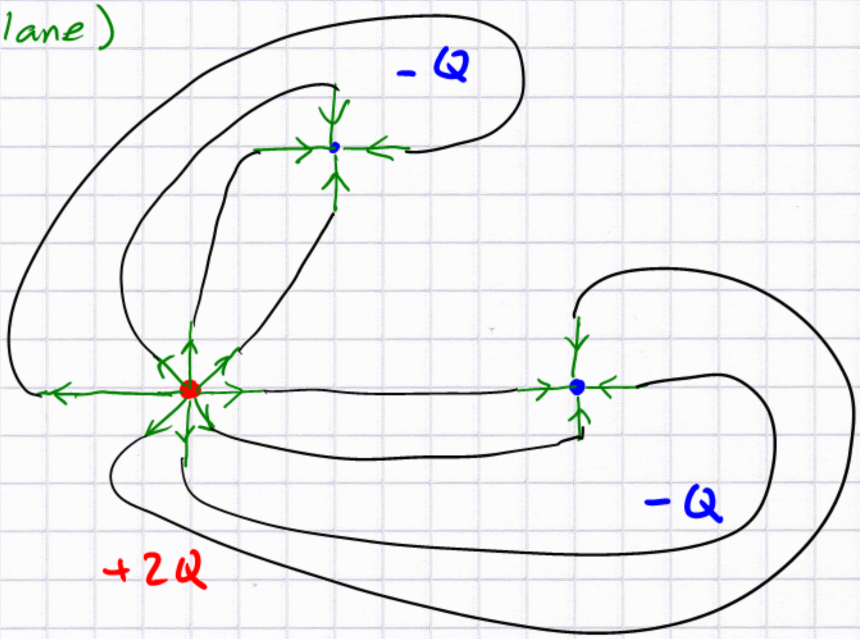
Gauss' law → $\vec{E}$ fields for macroscopic charged objects

Given a collection of point charges Q_i $i = 1, 2, \ldots$

we can obtain $\vec{E}_{net}$ by adding contributions for $i = 1, 2, \ldots$

Example (in a plane)

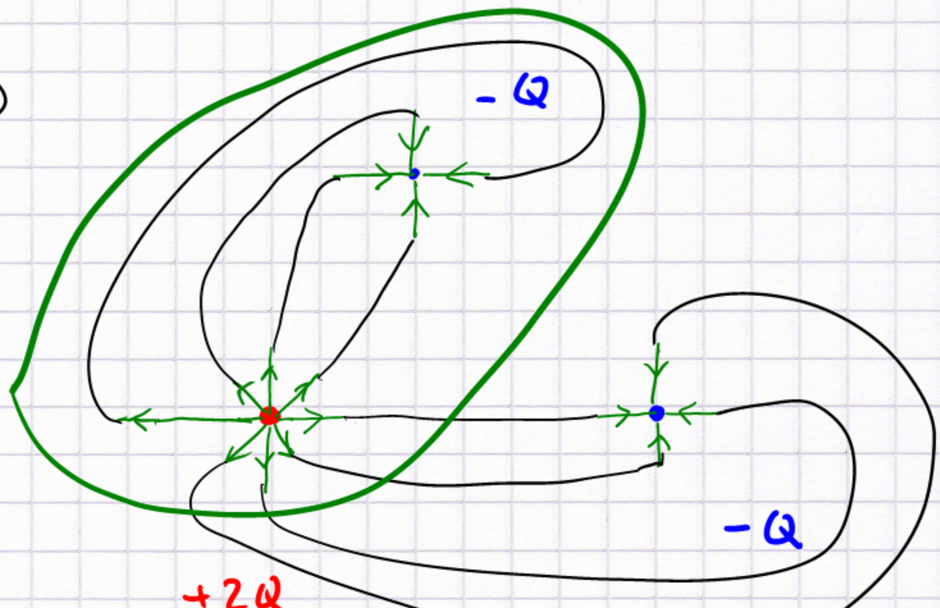
field lines



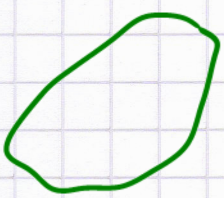
?

twice as many lines leave +2Q than enter -Q

we could try it out in charges-and-fields.swf (arrow plot + field probe)



4 lines correspond to Q



contains

+2Q - Q = +Q

4 field lines leave the  area

This is true for any such closed contour we draw:

HOW MANY LINES LEAVE/ENTER → MEASURE OF NET CHARGE

Gauss' law uses this idea:

