Potential Energy vs Force

We defined work as the area under the $F(x)$ curve

Then we defined $-W$ as the difference $(PE)_{fin} - (PE)_{in}$

basically $PE = -W + \text{const.}$

we can add a constant since physics depends on (PE) differences

$PE = -$ area under $F(x)$ curve $+$ const.

$$V(x_f) - V(x_i) = -\int_{x_i}^{x_f} F(x)\,dx$$

$\therefore$ PE = neg. antiderivative of $F(x)$

$$\therefore \quad F(x) = -\frac{dV}{dx}$$

Example: 1) spring + mass ; $x_0 = 0$ $\qquad V_H(x) = \frac{1}{2} k x^2$

(Hooke's law) $\qquad F_H(x) = -kx$

2) Gravitational PE: $\quad V_G(r) = -\frac{GmM}{r}$

$$F_G(r) = -\frac{dV_G}{dr} = -\frac{d}{dr}\left(-\frac{GmM}{r}\right) = \frac{d}{dr}\left(\frac{GmM}{r}\right)$$

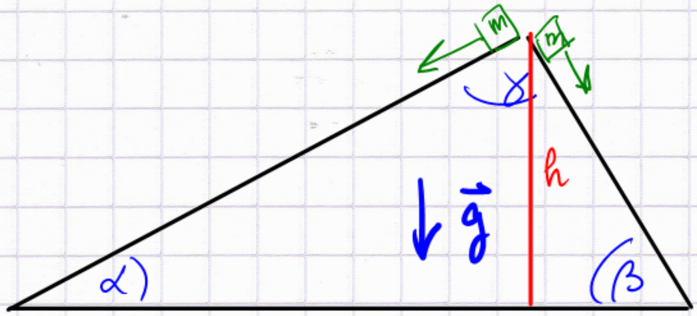
$$= -\frac{GmM}{r^2} \quad \checkmark$$

Applications: a) circular orbit for m going around M

$\longrightarrow \quad V_G(r) = -\frac{GmM}{r_0} \quad \rightarrow$ static: no change in radial motion

b) elliptic orbit: radial distance changes $\rightarrow$ radial KE changes but $KE_{rad} + PE = \text{const}$

Come back to gravity at earth's surface (red+blue ski runs)



Left: $g_{\parallel} = g \sin\alpha$

right: $g_{\parallel} = g \sin\beta$

$\Delta V = mgh$ $\quad\therefore$ v_f (@ bottom) follows from

$$\frac{m}{2} v_f^2 = mgh \rightarrow v_f = \sqrt{2gh}$$

how is this possible? Use Kinematic eqs:

$v_L(t) = g \sin\alpha \, t$ $\qquad$ $v_R(t) = g \sin\beta \, t$

If $v_L(t_{f_L}) = v_R(t_{f_R}) = v_f = \sqrt{2gh}$ then $t_{f_L} \neq t_{f_R}$

The skiers don't arrive at the same time, but with the same speed.

$s_L = \frac{1}{2} g \sin\alpha \, t_{f_L}^2$ $\qquad$ $s_R = \frac{1}{2} g \sin\beta \, t_{fR}^2$

$s_L = h/\sin\alpha$ $\qquad$ $s_R = h/\sin\beta$

$\therefore \quad h = \frac{1}{2} g \, t_{f_L}^2 \sin^2\alpha$ $\qquad$ $h = \frac{1}{2} g \, t_{fR}^2 \sin^2\beta$

$t_{f_L} = \sqrt{\frac{2h}{g}} \frac{1}{\sin\alpha}$ $\qquad$ $t_{f_L} \neq t_{fR}$ $\qquad$ $t_{fR} = \sqrt{\frac{2h}{g}} \frac{1}{\sin\beta}$

$\therefore$ the prior conclusion is consistent with this!

Let's use the constant-acc. kinematic eqn with time eliminated (equivalent to energy conservation):

$$v_f^2 = v_i^2 + 2a\,\Delta x$$

$$\left.\begin{aligned} v_{f_L}^2 &= 2g\sin\alpha\left(\frac{h}{\sin\alpha}\right) = 2gh \\ v_{f_R}^2 &= 2g\sin\beta\left(\frac{h}{\sin\beta}\right) = 2gh \end{aligned}\right\}$$

energy conservation: $mgh \longrightarrow \frac{1}{2}mv_f^2$

The message: the skier on the steeper slope arrives earlier, but with the same final speed

S/he arrives earlier, since there is less distance traveled.

$t_f \sim \dfrac{1}{\sin\vartheta}$ but v_f is the same!!

To be learned:

our intuition ("steeper slope $\rightarrow$ we get faster")

is partially right (earlier arrival)

but don't jump to the conclusion about v_f.

Energy conservation (and kinematic eqn) $\longrightarrow$

$v_{f_L} = v_{f_R}$ from $mgh \rightarrow \frac{1}{2}mv^2$