Potential Energy vs Force

We defined work as the area under the $F(x)$ curve

Then we defined $-W$ as the difference $(PE)_{\text{fin}} - (PE)_{\text{ini}}$

basically: $PE = -W + \text{const}$.

We can add a constant since physics depends on $(PE)$ differences

$PE = \text{area under } F(x)\text{ curve} + \text{const}$

$V(x_f) - V(x_i) = -\int_{x_i}^{x_f} F(x) \, dx$

$\therefore PE = \text{neg. anti-derivative of } F(x)$

$\therefore F(x) = -\frac{dV}{dx}$

Example: 1) spring + mass \( x_0 = 0 \)

\[ V_H(x) = \frac{1}{2} k x^2 \]

(Hooke's law)

\[ F_H(x) = -k x \]

2) Gravitational PE:

\[ V_G(r) = -\frac{G m M}{r} \]

\[ F_G(r) = -\frac{dV_G}{dr} = -\frac{d}{dr} \left( -\frac{G m M}{r} \right) = \frac{d}{dr} \left( \frac{G m M}{r} \right) \]

\[ = -\frac{G m M}{r^2} \]

Applications: a) circular orbit for $m$ going around $M$

$\rightarrow V_G(r) = -\frac{G m M}{r_0}$ \( \rightarrow \text{static: no change in radial motion} \)

b) elliptic orbit: radial distance changes $\rightarrow$ radial KE changes but $KE_{\text{rad}} + PE = \text{const}$
Come back to gravity at earth's surface (red+blue ski runs)

Le(t) : \( g_{\parallel} = g \sin \alpha \)
right : \( g_{\parallel} = g \sin \beta \)

\[ \Delta V = mgh \]
\[ \therefore \quad v_f (a \text{ bottom}) \quad \text{follows from} \]
\[ \frac{m}{2} v_f^2 = mgh \quad \Rightarrow \quad v_f = \sqrt{2gh} \]

how is this possible? \quad \text{Use Kinematic eqs :}

\[ v_L(t) = g \sin \alpha t \]
\[ v_R(t) = g \sin \beta t \]

If \( v_L(t_f) = v_R(t_f) = v_f = \sqrt{2gh} \) then \( t_{fL} = t_{fR} \)

The skiers don't arrive at the same time, but with the same speed.

\[ s_L = \frac{1}{2} g \sin \alpha \quad t_{fL}^2 \]
\[ s_R = \frac{1}{2} g \sin \beta \quad t_{fR}^2 \]

\[ s_L = h / \sin \alpha \]
\[ s_R = h / \sin \beta \]

\[ \therefore \quad h = \frac{1}{2} g \quad t_{fL}^2 \sin^2 \alpha \]
\[ h = \frac{1}{2} g \quad t_{fR}^2 \sin^2 \beta \]

\[ t_{fL} = \sqrt{\frac{2h}{g}} \quad \frac{1}{\sin \alpha} \quad \leftrightarrow \quad t_{fR} = \sqrt{\frac{2h}{g}} \quad \frac{1}{\sin \beta} \]

\[ \therefore \quad \text{the prior conclusion is consistent with this!} \]
Let's use the constant-acc. Kinematic eqn with time eliminated (equivalent to energy conservation):

\[ v_f^2 = v_i^2 + 2a \Delta x \]

\[ v_{fL}^2 = 2g \sin\alpha \left( \frac{h}{\sin\alpha} \right) = 2gh \]

\[ v_{fR}^2 = 2g \sin\beta \left( \frac{h}{\sin\beta} \right) = 2gh \]

\[ \text{energy conservation: } \quad \frac{1}{2}mv_f^2 \]

The message: the skier on the steeper slope arrives earlier, but with the same final speed.

s/he arrives earlier, since there is less distance traveled.

\[ t_\frac{f}{L} = \frac{1}{\sin\alpha} \quad \text{but } v_\frac{f}{L} \text{ is the same!!} \]

To be learned:

our intuition ("steeper slope \(\rightarrow\) we get faster")

is partially right (earlier arrival)

but don't jump to the conclusion about \(v_\frac{f}{L}\).

Energy conservation (and Kinematic eqn) \(\rightarrow\)

\[ v_{fL} = v_{fR} \quad \text{from } \quad mgh \rightarrow \frac{1}{2}mv_f^2 \]