Elasticity

Rod clamped at one end is pulled at the other by a force $\vec{F}$

acts like a spring with huge constant k ?

Hooke: $|\vec{F}| = k\Delta L$ really ?

yes, for small ΔL ;

then less force required to stretch

within elastic limit (EL):

rod restored to length L when $F \to 0$

beyond EL: returns to larger length L'

material property ? (take out L, A)

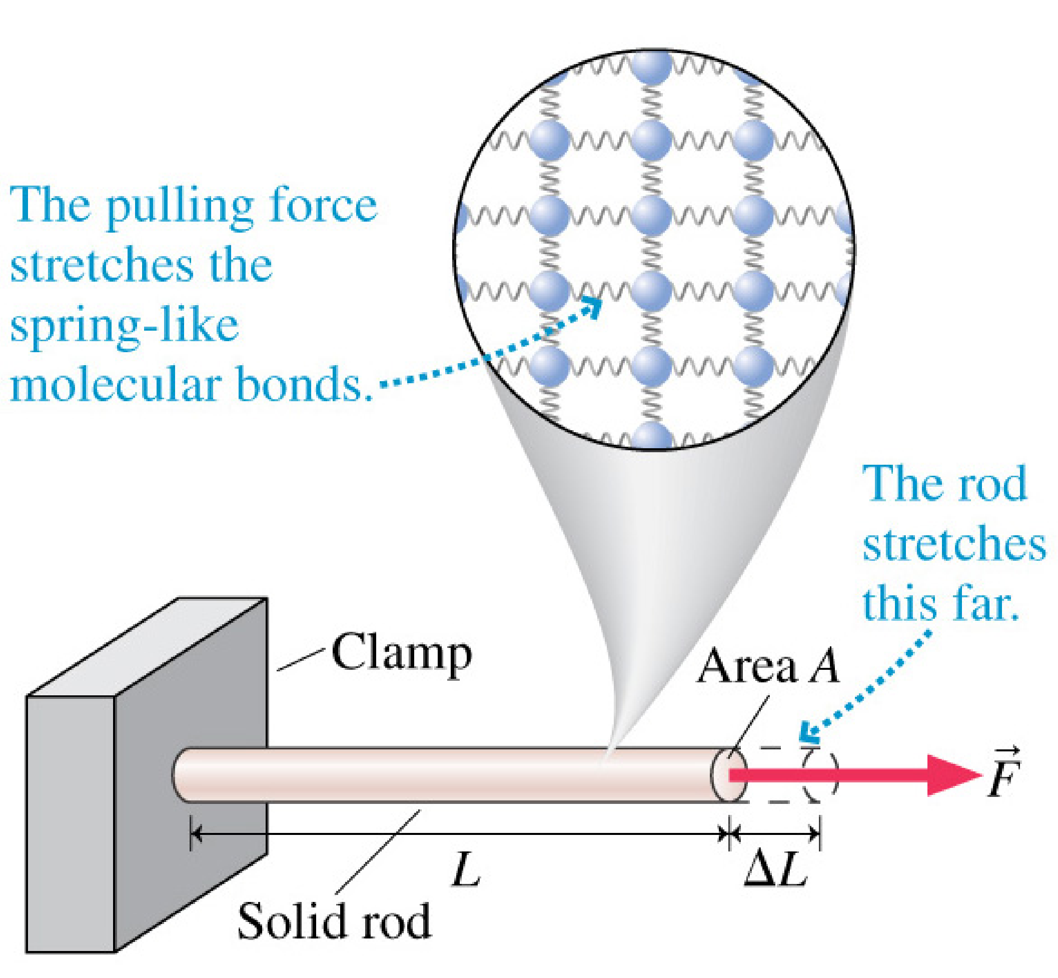
define: strain = $\frac{\Delta L}{L}$ tensile stress = $\frac{F}{A}$

