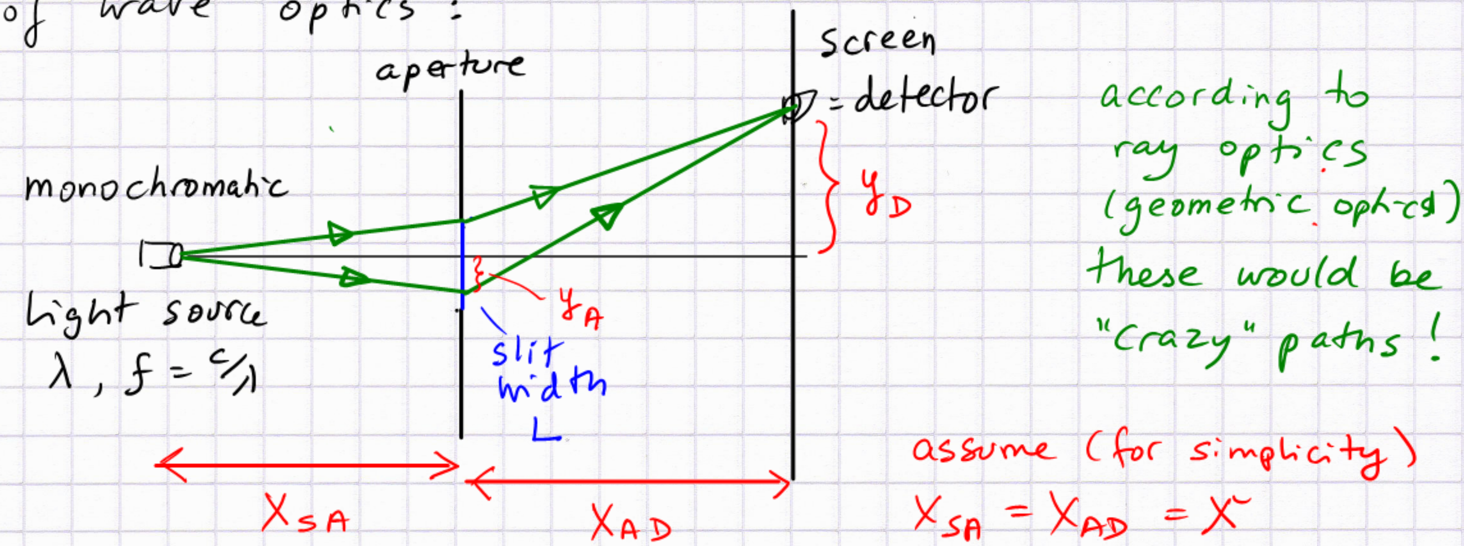
Fraunhofer (single-slit) diffraction

→ Feynman's sum over paths method

The wave-particle duality can be illustrated for light (photons = particles with energy $E = hf$, with wave properties).
We use an aperture of small size (several wavelengths λ) to see how geometric optics can emerge as a limit of wave optics:



Feynman's idea: assume that photons travel along all possible straight-line connections between S and D.

For each connection observe how time passes → photons travel with speed c; paths have different lengths $s = s_1 + s_2$ ($s_1 \hat{=}$ source → mirror, $s_2 \hat{=}$ mirror → detector);

The paths are labeled by the vertical position y_A where they pass through the slit (aperture)

$$s_1 = \sqrt{X^2 + y_A^2} \quad ; \quad s_2 = \sqrt{X^2 + (y_D - y_A)^2}$$

We choose N paths by segmenting the slit: $h = \frac{L}{N}$,
(N = even), $y_{A,n} = -\frac{L}{2} + (n + \frac{1}{2})h$ $\quad N = \quad n = 0..N-1$

Feynman's idea: for each path calculate a complex-valued amplitude $A_n = e^{i2\pi ft} = \cos(2\pi ft) + i\sin(2\pi ft)$

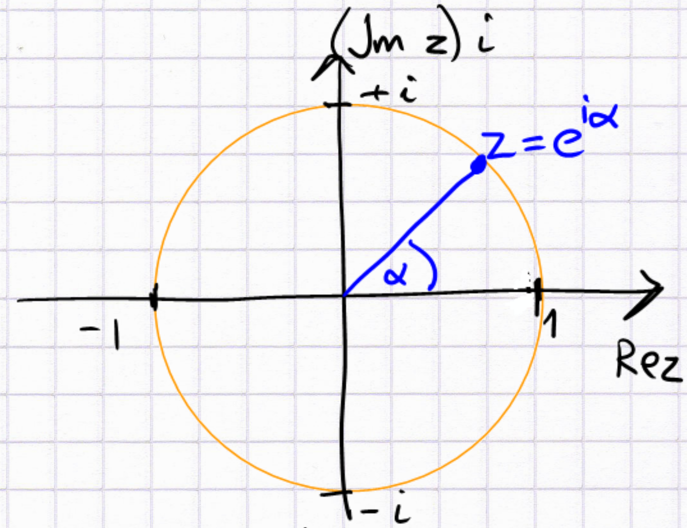
Here $i = \sqrt{-1}$ is the imaginary unit introduced to solve $z^2 + 1 = 0$ ($z = \pm i$ is the solution)

Complex numbers $z = x + iy$ have independent real and imaginary parts.

