Work

To integrate Newton's 2^{nd} law, i.e., to determine the complete motion $[\vec{r}(t), \vec{v}(t), \vec{a}(t)]$ from the force law $\vec{F}(\vec{r})$, and the initial conditions $\vec{r}_0(t_0)$, $\vec{v}_0(t_0)]$ can be difficult (cf. Kepler motion)

Ask a simpler question:

How does the speed of an object change under the action of a force over a distance segment of the motion?

1d motion with constant force $a = \frac{F}{m}$:

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$= v_i^2 + 2\frac{F}{m}\Delta x$$

Can this be generalized $\rightarrow F = F(x)$?

$\rightarrow x \rightarrow \vec{r} = x\hat{\imath} + y\hat{\jmath}$

Remember: lever arm principle $\longrightarrow$

We can get the same "action" from a small vs big force if the small force acts over a proportionately longer distance Δx

Define work in 1d:

$$W = F \Delta x$$

Note: it is 1d, but a sign is included:

1) F acts in the direction of displacement:

$$F > 0, \quad \Delta x > 0 \quad \rightarrow \quad W = F \Delta x > 0$$

2) F acts against the displacement:

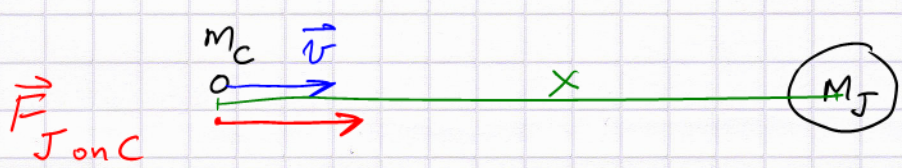
$$F < 0, \quad \Delta x > 0 \quad \rightarrow \quad W = F \Delta x < 0$$

Negative work: particle is pushing against a force field (slowing down) $\rightarrow$ the force field is doing negative work on the particle

Point of view: force does work on the massive (inert) object.

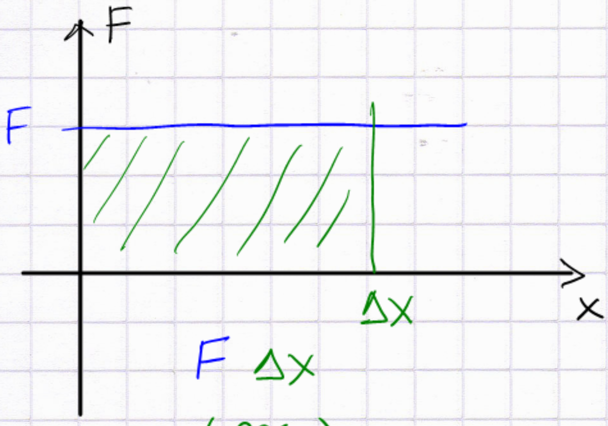
Complication 1: What if the force changes with distance $F = F(x)$?

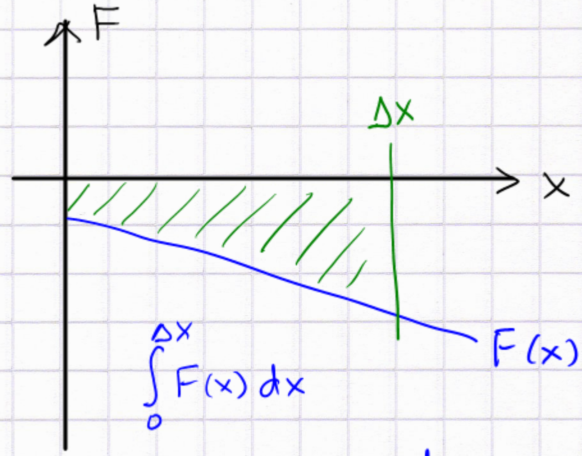
Example: comet heads directly for impact on Jupiter: F increases as it gets closer!



$$F_{J\,on\,C} = \frac{G M_J m_c}{x^2}$$

Work = area under F(x) curve





$\int_0^{\Delta x} F(x)\,dx$ = area under curve (with sign)

$$W = \int_0^{\Delta x} F(x)\,dx$$ = area between curve, x-axis, $x = \Delta x$ vertical line, y-axis

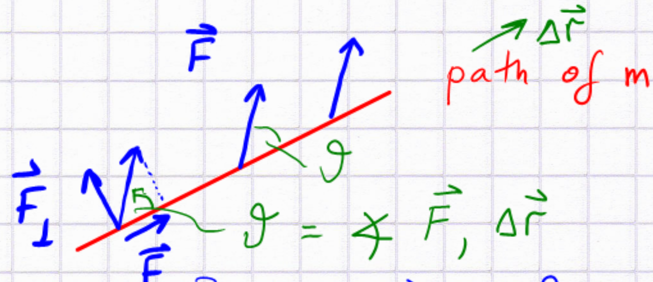
Complication 2 $\qquad \vec{F} = \vec{F}(\vec{r})$ force field

Only the component of the force along the motion does contribute!

$$\vec{F} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

$\vec{F}_{\parallel}$: speeds up particle

$\vec{F}_{\perp}$: wants to deflect particle (doesn't change the speed, such as a purely centripetal force)



$\vartheta = \sphericalangle\, \vec{F}, \Delta\vec{r}$

$F_{\parallel} = |\vec{F}| \cos\vartheta$

$W = F\,\Delta x \longrightarrow F\,\Delta r\,\cos(\sphericalangle\,\vec{F}, \Delta\vec{r})$

"dot product" of two vectors: $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y \; (+A_z B_z)$

(can be negative!) $\qquad = A\,B\,\cos(\sphericalangle\,\vec{A}, \vec{B})$