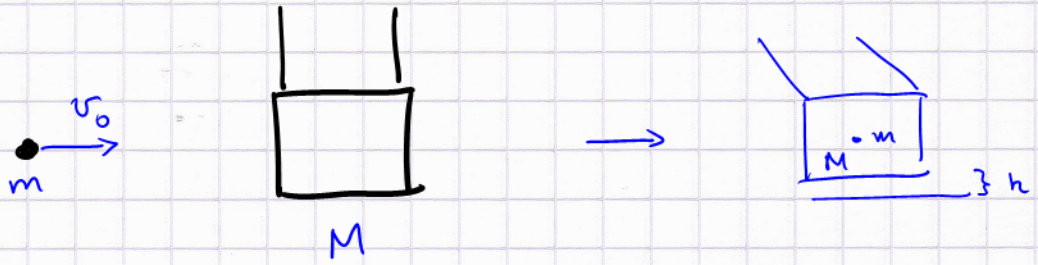
Ballistic pendulum



completely inelastic collision (bullet comes to a halt inside the sand box)

Momentum conservation during collision:

$$m v_0 + M \cdot 0 = (M+m) V_f \quad \therefore V_f = \frac{m}{m+M} v_0$$

The motion of $(M+m)$ in the gravitational field:

energy conversion

$$\frac{1}{2}(M+m) V_f^2 = (M+m) g h$$

(gh: measured quantity)

$$\frac{1}{2}(M+m)\left(\frac{m}{M+m} v_0\right)^2 = (M+m) g h$$

$$\frac{m^2}{(M+m)^2} v_0^2 = 2gh \quad \therefore v_0 = \frac{M+m}{m}\sqrt{2gh}$$

$$\approx \frac{M}{m}\sqrt{2gh}$$

easily measured final height → determine bullet speed as it enters the sand box.

How much mechanical energy was lost (converted to heat)? Q

$$E_0 = \frac{1}{2} m v_0^2 \qquad E_{mech}^{fin} = \frac{1}{2}(M+m) V_f^2 = \frac{1}{2} m v_0^2 \left(\frac{m}{m+M}\right)$$

$$Q = E_0 - E_{mech}^{fin} = \frac{1}{2} m v_0^2 \left(1 - \frac{m}{m+M}\right) = E_0 \left(\frac{m+M-m}{m+M}\right) = \frac{M}{M+m} E_0$$

$M > m$ → almost all E_0 is converted to heat!

Inelastic Collision

A glider (mass m) moves with speed v_0 and hits a stationary glider (mass M = m). They stick after the collision (glue, gum, magnets, coupling, ...)

Q1: how fast is the combined system?

A: total momentum is conserved

$$m v_0 + M \cdot 0 = (m+M) V = 2m V$$

$$\therefore \quad V = \frac{m v_0}{2m} = \frac{1}{2} v_0$$

Q2: what follows for the mechanical energy?

A: $E_{mech}^{before} = \frac{1}{2} m v_0^2$

$$E_{mech}^{after} = \frac{1}{2}(M+m) V^2 = \frac{1}{2}(2m) V^2 = m V^2$$

$$= m \left(\frac{v_0}{2}\right)^2 = \frac{1}{4} m v_0^2 = \frac{1}{2} E_{mech}^{before}$$

This collision is inelastic! One half of the original mechanical energy, i.e., $\frac{1}{2} E_{mech}^{before}$ is converted into something else.

What is the something else? heat?

The Q-value is $\frac{1}{2} E_0$ for the case m = M.

Remember: for the bullet (m) hitting a pendulum (M = sandbox; M >> m): $Q \lesssim E_0$.