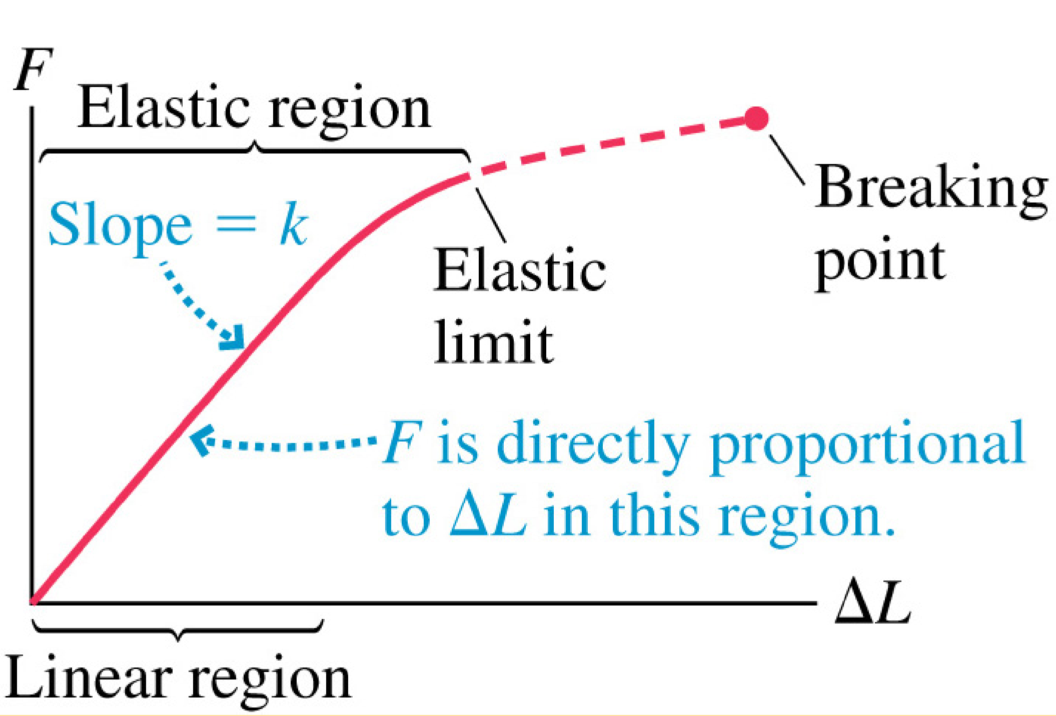
strain=dimensionless, $[F/A] = \mathrm{N/m^2}$

How can we generalize Hooke's law ?

$$\frac{F}{A} = Y\frac{\Delta L}{L} \text{ ?}$$

Y = Young's modulus; $[Y] = \mathrm{N/m^2}$





Tensile Stress

$F = k\Delta L \rightarrow$

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

apply Hooke to bond layer in longitudinal direction:

# of parallel springs ~ A

force per bond ~ F/A

Now the RHS:

