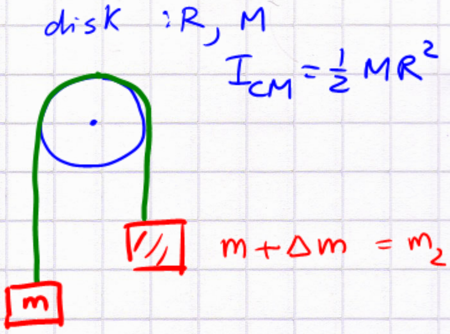
Combined linear + rotational motion

Atwood machine



- We discussed it before in the $m \gg M$ limit
- assume: no slippage of rope over disk!

  $\therefore$ linear motion of the masses is locked to rotation of the disk

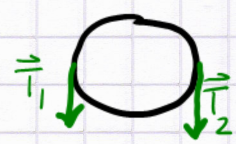
what do we expect? $\rightarrow$

$m + \Delta m$	moves down
m	moves up
disk	rotates CW

tension in string the same on both sides?

@ m_1: moves up; call $\uparrow \hat{j}$ positive motion

$$m a_1 = T_1 - mg \qquad ①$$



@ m_2: moves down; call $\downarrow \hat{j}$ positive motion

$$(m + \Delta m)\, a_2 = (m + \Delta m) g - T_2 \qquad ②$$

rotation about CM:

$$\tau_1 = R T_1 > 0$$ (CCW motion would result)

$$\tau_2 = -R T_2 < 0$$

$$\tau_{net} = \tau_1 + \tau_2 = R(T_1 - T_2)$$

presumably < 0!

$$\frac{1}{2} MR^2 \alpha = \tau_{net} = R(T_1 - T_2) \qquad ③$$

What else do we know?

1) string does not stretch $\therefore$ $a_1 = a_2 = a$ (signs are taken care of)

2) $-(R\alpha) = a$ why? $R\omega = v_y$ $\left(a = \frac{dv}{dt}\right)$

$$R\Delta\vartheta = \Delta y$$

$$R(\vartheta - \vartheta_0) = y - y_0$$

but $a > 0$, while $\alpha_z < 0$ due to $\tau_z < 0$!

collect eqs : $ma = T_1 - mg$ (1)

$$(m+\Delta m)a = (m+\Delta m)g - T_2 \quad (2)$$

$$\frac{1}{2}MR^2\left(-\frac{a}{R}\right) = R(T_1 - T_2) \quad (3)$$

We want $\underline{\underline{a}}$ (acceleration of either mass & ang. acc. α)

T_1 and T_2 are unknown, but we have 2 extra eqs.

solve (1) for T_1 ; (2) for T_2 ; insert in (3) :

(1) : $T_1 = m(a+g)$; (2) $T_2 = (m+\Delta m)(g-a)$

(3) : $\frac{1}{2}MR\,a = -R(T_1 - T_2)$ $\therefore$ $\frac{1}{2}Ma = T_2 - T_1$

$$\therefore \quad \frac{1}{2}Ma = +(m+\Delta m)(g-a) - m(g+a)$$

$$= \Delta m\, g - (2m+\Delta m)a$$

$$\left(2m+\Delta m+\frac{1}{2}M\right)a = \Delta m\, g \quad \therefore \quad \boxed{a = \frac{\Delta m}{2m+\Delta m+\frac{M}{2}}\,g}$$

Remarks: The previous result : $a = \dfrac{\Delta m}{2m+\Delta m}g$

was interpreted as :

- The mass difference Δm drives the Atwood machine ($\Delta m\, g$ is the net gravitational force)
- The Atwood machine has inertia : $(m+\Delta m) + m = 2m + \Delta m$
- The full Atwood machine has linear inertia $2m + \Delta m + \frac{M}{2}$
- The tensions T_1 and T_2 are NOT the same.