Doppler effect

· sound · also EM waves ?

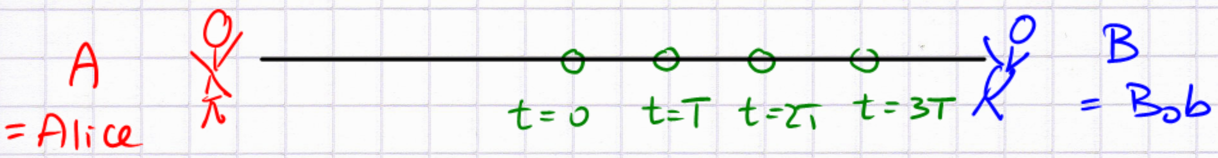
· moving sources of waves vs. · moving observers

· waves that require a medium for propagation (sound)

① begin with sound; stationary medium (air)

stationary observer(s), moving source

sound source moving left → right at v_{src}: ⟼



pick snapshots at $t = (0, 1, 2, 3, \dots)\,T$ where

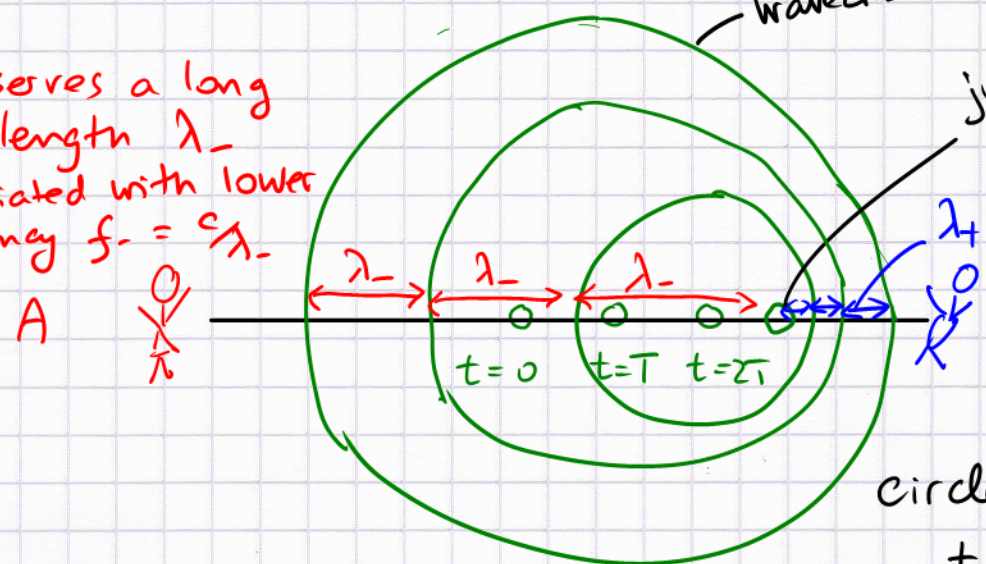
$$T = \frac{1}{f_0} = \text{period of sound wave.}$$

why? since we want to look at wave crests emitted at these times

wavecrest from $t = 0T$

just about to emit a wave crest at $t = 3T$

A observes a long wavelength λ_- associated with lower frequency $f_- = c/\lambda_-$



circles centered on $t_i = iT$ respectively

B observes a short wavelength λ_+ associated with high frequency $f_+ = \frac{c}{\lambda_+}$

$c \approx 343\ \frac{m}{s}$

Now derive the changed wavelengths

1) radii of the circles $s_i = c t_i = c\, i T = i \frac{c}{f_0}$

2) @ B we see $3\lambda_+$ corresponds to $s_3 = 3\frac{c}{f_0}$ as the position of the first wave crest (using the $t=0$ source location as origin),

while the source is now located at:

$x_{src} = v_{src} \cdot t_3 = v_{src} \cdot 3T$

Thus, $3\lambda_+ = s_3 - x_{src} = 3\frac{c}{f_0} - 3 v_{src} \frac{1}{f_0}$

$$\therefore \quad \lambda_+ = \frac{c - v_{src}}{f_0} \qquad \text{But: } \lambda_+ = \frac{c}{f_+}$$

$$\therefore \quad \frac{\cancel{c}}{f_+} = \frac{\cancel{c}\,(1 - v_{src}/c)}{f_0} \qquad \therefore \quad \boxed{f_+ = \frac{f_0}{1 - \frac{v_{src}}{c}}}$$

B observes an increased frequency f_+ from the approaching source ($v_{src} > 0$ in the formula)

The result for A can be obtained either by the substitution $v_{src} \longrightarrow -v_{src}$, or by repeating the arguments for λ_-