# of springs in series ~ L

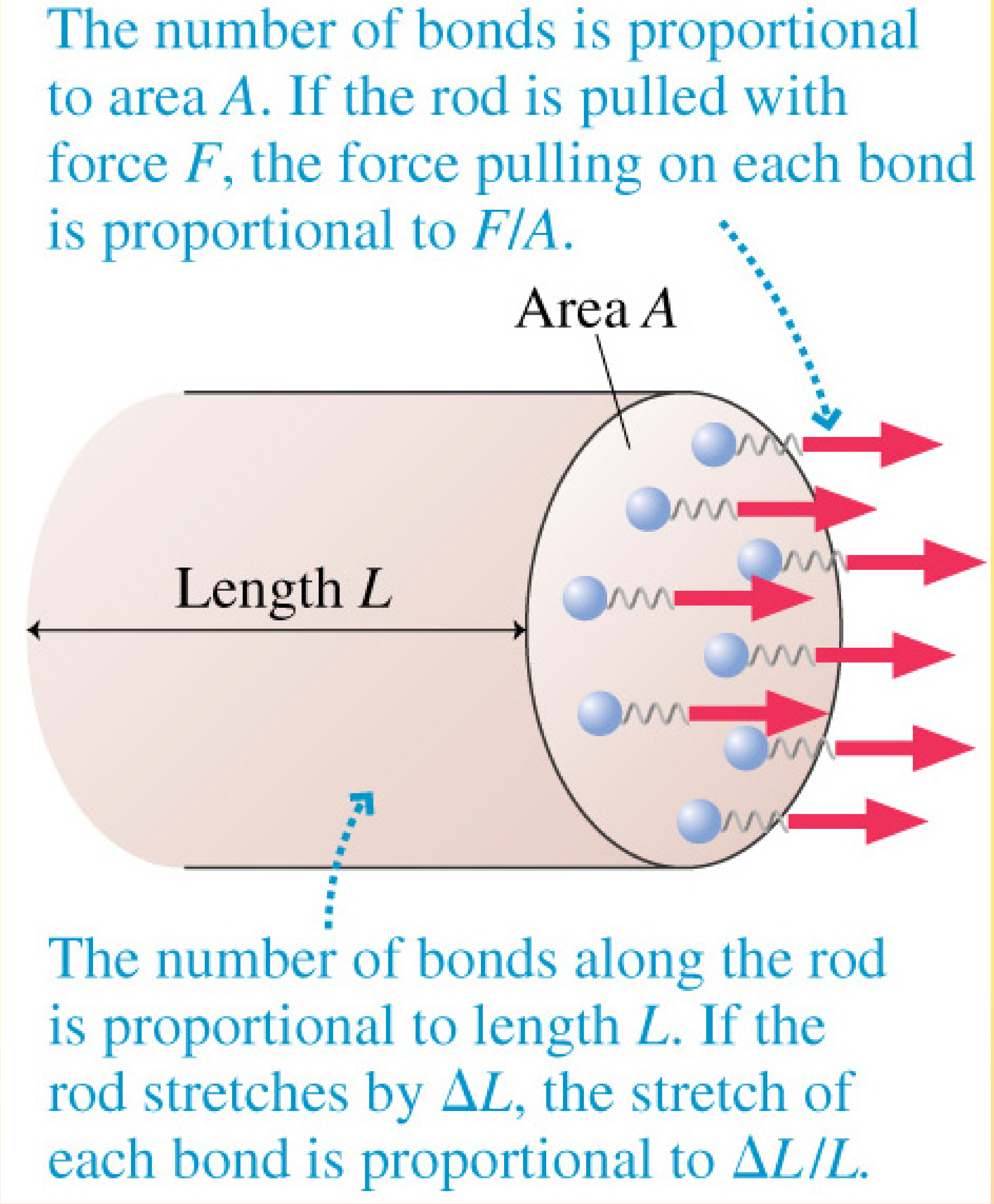
stretch per bond ~ $\Delta L/L$

'bond spring constant' k:

$$Y = k\frac{L}{A}$$

Metals:

