \cup C)$$

Try to prove $\overline{A \cap B} = \bar{A} \cup \bar{B}$

$$\overline{A \cap B} = \{x \in U : x \notin A \cap B\}$$
$$= \{x \in U : \neg(x \in A \wedge x \in B)\}$$
$$= \{x \in U : \neg(x \in A) \vee \neg(x \in B)\}$$
$$= \{x \in U : x \notin A \vee x \notin B\}$$
$$= \{x \in U : x \in \bar{A} \vee x \in \bar{B}\}$$
$$= \{x \in U : x \in \bar{A} \cup \bar{B}\}$$
$$= \bar{A} \cup \bar{B}$$

We can generalize intersections and union to more than two sets

A *union* of a collection of sets is the set whose elements are elements that are a member of at least one of the sets

An *intersection* of a collection of set is the set whose elements are elements that are members of every set in the collection

$\{1,2\} \cup \{1,2,3\} \cup \{1,2,4\} \cup \{5\} = \{1,2,3,4,5\}$

$\{1,6,7\} \cap \{2,6,7\} \cap \{3,6,9\} = \{6\}$

For $A_1 \cup A_2 \cup \cdots \cup A_n$ write $\bigcup_{i=1}^{n} A_i$

For $A_1 \cap A_2 \cap \cdots \cap A_n$ write $\bigcap_{i=1}^{n} A_i$

Let $A_i = \{1,2,3,\ldots,i\}$

$$\bigcup_{i=1}^{n} A_i = A_n = \{1,2,3,\ldots,n\}$$

$$\bigcup_{i=1}^{\infty} A_i = \{1,2,3,\ldots\} = \mathbb{Z}^+$$

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$A_5 = \{5, 6\}$

$B_i = \{i, i-1\}$

Let $A_i = \{i, i+1\}$ and $B_i = \{i-1, i\}$ for $i = 1, 2, \ldots$

Find $\bigcup_{i=1}^{\infty} A_i$, $\bigcup_{i=1}^{\infty} B_i$, $\bigcap_{i=1}^{\infty} A_i$, $\bigcap_{i=1}^{\infty} B_i$

$\bigcup_{i=1}^{\infty} A_i = \{1, 2, \cdots\} = \mathbb{Z}^+$

$\bigcup_{i=1}^{\infty} B_i = \{0, 1, 2, \cdots\} = \mathbb{N}$

$\bigcap_{i=1}^{\infty} A_i = \phi$ $\qquad$ $\bigcap_{i=1}^{\infty} B_i = \phi$

$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cdots\cdots$

$n \in \mathbb{Z}^+$ $\qquad$ $n \in A_{n-1}$ $\qquad$ $n \in A_n$

$n \notin A_{n+1}$