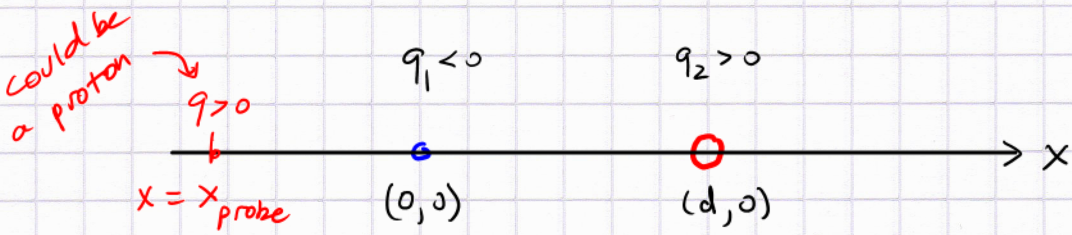
Problem 17.24 Two point charges [$q_1 = -1.5\ C$, at $(0,0)$] and [$q_2 = +5.0\ C$, at $(1.2, 2.5)\ m$] are fixed. Where can one put an e^- in the plane (x,y), such that it will be in equilibrium?

Solution: The distance between q_1 and q_2 is $d = \sqrt{1.2^2 + 2.5^2}\ m$ $(d = 2.77\ m)$

First, solve the problem in one dimension:



For which x range is it possible to get a cancellation of the two Coulomb forces?

$$F_{on\ q} = F_{q_1\ on\ q} + F_{q_2\ on\ q}$$

Assume $q > 0$ = probe

The ranges: (A) $x < 0$: $\begin{cases} F_{q_1\ on\ q} > 0 & \text{(to the right)} \\ F_{q_2\ on\ q} < 0 & \text{(to the left)} \end{cases}$

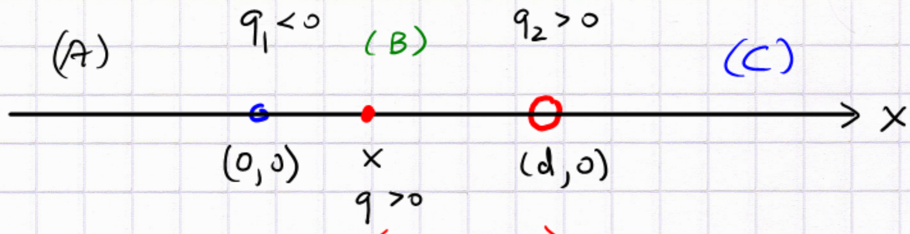
cancellation is possible!

(B) $0 < x < d$: $\left. \begin{array}{ll} F_{q_1\ on\ q} < 0 & \text{(to the left)} \\ F_{q_2\ on\ q} < 0 & \text{(to the left)} \end{array} \right\}$ always add up to bigger force

(C) $x > d$: $\left. \begin{array}{ll} F_{q_1\ on\ q} < 0 & \text{(to the left)} \\ F_{q_2\ on\ q} > 0 & \text{(to the right)} \end{array} \right\}$ cancellation is possible

The region between the charges (B) is excluded, but nevertheless, we will use x as a coordinate (with sign) for x. The distances^2 to the charges: $d_1^2 = x^2$, and $d_2^2 = (d-x)^2$ enter Coulomb's law.

Force-free condition (equilibrium):



distance $|d-x|$ works for all x!

distance $|x|$ also works for all x

$$\underbrace{\frac{K|q_1 q|}{x^2} = \frac{K|q_2 q|}{(d-x)^2}}$$

1) magnitudes statement, has no sign info, that's why we discussed carefully regions (A), (B), (C)

2) sign of probe charge becomes irrelevant, especially when cancellation of forces is what we are looking for.