2 → stationary source, moving observer

Naively, we might think: replace v_{src} by v_{obs}

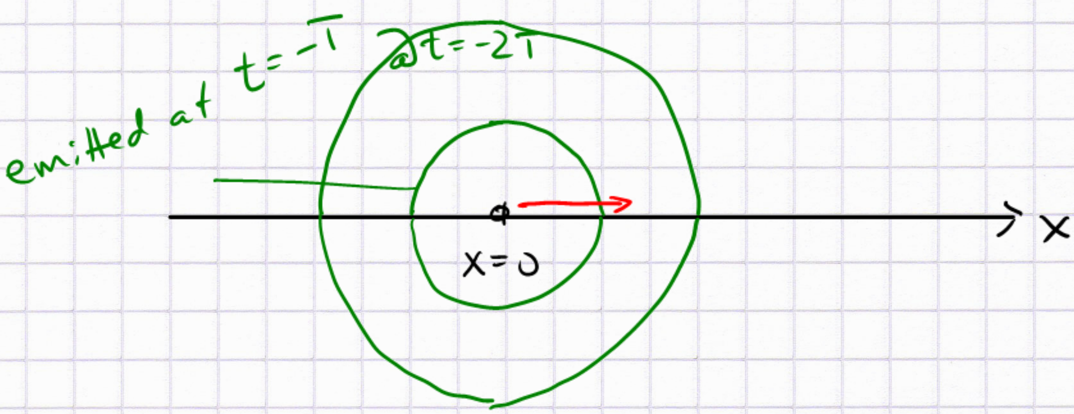
However, this is incorrect, the formulae

$$f_\pm = \left(1 \pm \frac{v_{obs}}{c}\right) f_0$$

differ from the moving source case somewhat (at order $\mathcal{O}\left(\left(\frac{v}{c}\right)^2\right)$)

How does this result come about?

Consider a source @ $x=0$ emitting wave crests at $t = 0, T, 2T, \ldots$ and an observer moving according to $x = v_0 t$. Assume $v_0 < c$



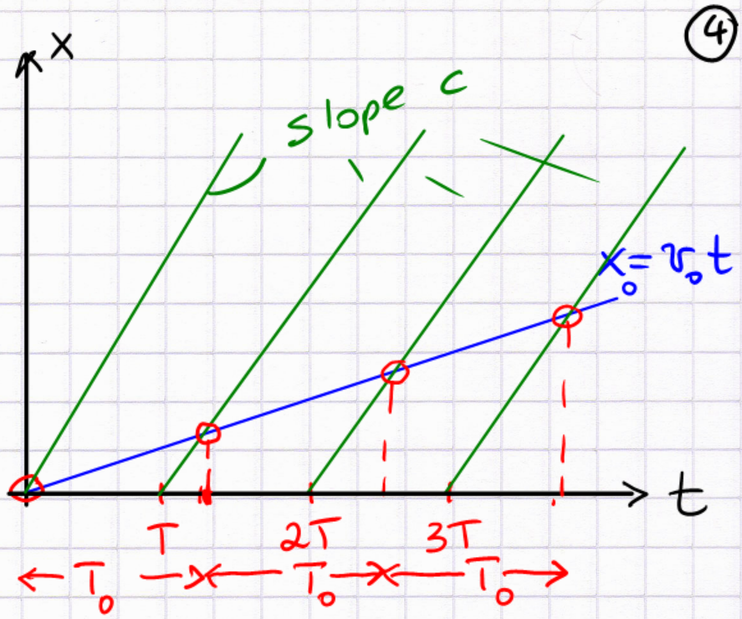
As the observer leaves the source $(x=0)$ a crest leaves (and runs ahead of him). When will the observer record the next crest? then the next crest, etc.? Those times will yield the observed period, and associated frequency!

Position-Time graph

observer: $x_o = v_o t$ (slope v_o)

wave crests # n:

$$x^{(n)} = c\,(t - nT)$$



T_o can be obtained as the time when the crest emitted at $t = T$ catchos up with the observer

For $t = T_o$ we have $v_o T_o = c\,(T_o - T)$ $\quad\therefore\quad (v_o - c)\,T_o = -cT$

$$\left(1 - \frac{v_o}{c}\right) T_o = T$$

Now use $f_o = \frac{1}{T_o}$ and $f = \frac{1}{T}$: $\left(1 - \frac{v_o}{c}\right)\frac{1}{f_o} = \frac{1}{f}$

$$\therefore\quad f_{obs} = \left(1 - \frac{v_{obs}}{c}\right) f$$

observer moves away ($v_{obs} > 0$) $\therefore$ reduced frequency

Note that $f_- = \dfrac{f_o}{1 + \frac{v_s}{c}}$ for a source moving away

agrees only to first order: $\dfrac{1}{1 + \frac{v_s}{c}} = 1 - \dfrac{v_s}{c} + \left(\dfrac{v_s}{c}\right)^2 \mp \dots$

The naive expectation that it doesn't matter who is moving (source or observer) is wrong when v_s is a sizeable fraction of c! $\longrightarrow$ effect from medium in which waves propagate

Light waves (EM waves) also experience Doppler shifts. A different formula, derived in Special Relativity!