Special case: (Euler's th.)

$e^{i\alpha} = \cos\alpha + i\sin\alpha$

define a unit circle in the complex plane for $\alpha = 0 .. 2\pi$



$\alpha = 0$: $e^{i\alpha} = 1$

$= \pi/2$ $\quad = i$

π $\quad = -1$

$3\pi/2$ $\quad = -i$

The final amplitude for a possible photon path depends on $f \cdot t_{fin} = \frac{c}{\lambda} t_{fin} = \frac{s}{\lambda}$ $\quad s = ct_{fin}$ = path length

For a photon of frequency f, wavelength λ:

$$A_n = e^{i2\pi f t_{fin}} = e^{i2\pi \frac{s}{\lambda}}$$

Sum over many possible paths → interference

$$A_{tot} = \sum_{\text{paths } n} A_n = \sum_n e^{i2\pi s_n/\lambda}$$

Light intensity: $I \sim |A_{tot}|^2$

What is happening at a dark fringe?

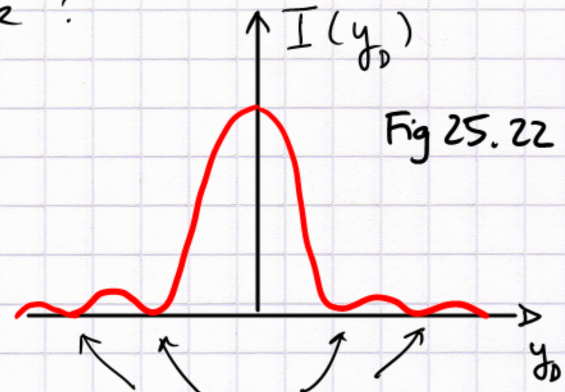
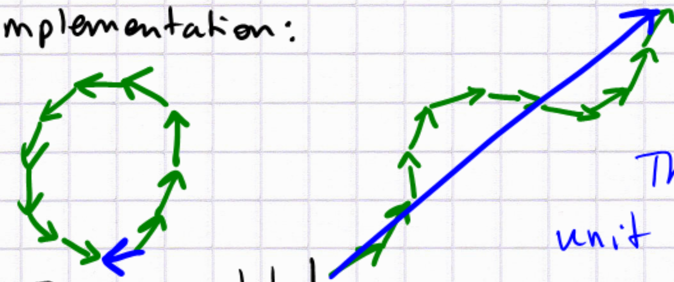


Fig 25.22

(almost) no photons arrive here

At certain detector heights y_D the contributions from all paths considered add up to result in a "zero-length" vector

Maple-implementation:



FraunhoferFeynman.html

The result of adding $N=10$ unit vectors $A_n = e^{i 2\pi s_n/\lambda}$ can be a short or a long vector

Square length of the resultant vector yields the intensity.

When the resultant vector is very short, then no photons arrive. The photons "know" the wave interference result and accumulate at the bright spot(s): constructive vs destructive interfering photon paths.

When the dimensions (slit/aperture size) become large compared to the wavelength of light the side maxima move closer + merge into the main spot

The limits of geometric optics were discovered in the 1880ies by Abbé while trying to build better microscopes.

In photography a small aperture yields a large depth (distance range imaged sharply). One loses light $\rightarrow$ needs longer exposure time or more sensitive recording device (CCD; high-speed film). However, below f-16 (mm?) the best optics (Leitz, Nikon, Zeiss, Canon, ...) would create blur due to diffraction!

Fraunhofer diffraction is more sophisticated than double-slit diffraction (which we can easily produce with water waves)

Big question: can we probe the wave nature of matter particles (electrons, protons, nuclei, atoms, molecules, viruses, baseballs, ...) by sending them through slits ??

A: for many objects on the list this has been accomplished in one form or another.

For atoms: slits are created by standing-light waves
→ Prof. A. Kumarakrishnan's (Kumar's) lab in Petrie.

→ internet: matter wave interference

C_{60} buckyballs (fullerenes) → megamolecules (clusters)
60 Carbon atoms
Soccerball structure
hexagon / pentagon

American Journal of Physics
Am. J. Phys. 71, 339 (2003)
by A. Zeilinger + 2 co-authors

mass $= 1.2 \times 10^{-24}$ kg ; diameter of ball ~ 1 nm

speed $= 200$ m/s

wavelength (de Broglie) $\lambda = \dfrac{h}{mv} \Rightarrow 2.8 \times 10^{-12}$ m $\ll$ diameter

→ pushing the boundaries of observing quantum phenomena on the borderline to macroscopic objects
(we perceive our world as a classical physics-world)