1.1 #38 f)

Construct truth table for $(p \wedge q) \vee \neg r$

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \vee \neg r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

1.3 #18

Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology

p	q	$\neg p$	$p \rightarrow q$	$\neg p \wedge (p \rightarrow q)$	$\neg q$	$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$
T	T	F	T	F	F	T
T	F	F	F	F	T	~~F~~ T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

NOT a tautology

$$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q \equiv (\neg p \wedge (\neg p \vee q)) \rightarrow \neg q$$
$$\equiv \neg p \rightarrow \neg q$$
$$\equiv q \rightarrow p$$
$$\equiv p \vee \neg q$$

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1.4 #16 d)
Determine the truth value of $\forall x (x^2 \neq x)$ where x is from the domain consisting of all real numbers

This statement is FALSE. Take $x=0$ (or $x=1$) as a counterexample.

Note the negation of the statement is $\exists x (x^2 = x)$ which is TRUE by taking $x=0$ (or $x=1$)

1.5 #30 d)
Express $\neg \exists y (\exists x R(x,y) \vee \forall x S(x,y))$ so that no negation is outside a quantifier

$$\neg \exists y (\exists x R(x,y) \vee \forall x S(x,y))$$

$$\equiv \forall y \neg (\exists x R(x,y) \vee \forall x S(x,y))$$

$$\equiv \forall y (\neg \exists x R(x,y) \wedge \neg \forall x S(x,y))$$

$$\equiv \forall (\forall x \neg R(x,y) \wedge \exists x \neg S(x,y))$$

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1.6 #24 a)
Identify the error

1. $\forall x\,(P(x) \vee Q(x))$ Premise
2. $P(c) \vee Q(c)$
3. $P(c)$
4. $\forall x\,P(x)$
5. $Q(c)$
6. $\forall x\,Q(x)$
7. $\forall x\,(P(x) \vee \forall x\,Q(x))$

1.7 #16
Prove that if $x, y,$ and z are integers and $x+y+z$ is odd, then at least one of x, y, or z is odd.

Proof by contrapositive
Assume that not at least one of $x, y,$ or z is odd. That means $x, y,$ and z are all even. So $x = 2k$, $y = 2\ell$, and $z = 2m$ for some $k, \ell, m \in \mathbb{Z}$. Then

$$x+y+z = 2k+2\ell+2m = 2(k+\ell+m)$$

and $x+y+z$ is even.

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1.8 #32

Prove that there are no integer solutions to

$$2x^2+5y^2=14$$

Since $x^2=(-x)^2$ and $y^2=(-y)$ we only need to consider nonnegative integers. Furthermore if $x\geq 3$, then $2x^2 \geq 2\cdot 3^2 = 18 > 14$ and if $y \geq 2$, then $5y^2 \geq 5\cdot 2^2 = 40 > 14$. Thus we only need to consider $0 \leq x \leq 2$ and $0 \leq y \leq 1$ we check

$$2\cdot 0^2+5\cdot 0^2 = 0 \neq 14$$
$$2\cdot 0^2+5\cdot 1^2 = 5 \neq 14$$
$$2\cdot 1^2+5\cdot 0^2 = 2 \neq 14$$
$$2\cdot 1^2+5\cdot 1^2 = 7 \neq 14$$
$$2\cdot 2^2+5\cdot 0^2 = 8 \neq 14$$
$$2\cdot 2^2+5\cdot 1^2 = 13 \neq 14$$

and therefore there are no solutions.

2.1 #28

Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$.

Take an arbitrary element $(a,b) \in A \times B$. This means $a \in A$ and $b \in B$. Because $A \subseteq C$ we can say that $a \in C$. Similarly since $B \subseteq D$ we conclude that $b \in D$. It follows that $(a,b) \in C \times D$ and therefor $A \times B \subseteq C \times D$.

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2.2 #32 b)
Can you conclude that $A = B$ if A, B, and C are sets such that $A \cap C = B \cap C$?

No, take $C = \emptyset$. Then $A \cap \emptyset = \emptyset = B \cap \emptyset$ but A and B can be any set.

An explicit counter example is $A = \{1, 2\}$, $B = \{3, 4\}$ and $C = \emptyset$.

Also not C does not have to be empty we could also take $A = \{1, 2\}$, $B = \{1, 3\}$ and $C = \{1\}$. Then

$$A \cap C = \{1\} = B \cap \{1\}$$

but $\{1, 2\} = A \neq B = \{1, 3\}$

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