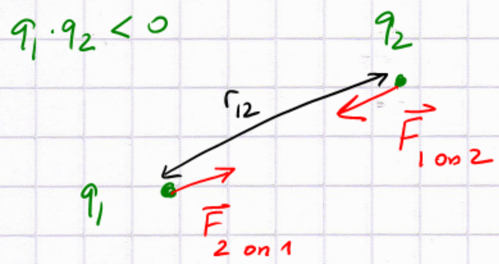
Coulomb's law and the electric dipole



suppose $q_1 = +e = 1.60 \times 10^{-19}\,C$ is a proton charge

and $q_2 = -e = -1.60 \times 10^{-19}\,C$ is an electron charge

$$|\vec{F}_{1\,on\,2}| = |\vec{F}_{2\,on\,1}| = K\frac{|q_1||q_2|}{r_{12}^2}$$

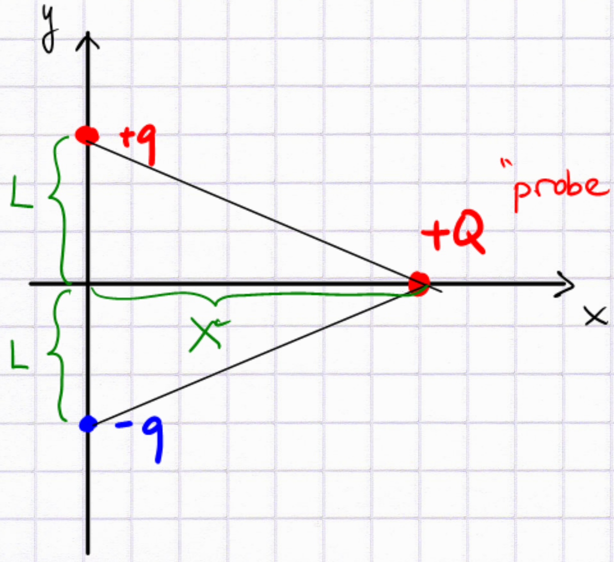
$$K = \frac{1}{4\pi\varepsilon_0} \approx 9.0 \times 10^9\,\frac{Nm^2}{C^2}$$

r_{12} = distance

what if we have more charges ?

Electric dipole → two opposite charges $\pm q$ are given at fixed locations $\pm L\hat{j}$

A separate charge $+Q$ is placed somewhere else to probe the electric force :



"probe" charge located @ $+X\hat{\imath}$

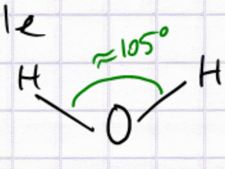
Electric force = 2 contributions

a) repulsion from $+q$

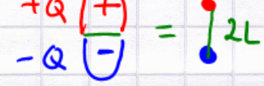
b) attraction towards $-q$

Why is this important ?

Water molecule

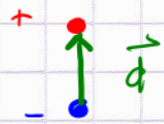
H_2O = 

from a large distance



If one wants to know how H_2O molecules (or other molecules) interact with each other in water, which means forming a liquid they have a typical separation larger than the size of the molecule

then: → the details don't matter too much

→ 

we have a total positive charge of $Q = +10\, q_{proton} = +10e$

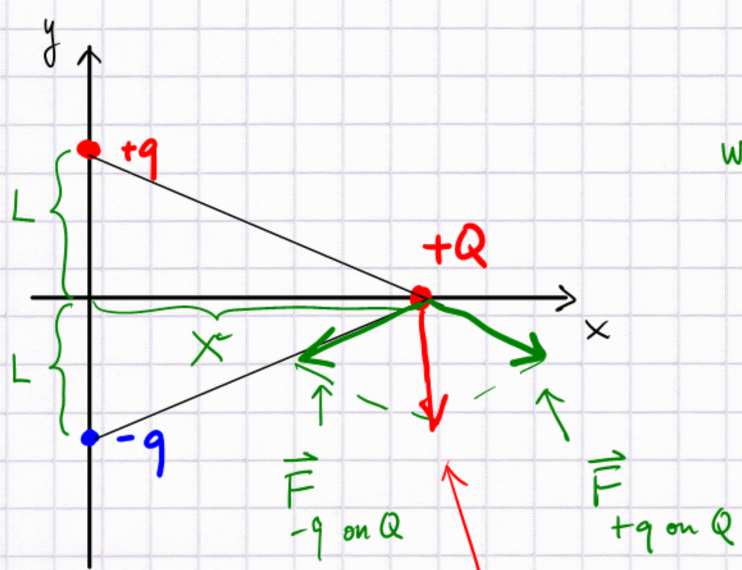
$Q = -10\, e$ (electrons)