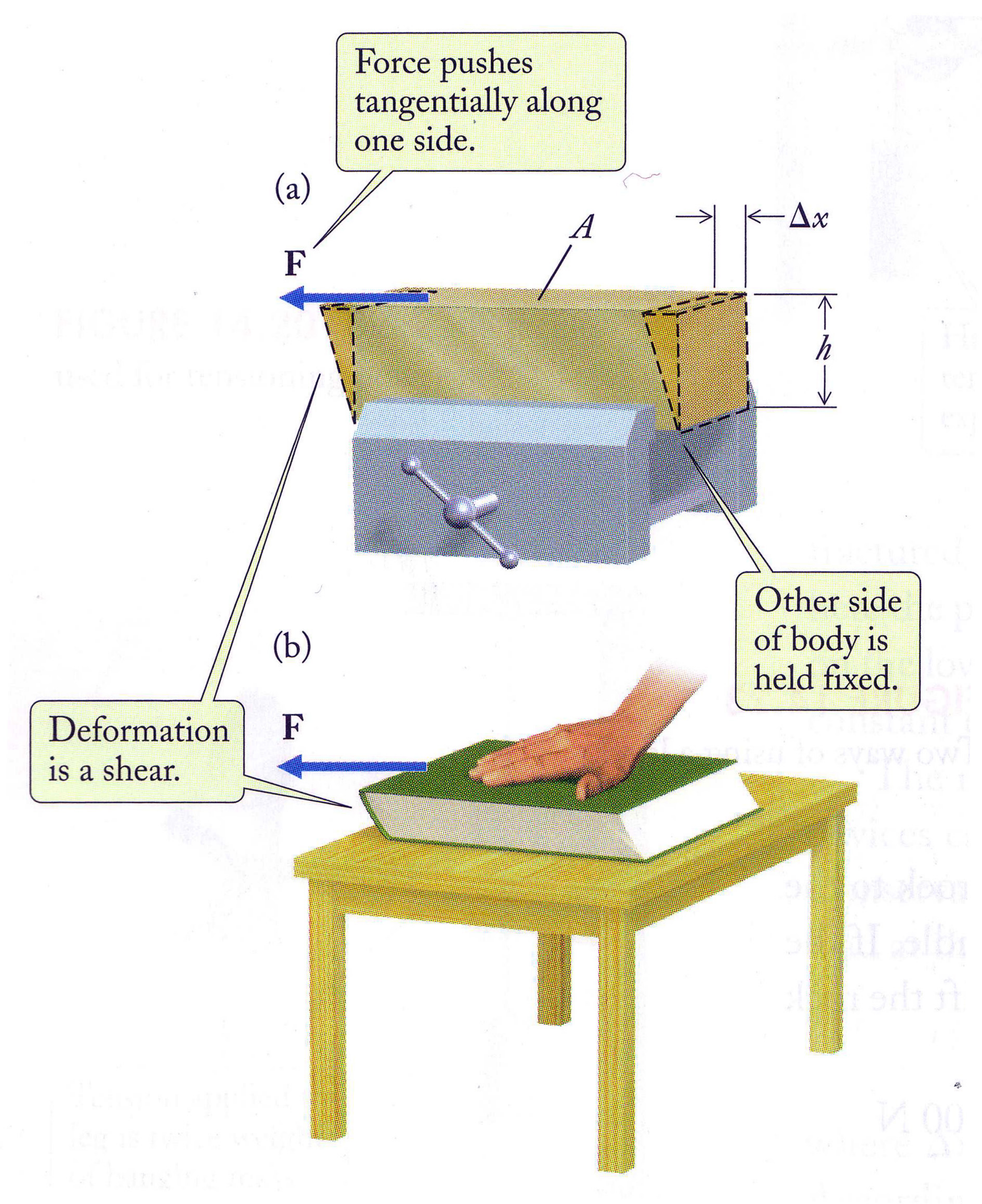
Y is multiple of $10^{10}\ \frac{\text{N}}{\text{m}^2}$



Shear Stress

cross section: rectangle → parallelogram (sliding atom layers)

force is tangential, applies to top area (hand covering entire book)



$$\frac{F}{A} = S\frac{\Delta x}{h}$$

why is the shear strain $\Delta x/h$?

longer h: same Δx requires less displacement of atomic layers (bonds)

S = shear modulus

Measurements:

