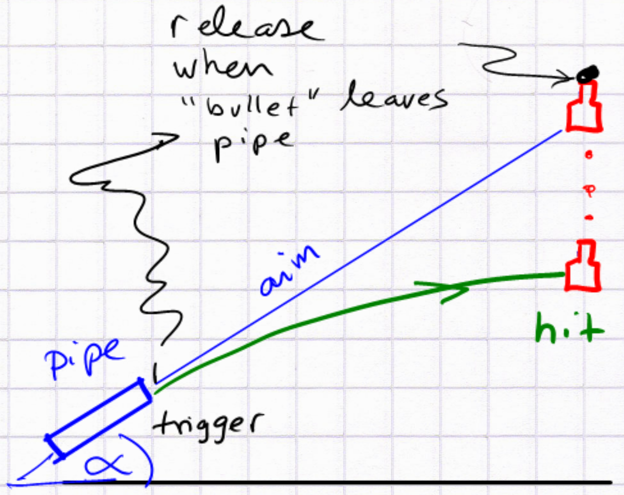
Projectile motion



Understand the problem:

a pipe shoots a dowel at an object that is released to fall under gravity just as the dowel leaves the pipe. the dowel hits the object IRRESPECTIVE of the muzzle speed!

2 motions to describe:

bottle : $y_B(t) = y_0 - \frac{1}{2} g t^2$

Dowel (bullet) = projectile → 2d motion

$\vec{v}_0 = v_{0,x}\,\hat{i} + v_{0y}\,\hat{j}$ $\qquad v_{0,x} = v_0 \cos\alpha \;;\; v_{0y} = v_0 \sin\alpha$

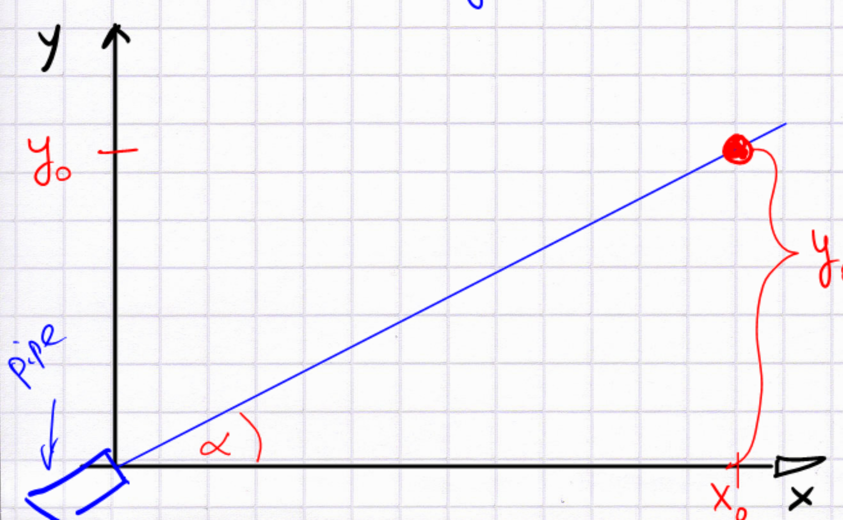
(initial condition ; v_0 = muzzle speed → depends on blow

$\hat{i}$ motion : $x_D = v_{0,x}\, t$

$\hat{j}$ motion : $y_D = 0 + v_{0,y}\, t - \frac{1}{2} g t^2$

$t=0$: common start point for both falling bottle + released dowel.

Careful drawing:



$\frac{y_0}{x_0} = \tan\alpha$

travel time ?

$x_0 = v_{0x}\, t_{hit} = v_0 \cos\alpha\; t_{hit}$

$\therefore \quad \boxed{t_{hit} = \frac{x_0}{v_0 \cos\alpha}}$

Question: • for the time t_{hit} [determined by the x-motion of dowel]

how far has the bottle dropped in height?

A:
$$y_B(t_{hit}) = y_0 - \frac{1}{2} g t_{hit}^2$$
$$= y_0 - \frac{1}{2} g \frac{x_0^2}{v_0^2 \cos^2\alpha}$$

Question: which height has the dowel reached at this time?

A:
$$y_D(t_{hit}) = v_{0y} t_{hit} - \frac{1}{2} g t_{hit}^2$$
$$= (v_0 \sin\alpha) \frac{x_0}{v_0 \cos\alpha} - \frac{1}{2} g \frac{x_0^2}{v_0^2 \cos^2\alpha}$$
$$= x_0 \tan\alpha - \frac{1}{2} g \frac{x_0^2}{v_0^2 \cos^2\alpha}$$

So, is it true that $y_D(t_{hit}) = y_B(t_{hit})$?

Yes, since $\tan\alpha = \frac{y_0}{x_0}$ and $y_D(t_{hit}) = y_0 - \frac{g}{2} \frac{x_0^2}{v_0^2 \cos^2\alpha}$
$$= y_B(t_{hit})$$

Thus, irrespective of the muzzle speed v_0:

- the dowel will hit the bottle.
- for smaller v_0 it will hit a lower height, since the time of flight t_{hit} is longer!