Uniform Circular Motion

Definitions:

Period T = time to go once around

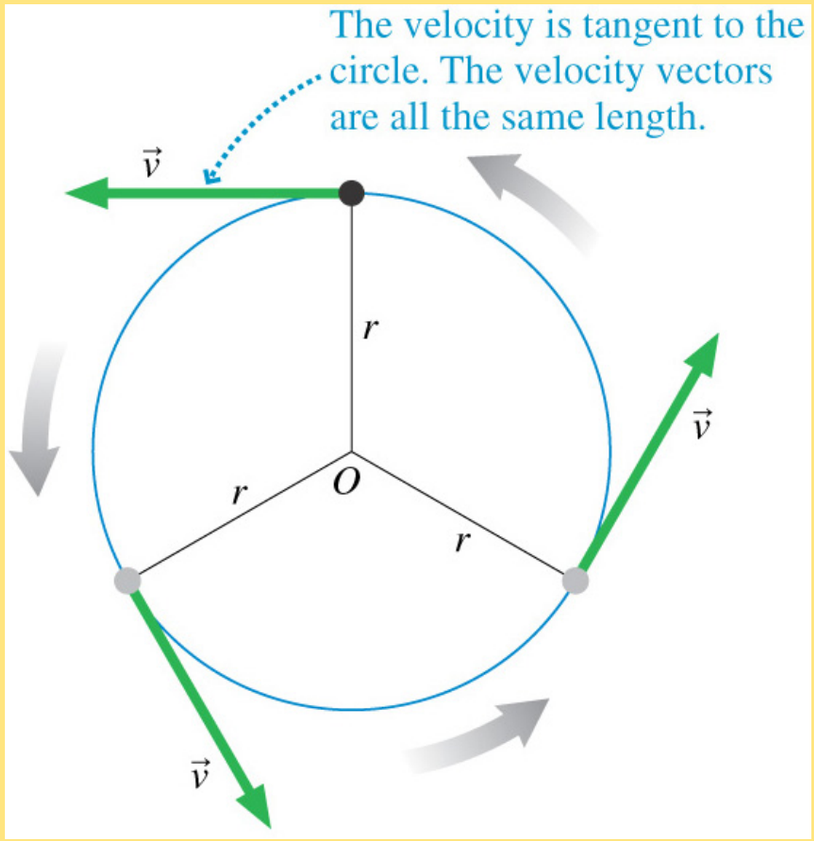
Speed $v = \frac{2\pi r}{T}$

Revolution $1\text{rev} = 360^\circ = 2\pi \text{ rad}$

Radians $1\text{rad} = 1\text{rad} \times \frac{360^\circ}{2\pi \text{ rad}} \approx 57.3^\circ$

Arc length $s = r\theta$, when θ measured in rad

Angular position θ changes with time t



Angular velocity ω is constant in t

In general: $\omega \equiv \frac{d\theta}{dt}$ a function of time t

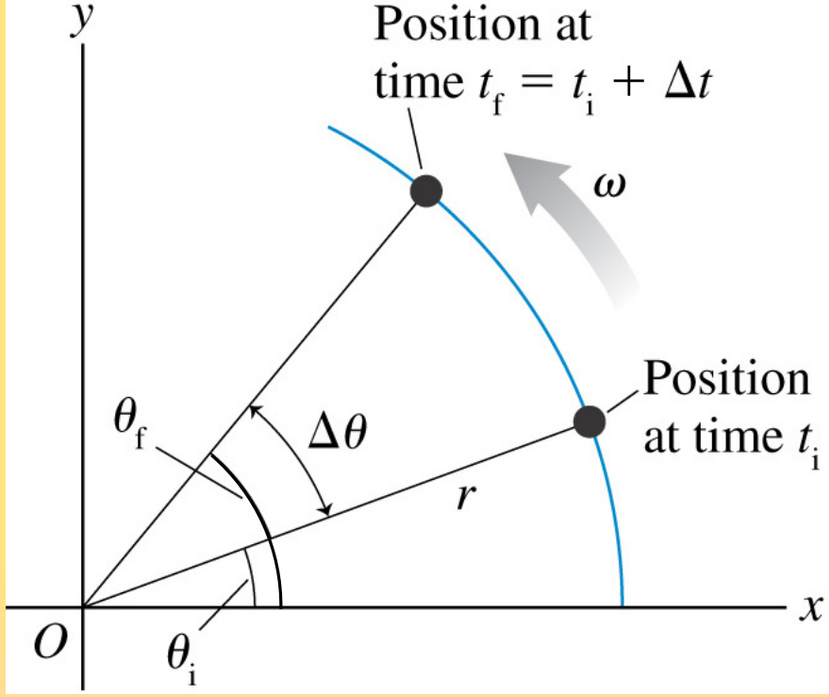
$x = r\cos\omega t$, $y = r\sin\omega t$, $\theta = \omega t$

position vector $\vec{r}(t) = ?$

$\vec{r}(t) = r\cos\omega t\,\hat{\imath} + r\sin\omega t\,\hat{\jmath}$ $\quad \vec{v}(t) = ?$

$\vec{v}(t) = -r\omega\sin\omega t\,\hat{\imath} + r\omega\cos\omega t\,\hat{\jmath}$ $\quad \vec{a}(t) = ?$

$\vec{a}(t) = -r\omega^2\cos\omega t\,\hat{\imath} - r\omega^2\sin\omega t\,\hat{\jmath} = -\omega^2\vec{r}$ (!)



Calculate $v = |\vec{v}|$, $a = |\vec{a}|$

$v^2 = \omega^2 r^2$, i.e., $v = \omega r$; $\quad a = \omega^2 r = \frac{v^2}{r}$ centripetal acc.