$(e = 1.60 \times 10^{-19}\ C)$

$\vec{d}$ is a vector pointing from the negative to the positive charge → measures by how much the charge is separated

Molecules with an electric dipole moment can form chains

Now go into some detail



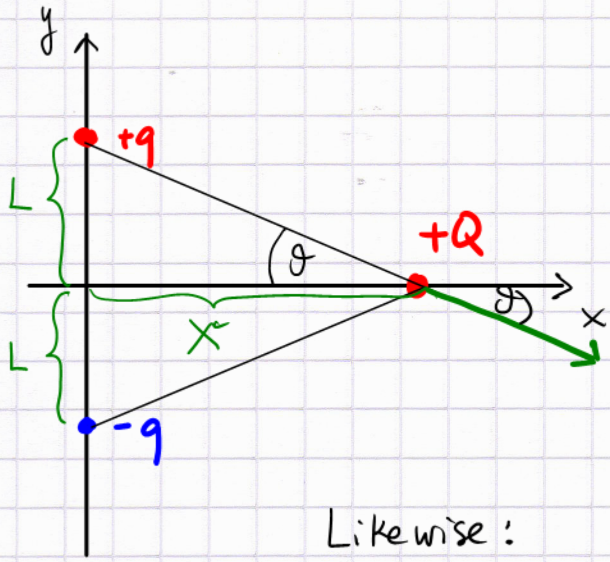
we need to add the two forces

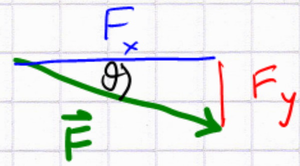
- x-components cancel
- y-components add

$$\vec{F}_{net\ on\ Q} = \vec{F}_{-q\ on\ Q} + \vec{F}_{+q\ on\ Q}$$

addition (superposition) principle

More detail:





$$F_{+q \text{ on } Q,\, x} = +\left|\vec{F}_{+q \text{ on } Q}\right| \cos\vartheta$$

points right

$$F_{+q \text{ on } Q,\, y} = -\left|\vec{F}_{+q \text{ on } Q}\right| \sin\vartheta$$

points down

Likewise:

$$F_{-q \text{ on } Q,\, x} = -\left|\vec{F}_{-q \text{ on } Q}\right| \cos\vartheta$$

$$F_{-q \text{ on } Q,\, y} = -\left|\vec{F}_{-q \text{ on } Q}\right| \sin\vartheta$$

Net force along $-\hat{j}$:

$$\vec{F}_{net} = -2\left|\vec{F}_{q \text{ on } Q}\right| \sin\vartheta\ \hat{j}$$

$$= -2\left|\vec{F}_{q \text{ on } Q}\right| \frac{L}{\sqrt{x^2+L^2}}\ \hat{j}$$

$$\left|\vec{F}_{q \text{ on } Q}\right| = K\,\frac{|q\,Q|}{x^2+L^2}$$

$$F_{net} = K\,|q\,Q|\,\frac{2L}{(x^2+L^2)^{3/2}} \xrightarrow[x \gg L]{} \frac{K\,(2L|q|)\,|Q|}{x^3}$$

interesting result: on the axis $\perp$ to the dipole
perpendicular

the force at large distance x

falls like $\frac{1}{x^3}$

- very naive: force should be zero? $\pm q$ cancel
- less naive: it has to fall faster than $\frac{1}{x^2}$, as net charge $= 0$