Constant-acceleration kinematics

From free-fall observations (or better: Atwood machine, etc.):

$$\Delta x = x - x_0 \sim c\,t^2 \quad \text{if the particle started at rest}$$

c ↑ $\frac{1}{2} g$ to be more precise, or $\frac{1}{2} a$ for AM

For constant-velocity motion $(a = 0,\ v = v_0)$

$$\Delta x = x - x_0 \sim v_0 t$$

$\therefore$ combined case (particle is moving with v_0, then acc.)

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

Derive the velocity:

$$v(t) = \frac{dx}{dt} = v_0 + at$$

The acceleration:

$$\frac{dv}{dt} = a$$

This allows to solve many problems, but it is only a limited class of problems

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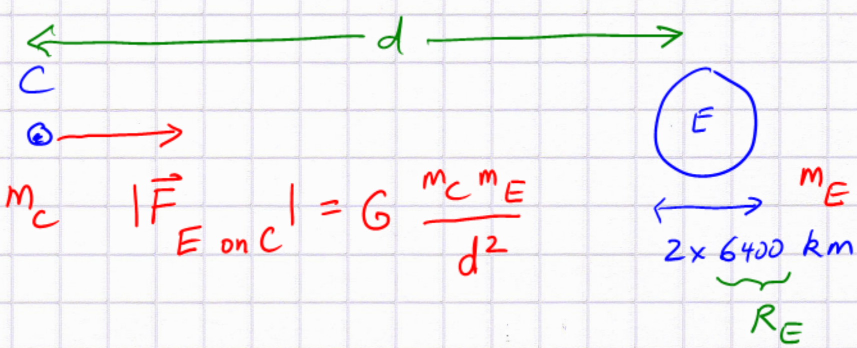
For a comet approaching earth this would not work:



$$|\vec{F}_{E\,\mathrm{on}\,C}| = G\,\frac{m_C m_E}{d^2}$$

As d decreases, $a_C = \dfrac{|\vec{F}_{E\,\mathrm{on}\,C}|}{m_C}$ increases! (not constant)

$$g = G\,\frac{m_E}{R_E^2} \approx 9.8\ \frac{m}{s^2}$$

works when fall distance $\Delta x \ll R_E$!

Space science: $g = g(h)$ accounts for this

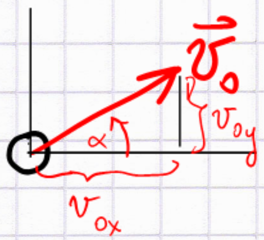
The equations for $x(t)$ and $v(t)$ can be combined to eliminate time to solve problems such as:

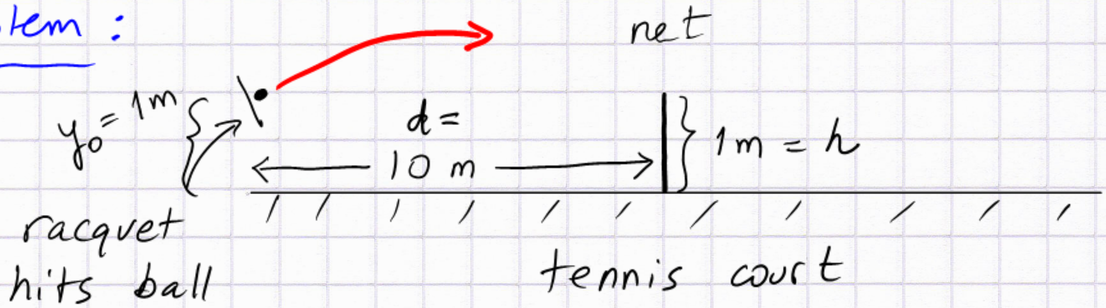
Given a constant-acc. motion for an object changing $v = v_0$ into $v = v_{fin}$ over distance Δx

$$(3.4) \qquad \boxed{v_f^2 = v_0^2 + 2 a \Delta x}$$

Here Δx is the displacement (during some time interval which was eliminated), and it can be large.

Challenge problem:





$\cos\alpha = \frac{v_{0x}}{v_0}$, $\sin\alpha = \frac{v_{0y}}{v_0}$

Will the ball make it over the net?

projectile motion:

$$\boxed{\begin{aligned} v_x(t) &= v_{0x} = v_0 \cos\alpha \\ v_y(t) &= v_{0y} - gt = v_0 \sin\alpha - gt \end{aligned}}$$

$$x(t) = \underbrace{x_0}_{=0} + v_{0x} t$$

$$y(t) = \underbrace{y_0}_{=1m} + v_{0y} t - \frac{1}{2} g t^2$$

want to know: how high is the ball at time when $x(t_1) = 10m$?

$\therefore\ t_1 = \frac{d}{v_{0x}}$ $\quad (d = 10m)$

Then $y(t_1) = 1 + v_{0y} \frac{d}{v_{0x}} - \frac{1}{2} g \left(\frac{d}{v_{0x}}\right)^2$ (in SI!)

given d, v_0, α, g this can be computed.