Guide to: standing waves on a string

Applications: notes produced by string instruments.

Basic idea: a left- and right-traveling wave of identical amplitude and frequency add to form a standing wave; for a string fixed at two endpoints a condition for allowed wavelengths (frequencies) emerges; the fundamental frequency is the first harmonic f_1, with integer multiples of this frequency it forms the spectrum of allowed frequencies for the string $f_n = n f_1 \quad n = 1, 2, ...$.
The fundamental has wavelength $\lambda_1 = 2L$, twice its length, for the higher harmonics $\lambda_n = 2L/n$.
Thus, $f_n = c_w/\lambda_n = n c_w/2L$. The propagation speed $c_w = \sqrt{F_t/\mu}$ depends on the string tension force and on the linear mass density for the string.

Derivations: 1) the addition of left- and right-traveling waves to form a product of a purely time-dependent amplitude times a spatial pattern (in the class notes, not in the book!)

2) the allowed wavelengths when the amplitude is forced to be zero at $x = L$ in addition to $x = 0$.
(in the class notes, not in the book!)

Examples: strings of given mass and length producing a fundamental note: derive the tension; change in tension leads to what change in the fundamental?

Equations: wavelength-frequency-propagation speed relation: $\lambda f = c_w$
right/left-traveling monochromatic wave: $y(x,t) = A \sin(kx \mp \omega t) \quad k = 2\pi/\lambda \quad \omega = 2\pi f$
trig addition theorem: $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin\alpha \cos\beta$
infinitely many solutions of $\sin(2\pi L/\lambda) = 0$ labeled by integer n: $2\pi L/\lambda = n\pi$ or: $\lambda_n = 2L/n$
string wavelength and sound wavelength are different; frequency is the same.

Problems: 12.46-59, 12.68; section 13.3 problems are for standing sound waves (wind instruments) for which the wavelength conditions can be different (open end vs closed end); the propagation speed is the speed of sound (which changes with temperature; cf.13.71). change in resonator length (13.68). Lasers, laser diodes, and quartz crystals work on the principle of using mirrors (or polished crystals) to define a standing-wave condition for light waves and radio waves respectively. Microwave technology does a similar thing.