Electric Potential → Voltage, after all

probe q + charge Q | charge Q (w/o probe)

$\boxed{\text{Force}} \longrightarrow \boxed{\text{Field}}$

neg. work $\downarrow$ | $\updownarrow$?

$\boxed{\text{potential energy}} \longrightarrow \boxed{\text{voltage}}$ = potential

probe q interacts with charge Q

go L → R:

multiply by probe charge q

(gravity: by mass m)

Motivation: The $\vec{E}$-field serves the purpose to describe what a charge Q does to space → a force effect on any charged particle q (the probe)

$\vec{E}(\vec{r})$ is a vector quantity, determines $\vec{F}_{\text{on } q}$

If we define $V(\vec{r})$ = potential, a scalar quantity for different positions $\vec{r}$ ⇒ easily figure out the KE of any charge q at different $\vec{r}$.

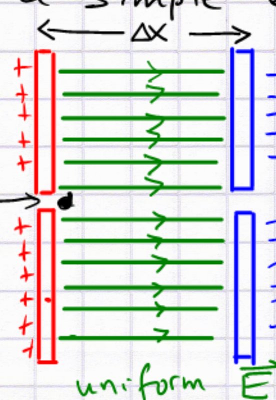
Electric potential: $V(\vec{r}) = \dfrac{U(\vec{r})}{q}$ unit = Volt $= \dfrac{J}{C} = \dfrac{Nm}{C}$

Use the parallel-plate capacitor as a simple example.

It provides a constant $\vec{E}$ field

introduce a positive probe $q > 0$ through hole in + plate

→ $\vec{E}$ accelerates it to the right over distance ΔX



Particle of charge q gains $PE = -W = -qE\Delta x$

divide out q: potential @ $+$ → define as $V(x=0)=0$

potential @ $-$ → define as $V(\Delta x) = -E\Delta x$

Potential difference between plates → $-E\Delta x = V(\Delta x)$

Was there a reason to define $V(x=0)=0$?

→ arbitrary choice

→ between plates: $\Delta V = -E\Delta x$

$$\frac{\Delta V}{\Delta x} = -E$$

For small separations $\frac{dV}{dx} = \lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -E$

generalize beyond the constant-field (parallel-plate cap.) example → $E_x(x) = -\frac{dV}{dx}$

where $V(x)$ is the local electric potential

Note: the change of motion of charge q does not depend on absolute values of electric potential, just on potential difference ≡ voltage applied

When $q>0$ goes from high potential → low potential

it converts electric PE → KE

For $q<0$ (e.g., electron) it is the other way around!!

Fig. 18.8 A, B

Why? higher potential $\equiv$ more positively charged region

$\Rightarrow$ a negative charge q is attracted to that $\Rightarrow$ speeds up
gains KE

The simple relation:

potential difference = "so many Volts" (battery, power supply)

leads to so much gain (or loss) in KE
for a charged particle (charge q)

$\Rightarrow$ define a "simpler" energy unit than $Nm = J$:

A charge unit $e = 1.60 \times 10^{-19}$ C accelerated by a voltage (= potential difference) of xx Volts

$\longrightarrow$ gain (or loss) in KE of xx eV

electron-volt $= 1.60 \times 10^{-19}$ J

top-notch particle accelerators $\nearrow$ Fermilab, Chicago
$\searrow$ LHC / CERN, Geneva

Tera-eV energies

KeV, MeV, GeV, TeV $\quad 10^3, 10^6, 10^9, 10^{12}$ eV

A standard battery (1.5 Volts) accelerates an e^- to 1.5 eV

In the H-atom the e^- is bound by 13.6 eV $\quad E_{1s} \sim Z^2$!

In the $U^{235/238}$ atom(s) the innermost (1s) e^- is bound by 116 keV.

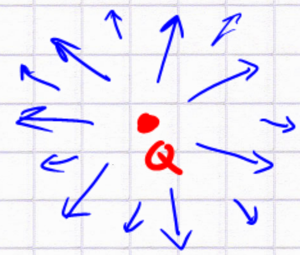
Potential of a point charge (nucleus $\approx$ sphere of radius xx 10^{-15} m looks like a point on 10^{-10} m scale)

$$E(r) = \frac{kQ}{r^2}$$

Q = nuclear charge = Z

= # of protons

$$\vec{E}(\vec{r}) = \frac{kQ}{r^2}\hat{r} = \frac{kQ}{r^3}\vec{r}$$



The $\vec{E}$ field is radial

The strength $\sim \frac{1}{r^2}$

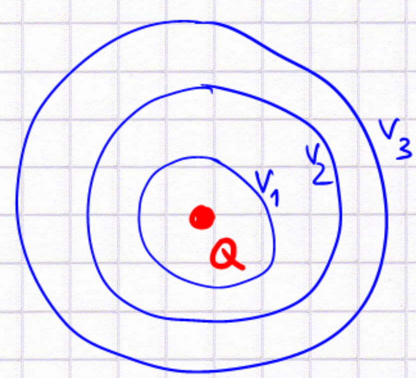
$$E(r) = -\frac{dV}{dr} \qquad \therefore \quad V(r) = \frac{kQ}{r}$$

why? $\frac{dV}{dr} = kQ\left(\frac{1}{r}\right)' = -\frac{kQ}{r^2}$

$$\therefore \quad E(r) = -\frac{dV}{dr} = +\frac{kQ}{r^2}$$

The function $V(r) = \frac{kQ}{r}$ is all we need to understand the radial vector field!

→ equipotential lines



Lines of equal potential

$\vec{E}$ is perpendicular to them

- moving a probe around such a line → no change in KE!!

→ look at potential values in charges-and-fields.swf software