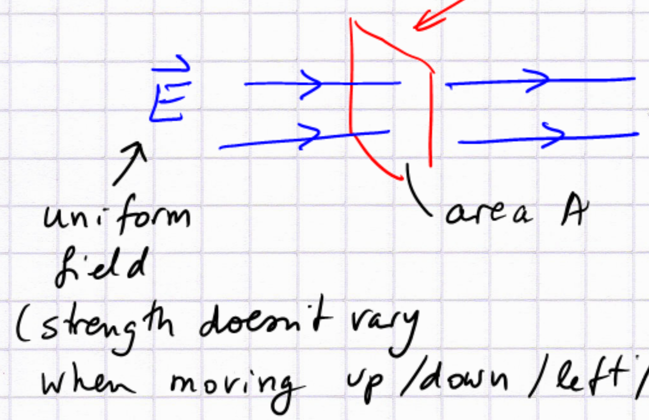
1) a probe surface (in our 2d planar example a closed contour) surrounds some charge

2) use a probe surface of appropriate symmetry:

for a point charge use a sphere centred on the charge

3) define electric flux Φ through the probe surface

Example: window pane permeated by $\vec{E}$ field:



pane + field perpendicular:

$$\Phi = E\,A$$

E: strength of field $|\vec{E}|$

A: area

Compare: illumination of a wall with a window
→ how much gets through the wall?
(depends on product of intensity and area)

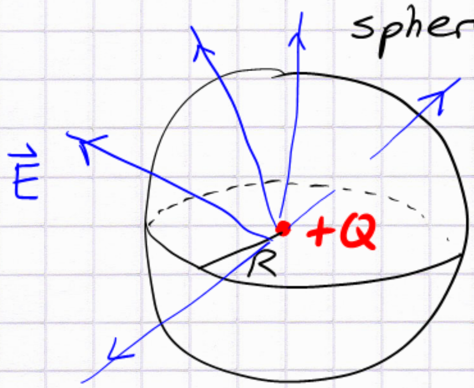
Q: What if the pane is tilted against the flux direction?

A: Use the dot product of the $\vec{E}$ field with the normal to the pane $\quad \Phi = E\,A\cos\vartheta$

Example : pane @ right angle to $\vec{E}$ → $\phi = 0$

4) Probe surfaces of interest are closed
(usually they surround charge)

sphere of radius R, $+Q$ sits @ centre



$\vec{E}$ field lines cross surface along normal → $\vartheta = 0$; $\cos\vartheta = 1$

$$\Phi = \underbrace{4\pi R^2}_{A} \, E_{\text{on surface}}$$

5) By Coulomb's law : $E_{\text{on surface}} = \dfrac{KQ}{R^2}$

Thus,
$$\Phi = 4\pi R^2 \frac{KQ}{R^2} = 4\pi \underbrace{K}_{\frac{1}{4\pi\varepsilon_0}} Q \equiv \varepsilon_0^{-1} Q = \frac{Q}{\varepsilon_0}$$

permeability $\varepsilon_0 \equiv \dfrac{1}{4\pi K}$

Flux is a scalar (real number, pos. or neg.)

For $Q > 0$ the flux is out of the sphere = positive

For $Q < 0$: $\vartheta = \pi$; $\cos\vartheta \to (-1)$

ϑ = direction angle between $\vec{E}$ and outward normal vector.

$$\to \phi = \left| \frac{-Q}{4\pi\varepsilon_0 R^2} \right| \cdot 4\pi R^2 \cdot (-1)$$

is negative

What have we learned so far?

Nothing new for a point charge?

→ Some geometric interpretation for why the force law is inverse-distance-squared!

- Given a point charge Q
- The electric field strength $\sim \frac{1}{R^2}$ R = distance from charge

(Coulomb's law, also true in Newtonian gravity)

- Gauss: flux through spheres of different radii:

$$\Phi = E A = E \, 4\pi R^2$$

but we have always the same charge enclosed, irrespective of value of R

$$\Rightarrow \quad \Phi = \frac{Q}{\varepsilon_0}$$

Therefore, $$\underbrace{E_{@R}}_{E@R} = \underbrace{E(R)}_{\text{"E of R"}} = \frac{Q}{\varepsilon_0} \cdot \frac{1}{4\pi R^2} = \frac{KQ}{R^2}$$

Gauss' law incorporates Coulomb's law, and becomes a useful tool for charge distributions, such as lines, planes, spheres