3) if a $0 < x < d$ solution is obtained to this 'force magnitude' equation, then it just means that both charges push the probe by an equal amount! (need to exclude it) $\rightarrow$ now solve (cancel $K|q|$)

$$\frac{|q_1|}{x^2} = \frac{|q_2|}{(d-x)^2} \quad \therefore \quad \left(\frac{d-x}{x}\right)^2 = \left|\frac{q_2}{q_1}\right| \equiv R$$

charge magnitude ratio

$$\therefore \quad \frac{d-x}{x} = \pm\sqrt{R}$$

$\leftarrow$ keep both possible solutions for now

$$d - x = \pm\sqrt{R}\, x \quad \therefore \quad x(1 \pm \sqrt{R}) = d$$

$$\boxed{x_{1/2} = \frac{d}{1 \pm \sqrt{R}}}$$

It looks like solutions are possible for both $x < 0$, and for $x > d$? [This will depend on the charge ratio: for $R < 1$ no $x < 0$ soln!]

Now analyze for our numbers: $R = \frac{5.0}{1.5} = 3.33$

$$x_{1/2} = \frac{2.77}{1 \pm 1.83}\ m \quad \therefore \quad x_1 = 0.979\ m = 0.98\ m$$

$$x_2 = -3.34\ m = -3.3\ m$$

Now check out x_1: it falls into the $0 < x < d$ range $\therefore$ it represents the case where both q_1 and q_2 push q equally in the same direction.

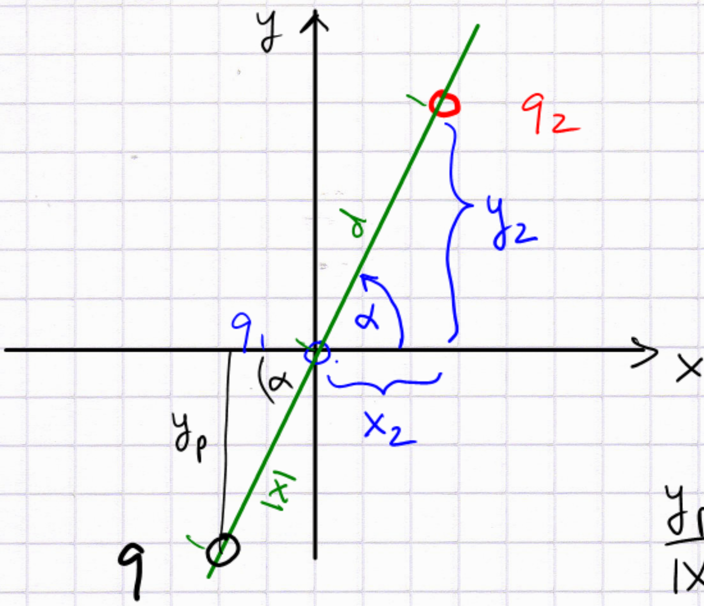
Answer: At $x = -3.3\ m$ any charged particle ($q < 0$ or $q > 0$) will be force-free.

Now ask: could we ever get a working positive root $x > d$? It all depends on R: the equilibrium point has to be on the side of the weaker charge. The stronger opposite charge then is further away and can be cancelled by a weaker closer charge.

The rest is a bit of geometry, but let's ask why we only consider points along a line?

A: Coulomb's force is radial, draw the probe charge at $y \neq 0$ $\Rightarrow$ cancellation is not possible by vector addition

We can't cancel x and y components at the same time.



$$\alpha = \tan^{-1}\left(\frac{y_2}{x_2}\right)$$

$$\alpha = \tan^{-1}\left(\frac{2.5}{1.2}\right) = 64.4^\circ$$

$$\frac{y_p}{|x|} = \sin\alpha = 0.902$$

$$y_p = 2.98 \cdot 0.902 \text{ m} = 2.69 \text{ m}$$
$$= 2.7 \text{ m}$$

↑ length, not coordinate!

$$x_p = |x| \cos\alpha = 1.29 \text{ m} = 1.3 \text{ m}$$

The probe q is in equilibrium at

P (−1.3, −2.7) m.