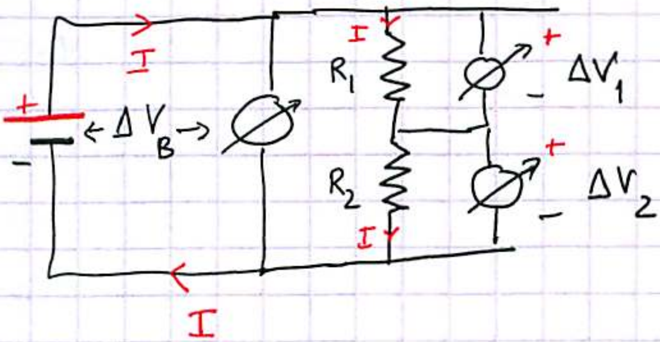
Kirchhoff's rules

We learned two important relations. For a capacitor the voltage drop $\Delta V_C = \frac{Q}{C}$ and for a resistor $\Delta V_R = R I$.

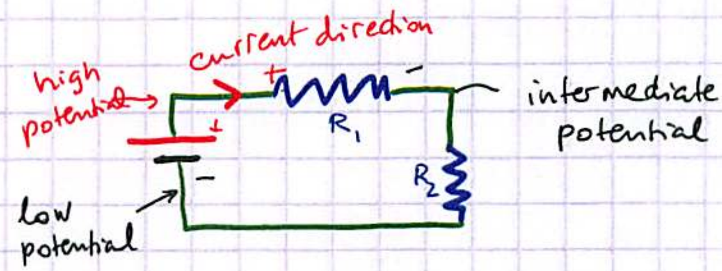
For a series combination of capacitors or resistors we learned the additivity rule for voltages



$$\Delta V_B = \Delta V_1 + \Delta V_2$$

Ohm's law (sloppy version) $\Delta V_R = R I$ (e.g. $\Delta V_1 = R_1 I$)

careful version: a loop



- the potential drops along the resistor: given a current direction, $\Delta V \mathrel{\hat{=}}$ from high to low
- with signs done carefully: $\Delta V = -RI$

voltages add up to zero: $\Delta V_B + \Delta V_1 + \Delta V_2 = 0$

$\boxed{\text{(1) Kirchhoff Loop Rule}}$

$$\Delta V_B - R_1 I - R_2 I = 0$$

$$\therefore \quad \Delta V_B = (R_1 + R_2) I$$

Here the battery EMF ($\mathcal{E} = \Delta V_B$) raises the charges from Low to High

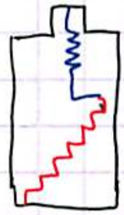
Dead battery?

When a battery gets old (or a rechargeable one is low)

we observe: no load → $\Delta V_B = \mathcal{E}$ = nominal value.

apply a load + measure voltage → it drops. WHY?

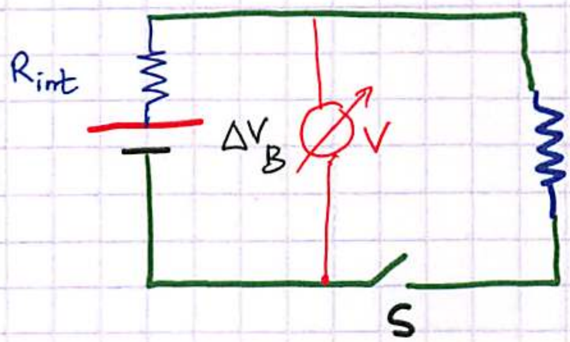
Real battery:



← internal resistance (in series)

$\mathcal{E}$ = EMF escalator, raises charges by $\mathcal{E} = \Delta V_B$

circuit:



Before S is closed: Voltmeter is very good, requires almost no current to measure → voltage drop across R_{int} is negligible → ΔV_B is displayed!

When S is closed: a current I flows. a voltage drop occurs across R_{int}. ⇒ $I = \frac{\Delta V_B}{R + R_{int}}$

Voltmeter reads less than ΔV_B due to the drop across R_{int}.

Old (discharged) battery: R_{int} gets bigger! external voltage collapses under load!

$$\text{Power} = \frac{\text{Dissipated Energy}}{\text{Unit time}}$$

Take a resistor with current I flowing → voltage drop.

In time Δt we have charge $\Delta q = I \, \Delta t$ dropping in energy by $\Delta E = \Delta q \; \Delta V = (I \, \Delta t) \, \Delta V$ due to the potential drop by $\Delta V = R I$ (magnitudes only, no sign)

Dissipated power $P = \dfrac{\Delta E}{\Delta t} = I \cdot \Delta V$ unit = volt · ampere = watt

$$P = I \cdot (RI) = R I^2$$

$$= \frac{\Delta V}{R} \cdot \Delta V = \frac{\Delta V^2}{R}$$

When using a resistor in a circuit → design consideration:

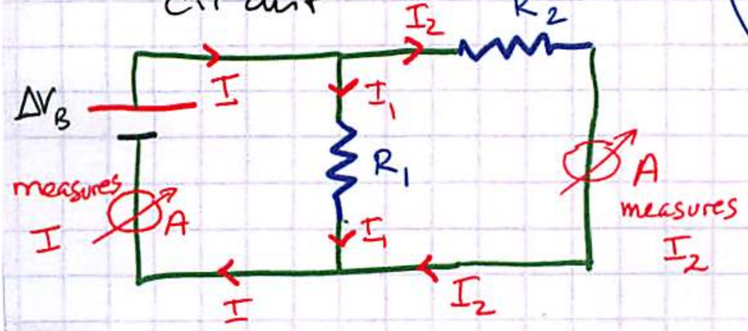
what current / voltage drop combination will occur?

is the resistor "big enough" to get rid off the heat?

1/10 Watt, 1/4 Watt, 1/2 Watt resistors, etc.

(2) Kirchhoff junction rule

Consider a branching circuit



Note how current is measured: it has to pass through the instrument. Ideal ammeter: $R_{int} \approx 0$

Two junctions in this circuit

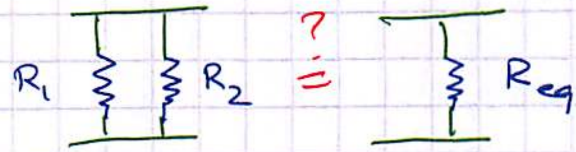
$$\sum I_{in} = \sum I_{out}$$

for each junction

Parallel resistors

Take the above circuit.

What is the equivalent load?



How do we solve this?

1) $I = I_1 + I_2$ (Kirchhoff junction rule)

2) $\Delta V_1 = \Delta V_2 = \Delta V_B$ (Kirchhoff loop w/o signs)

3) Ohm's law 3 times: $\Delta V_B = R_{eq} I$

$\Delta V_B = R_1 I_1 \qquad \Delta V_B = R_2 I_2$

solve (3) for currents and use in (1):

$$\frac{\Delta V_B}{R_{eq}} = \frac{\Delta V_B}{R_1} + \frac{\Delta V_B}{R_2} \quad \therefore \quad (\text{cancel } \Delta V_B)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

parallel resistors
→ same eqn as series capacitors

series resistors
→ same eqn as parallel capacitors

The idea of parallel resistors has other applications:

a radiator system pumps water through hoses with branching + hoses of different diameter

→ pipe with small cross section → bigger resistance
thicker pipe or parallel pipes → less resistance