Name: ______________________________

YorkU email: ______________________________

Student Number: ____________________

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- No calculators, cell phones or any other electronic devices can be used on this exam.
- Clear your desk of everything excepts pens, pencils and erasers.
- If you need scratch paper, the last page is blank.
- Without fully opening the exam, check that you have pages 1 through 10.
- Fill in your name, etc. on this first page.
- **Show all your work unless otherwise indicated.** Write your answers clearly! Include enough steps for the grader to be able to follow your work.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- You will be given exactly 60 minutes for this exam.

I have read and understand the above instructions: ______________________________

SIGNATURE

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

1. (5 points) The compound proposition $(p \vee q) \wedge (p \rightarrow (\neg q))$ is:
   - **A. Satisfiable.**
   - B. Unsatisfiable.
2. (5 points) The negation of $\exists x \exists y(x + y = 7)$ is:
   - A. $\forall x \exists y(x + 7 = 0)$
   - B. $\exists x \exists y(x + y \neq 7)$
   - **C.** $\forall x \forall y(x + y \neq 7)$
3. (5 points) A proposition logically equivalent to $p \rightarrow q$ is:
   - A. $\neg p \rightarrow \neg q$
   - B. $q \rightarrow p$
   - **C.** $\neg q \rightarrow \neg p$

Extra Work Space.

Fill in the Blanks. No work needed. No partial credit available.

4. (5 points) For $B = \{2, y\}$ the powerset is

$$\mathcal{P}(A) = \{\emptyset, \{2\}, \{y\}, \{2, y\}\} \underline{\qquad\qquad}.$$

5. (5 points) When x and y are taken for the domain consisting of all integers, the truth value of $\exists x \forall y(xy = 1)$ is False_________.

6. (5 points) The intersection $(1, 10) \cap [2, 4] \cap [3, 5]$ expressed as a single interval is $[3, 4]$_________.

Extra Work Space.

Standard Response Questions. Show all work to receive credit.

7. (10 points) Construct a truth table for the compound proposition $(\neg p) \to (p \vee q \vee r)$.

Solution:

p	q	r	$\neg p$	$p \vee q \vee r$	$(\neg p) \to (p \vee q \vee r)$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	F	F

8. (15 points) Show using a chain of logical equivalences that $\neg(p \vee r \vee (\neg p \wedge q))$ is equivalent to $\neg p \wedge \neg q \wedge \neg r$.

Solution:

$$\begin{aligned}
\neg(p \vee r \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg r \wedge \neg(\neg p \wedge q) \\
&\equiv \neg p \wedge \neg r \wedge (p \vee \neg q) \\
&\equiv (\neg p \wedge \neg r \wedge p) \vee (\neg p \wedge \neg r \wedge \neg q) \\
&\equiv F \vee (\neg p \wedge \neg r \wedge \neg q) \\
&\equiv \neg p \wedge \neg r \wedge \neg q \\
&\equiv \neg p \wedge \neg q \wedge \neg r
\end{aligned}$$

9. (20 points) Prove that $3n^2 + 3$ is even if and only if n is an odd integer.

Solution: First we show that if n is odd, then $3n^2 + 3$ is even. Suppose n is odd, then write $n = 2k + 1$ for some integer k. We then find that

$$\begin{aligned} 3n^2 + 3 &= 3(2k+1)^2 + 3 \\ &= 12k^2 + 12k + 3 + 3 \\ &= 12k^2 + 12k + 6 \\ &= 2(6k^2 + 6k + 3) \end{aligned}$$

and can conclude that $3n^2 + 3$ is even.

We will demonstrate that $3n^2 + 3$ is even implies that n is odd by proving the contrapositive. Assume n is even. So, we write $n = 2k$ for an integer k. In this case

$$\begin{aligned} 3n^2 + 3 &= 3(2k)^2 + 3 \\ &= 12k^2 + 3 \\ &= 12k^2 + 2 + 1 \\ &= 2(6k^2 + 1) + 1 \end{aligned}$$

and it follows that $3n^2 + 3$ is odd.

10. (10 points) Provide examples of two finite sets S and T so that $|\mathcal{P}(S)| = 16$ and $|S \times T| = 16$. List all members of set $S \times T$ for your chosen sets S and T.

Solution: One possible solution is $S = \{x, y, z, w\}$ and $T = \{1, 2, 3, 4\}$. The fact that $|\mathcal{P}(S)| = 16$ means $|S| = 4$ and $|S \times T| = 16$ means $|S||T| = 16$. In this case the members of $S \times T$ are:

$(x, 1)$	$(y, 1)$	$(z, 1)$	$(w, 1)$
$(x, 2)$	$(y, 2)$	$(z, 2)$	$(w, 2)$
$(x, 3)$	$(y, 3)$	$(z, 3)$	$(w, 3)$
$(x, 4)$	$(y, 4)$	$(z, 4)$	$(w, 4)$

11. (15 points) Let x and y be from the domain consisting of all rational numbers. Prove that the statement $\exists! x, \forall y (xy \neq 1)$ is true.

Solution: We first see when $x = 0$, for any $y \in \mathbb{Q}$ we have that $xy = 0 \cdot y = 0 \neq 1$. Thus we can conclude that $\exists x, \forall y (xy \neq 1)$ is true. We must now show the uniqueness.

Now take any $x \in \mathbb{Q}$ with $x \neq 0$. This means $x = \frac{a}{b}$ for $a, b \in \mathbb{Z}$ with both $a \neq 0$ and $b \neq 0$ (by the definition of a rational number $b \neq 0$, and $a \neq 0$ because $x \neq 0$). For such x we can then take $y = \frac{b}{a}$ which is also a rational number (hence in our domain) and see that $xy = 1$. Therefore we can conclude that $\exists! x, \forall y (xy \neq 1)$ is true as desired.

Extra Work Space.

Extra Work Space.