Electric Potential Energy

1) The $\vec{E}$ field

whether for

- point charge Q : $\left(\frac{KQ}{r^2}\right.$, radial direction $\left.\right)$
- line of charge, $\lambda = \frac{Q}{L}$: $\left(\frac{K\lambda}{x}, \perp \text{ to line}\right)$
- plate, $\sigma = \frac{Q}{A}$ $\left(\frac{\sigma}{2\varepsilon_0}, \perp \text{ to plate}\right)$
- parallel plate cap., $\sigma = \frac{Q}{A}$ $\left(\frac{\sigma}{\varepsilon_0}, \perp \text{ to plates}\right)$

  only one of the plate areas counts!

is analogous to the gravity result(s):

$\vec{g}$ at the surface of the earth, point radially towards centre

$$-\frac{GM}{r^2}\hat{r} = \frac{-GM}{r^3}\vec{r}$$

In gravity : $\vec{W} = m\vec{g}$ (weight) $\rightarrow$ $\vec{r}$-indep.!

$\vec{F}_G(\vec{r}) = m\left(\frac{-GM}{r^3}\right)\vec{r}$ (gravitational force)

In electrostatics : $\vec{F}_E = q\vec{E}$ $\nearrow$ indep of $\vec{r}$ for plate

In gravity we learned how to use energy conservation :

starting with work $W = F\,\Delta x$ $(= \vec{F}\cdot d\vec{r})$

$\rightarrow \int \vec{F}\cdot d\vec{r}$

we defined potential energy as $PE = -W$

(really ΔPE, since any constant can be added to PE)

and then used $KE + PE = \text{const}$

or $(KE + PE)_{fin} = (KE + PE)_{in}$ with $KE = \frac{1}{2}mv^2$ to solve 1d motion.

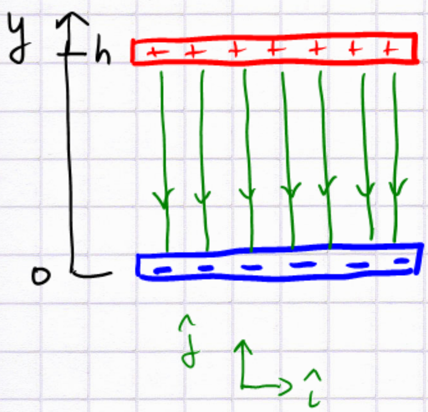
In electricity we will mostly deal with charged particles (electrons) moving in $\vec{E} = \vec{E}_0$ (constant) electric fields (analogous to free fall in $\vec{g}$)

$$\therefore \quad \Delta PE_{el} = -W_{el} = -q\underbrace{\vec{E}\cdot\Delta\vec{x}} = -qE\Delta x$$

$$\vartheta_{\vec{E},\Delta\vec{x}} = \begin{matrix}0\\ \pi\end{matrix} \quad \therefore \quad \cos\vartheta_{\vec{E},\Delta\vec{x}} = \begin{matrix}+1\\ -1\end{matrix}$$

Direction of motion will be aligned or counteraligned

Examples: 1) two parallel plates, connected to +/− of a battery
→ creates equal + opposite surface charge $\sigma = \frac{Q}{A}$



uniform $\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{j}$

suppose we start a negative charged particle at $y = 0$ (with $v_x = 0$, no x motion)

ignore gravity (strong E field)

$\vec{F}_E = q\vec{E}$ $\qquad q < 0$ (take an e^-)

electron: $q = -1.60 \times 10^{-19}$ C $\qquad \vec{F}_E = -e\vec{E} = +\frac{\sigma e}{\varepsilon_0}\hat{j}$

$m = 9.11 \times 10^{-31}$ kg

$$m a_y = \frac{\sigma e}{\varepsilon_0}$$

Suppose $\sigma = \frac{100\,nC}{mm^2}$ is given

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N m^2}$$

$$a_y = \frac{100 \times 10^{-9}}{10^{-6}} \; \frac{1.60\times10^{-19}}{8.85\times10^{-12}} \; \frac{1}{9.11\times10^{-31}} \qquad \frac{C}{m^2}\;\frac{C\,Nm^2}{C^2}\;\frac{1}{kg}$$

$$= 10^{-1}\;10^{-7}\;10^{31}\;\frac{1.60}{8.85\cdot 9.11}\;\frac{m}{s^2} \approx 2\times10^{21}\;\frac{m}{s^2}$$

Was it reasonable to ignore gravity?

Q : What is the KE as the e^- slams into the + plate?

kinematic eqn : $v_f^2 = v_i^2 + 2a\,\Delta x$

have : $v_i = 0$, a $\Rightarrow$ want v_f

The answer depends on the plate separation !! (Δy)

why? $\longrightarrow$ we have a constant force $F_{E,y}$

the work : $F_y \, \Delta y$

$$\Delta PE = -W = -F_y \, \Delta y$$

Example : take $\Delta y = 1\text{ cm} = 10^{-2}\text{ m}$

$$F_y = \frac{\sigma e}{\varepsilon_0} \longrightarrow \frac{100 \times 10^{-9}}{10^{-6}} \; \frac{1.60 \times 10^{-19}}{8.85 \times 10^{-12}} \; \frac{C}{m^2} \; \frac{C}{C^2}\,Nm^2$$

$$\approx 2.0 \times 10^{-9} \text{ N}$$

seems small? but what is m_e?

$$\Delta PE \longrightarrow 2.0 \times 10^{-9} \times 10^{-2} \text{ Nm} = 2.0 \times 10^{-11} \text{ Nm}$$

" Joule

$$\therefore \frac{1}{2} m \left(v_f^2 - \underbrace{v_i^2}_{=0} \right) = 2.0 \times 10^{-11} \text{ Nm}$$

$$v_f^2 = \frac{4.0 \times 10^{-11} \text{ Nm}}{9.11 \times 10^{-31} \text{ kg}} \approx \frac{4}{9}\, 10^{20} \frac{m^2}{s^2}$$

$$v_f \approx \frac{2}{3} \times 10^{10} \frac{m}{s}$$

Q : is this reasonable? $\longrightarrow$ NO, since it exceeds the speed of light

The message : $\frac{100\,nC}{mm^2}$ is a big surface charge density

(be careful with results - or when assigning problems)

Applications : vacuum tubes : heated filament allows electrons out of metal, they hover above the metal surface

Electric plates (grids) are used to accelerate electrons and also to deflect them

Electrons slamming into metal plate with high KE $\rightarrow$ some of that energy is converted to radiation $\rightarrow$ X ray tubes (dentist, medical imaging)

- old-fashioned TV (oscilloscope) picture tubes :
  deflected e^- hit phosphorous screen $\rightarrow$ well-controlled light emission

- modern screens (digital) : $\vec{E}$ fields are controlled at the pixel level, no more e^- beams, more sophisticated mechanisms to control light output

Note : Chapter 18.1 emphasized the PE between two point charges q_1 and q_2 : (Example 18.3)

$$\Delta PE_Q = + \frac{k q_1 q_2}{r}$$

Important in CHEM + PHYS