$S < Y$ (same units!) by typically a factor of 3

Lead (Pb) has S, Y a factor of 14 less than steel

Concrete can be pulled (Y comparable to lead), but does not shear

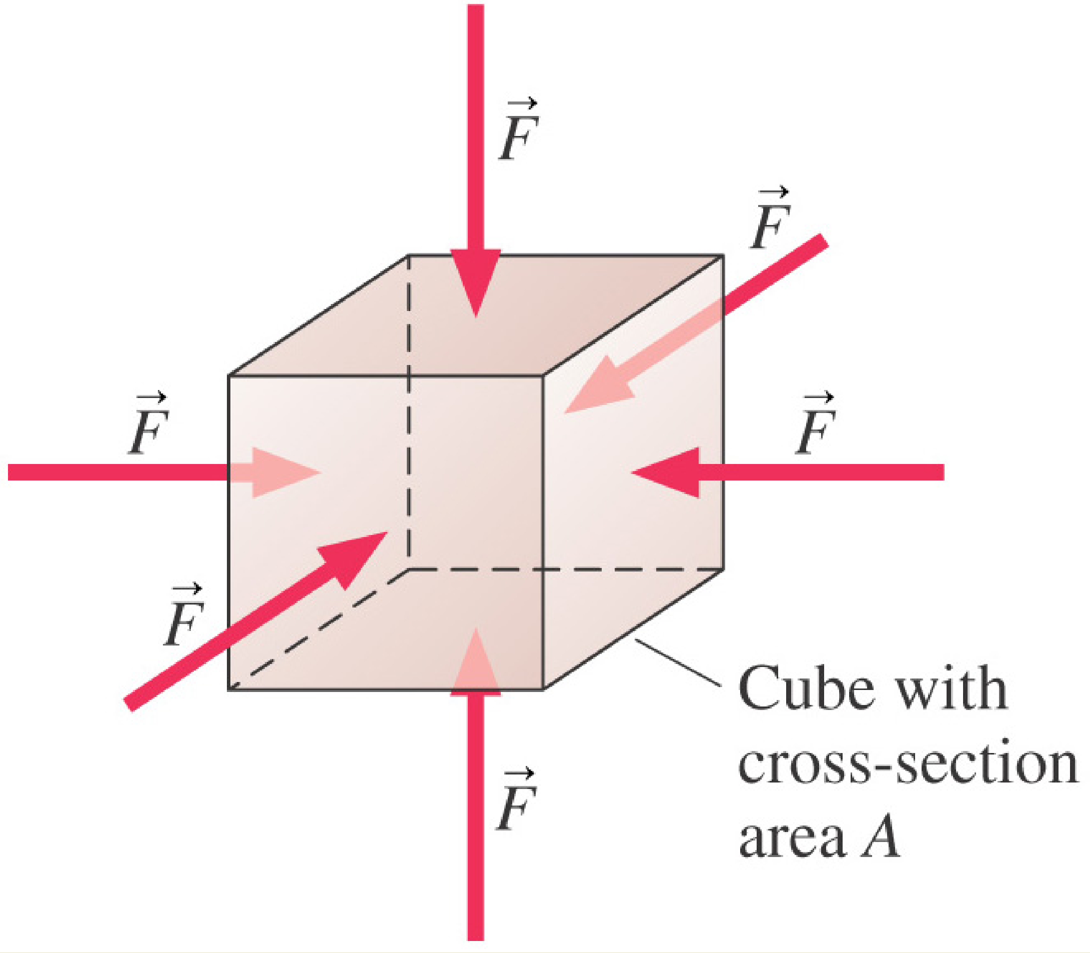
Civil engineering applications!

Bulk Modulus

Compress entire object by applying force to all faces. F/A = pressure, now.

How hard is it to reduce the volume?

Immerse object in deep water



$$\frac{F}{A} = -B\frac{\Delta V}{V}$$

B = bulk modulus; $\Delta V/V$ = volume strain; F/A = volume stress.

A positive stress causes a volume decrease ($\Delta V < 0$) $\Rightarrow$ $-$ sign.

Bulk modulus of liquids is well defined. B_{H_2O} is about $0.02 B_{Fe}$

Steel, brass: $S < B < Y$; softer metals (Pb, Al) $S < Y < B$

Bigger modulus: less strain from same stress

Elongation or shear of liquids requires very little force

Why do solids and liquids behave this way $\Rightarrow$
condensed matter physics: microscopic view based on electrostatic forces and many-body quantum mechanics