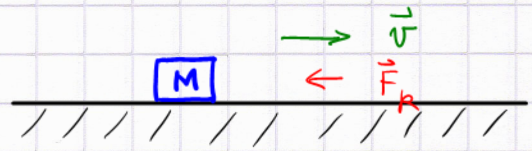
Drag vs Friction

Kinetic friction is a constant force $\quad |\vec{F}_k| = \mu_k |\vec{N}|$

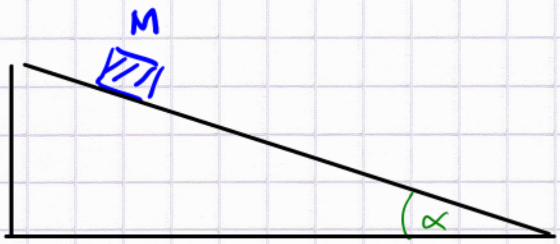
which opposes motion.



- $\vec{N} + M\vec{g} = 0 \quad$ (vertical direction)
- suppose M slides with initial velocity $\vec{v}_0$ (due to push)

then $\vec{F}_k \sim -\vec{v}$ will provide a constant deceleration until M comes to a halt

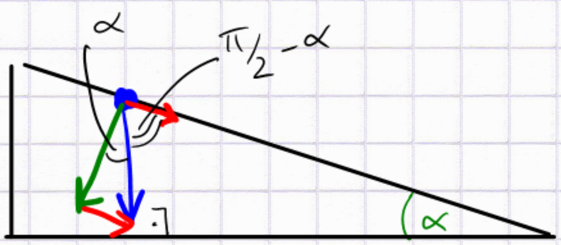
Another example: inclined plane with gravity



gravity is reduced to $Mg\sin\alpha$ along the incline.

why?

resolve $M\vec{g}$ into components



$$M\vec{g} = M\left(\vec{g}_{\parallel} + \vec{g}_{\perp}\right)$$

$$\frac{g_{\perp}}{g} = \cos\alpha \qquad \frac{g_{\parallel}}{g} = \sin\alpha$$

- $M\vec{g}_{\perp}$ is canceled by a normal force
- $M\vec{g}_{\parallel}$ accelerates the mass down the incline

↪ modified free fall with $\quad g_{eff} = g\sin\alpha$

$\longrightarrow 0 \quad$ for $\alpha = 0$

$\longrightarrow g \quad$ for $\alpha = \pi/2$

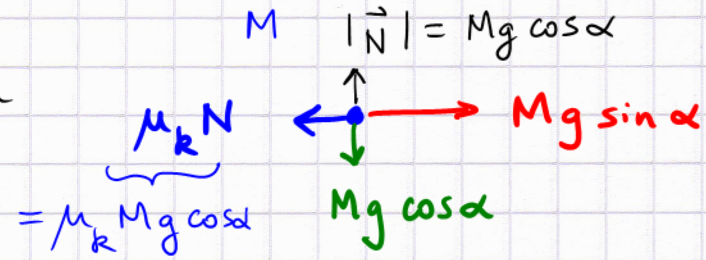
Now add friction:

Without friction the velocity change will be given as

$$v_f = v_0 + (g \sin\alpha)\, t_f \qquad \text{(linear increase)}$$

Kinetic friction opposes this motion. Free body diagram:

Rotated orientation by α:



Combined forward acceleration: $\boxed{g(\sin\alpha - \mu_k\cos\alpha)}$

velocity grows linearly in time:

$$v_f = v_0 + g(\sin\alpha - \mu_k\cos\alpha)\, t_f$$

Now discuss air drag

- opposes motion
- air in front of object to be displaced
- drag is not constant, but depends on the speed of M:

bigger speed $\rightarrow$ more drag

(example: performance cyclist: 60 km/h is a sustainable speed $\rightarrow$ body @ full throttle)

opposing wind $\rightarrow$ $v_{sust.}$ is reduced

$\rightarrow$ drag is qualitatively different from friction

- Depends on geometry of M

A = cross sectional area

ρ = density of air

model: $F_{drag} = 0.5\, \rho\, A\, v^2$ $\leftarrow$ quadratic dependence on speed

The quadratic dependence on speed is responsible for the strong correlation:

power provided by cyclist, engine $\longleftrightarrow$ achievable sustained max speed

For slow motion drag can be modeled as $\sim v$.

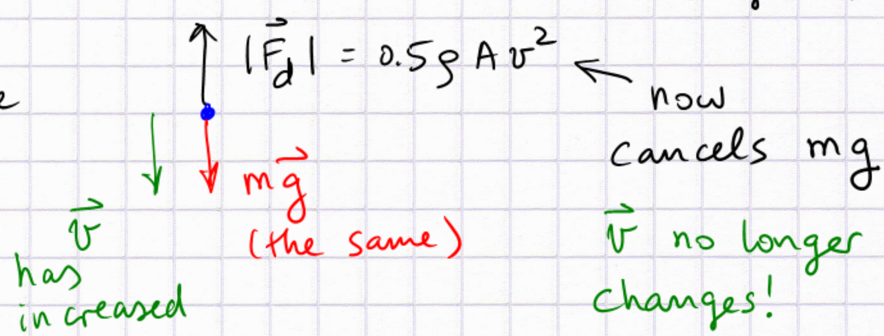
For small v (initial motion) drag is negligible

$\rightarrow$ free fall (or constant acceleration kinematics)

$v(t) = v_0 + at$ ← speed grows, drag kicks in

Suppose $a = g$. Parachutist accelerates in free fall, then opens parachute. (cross sectional area A goes up)

short time later:



↪ qualitative understanding:

1) small v: gravity accelerates object → drag small

$\rightarrow$ v increases $\Delta v = gt$

2) now v is growing until the drag force matches the downward acceleration $\rightarrow$ $m\vec{a} = \vec{F}_{net} = 0$

↪ terminal velocity:

$$mg = 0.5\rho A v_t^2$$

$$v_t = \sqrt{\frac{2.0\, mg}{\rho A}}$$

Extra:

full $v(t)$ result

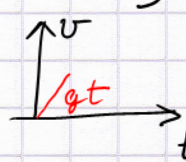
$$ma = mg - Cv^2$$

$$\boxed{\frac{dv}{dt} = g - \frac{C}{m}v^2}$$ solvable?

In Maple, in Parachute.mw (printout: Parachute.pdf) the function $v(t)$ that solves the above equation (Newton's 2nd law) is given

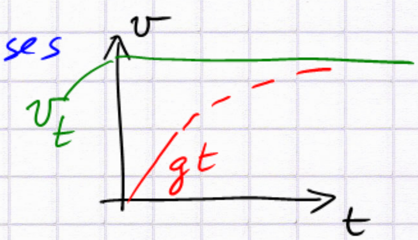
What does the solution show?

- For short times (small $v \rightarrow$ negligible v^2) it starts like free fall: $\boxed{v(t) \underset{\text{small } t}{\simeq} gt}$



- When $v(t)$ grows the 2nd term on the RHS kicks in $\rightarrow$ less acceleration

  $\rightarrow$ slope in $v(t)$ decreases



- Eventually, the gravitational acceleration g is cancelled by the drag acceleration

  $\rightarrow$ a "free" particle $\Rightarrow$ constant velocity $= \text{terminal } v_t$

Parachutist @ v_t: in force equilibrium, $F_{net} = 0$

$a(t) = \frac{dv}{dt} = 0$