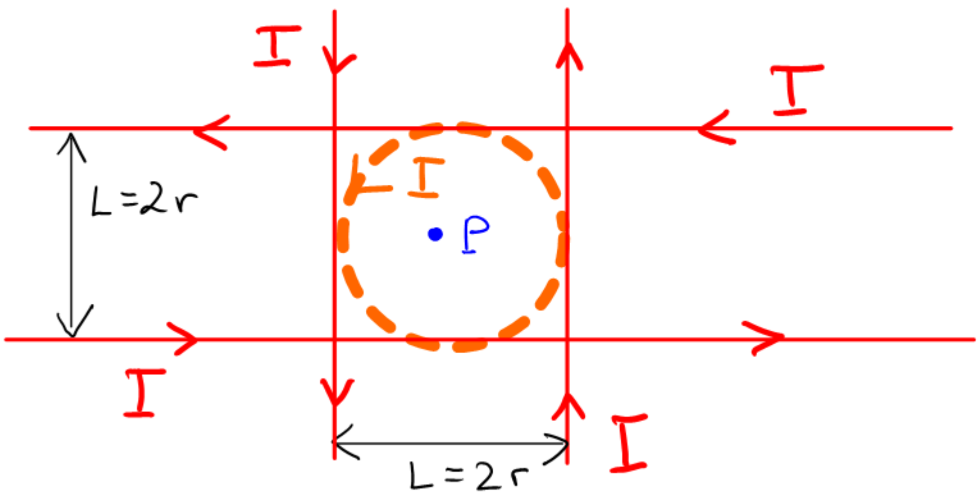
PhysicsTutor

Magnetic field

Problem:

- Four long straight wires, each with current *I* overlap to form a square with side 2*r*.
- Find the magnetic field at the center of the square, and compare it with the result from the formula for a circular loop of current *I*.



Relevant ideas:

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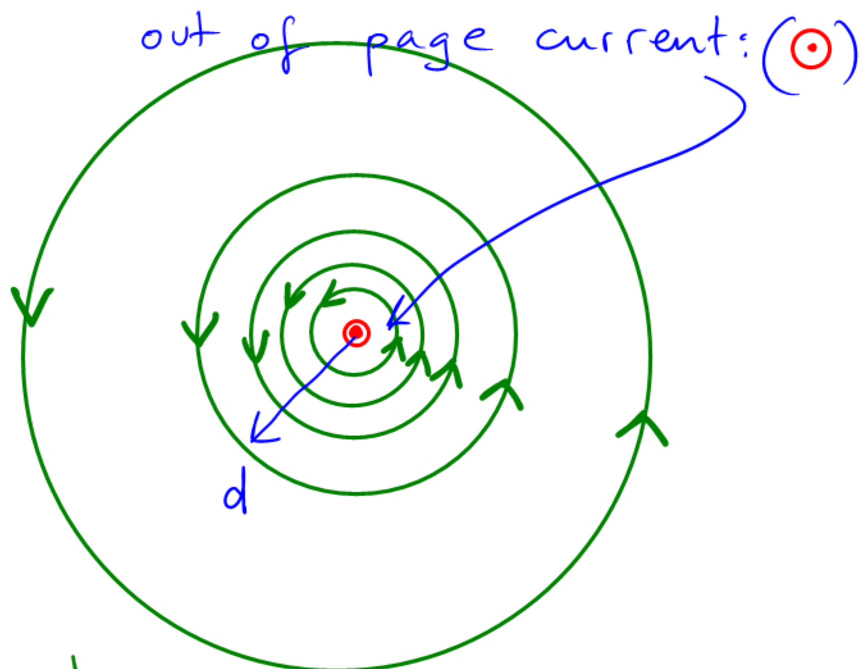
- A long current-carrying wire is surrounded by a magnetic field whose strength drops as 1/d.

$$B = \frac{\mu_0 I}{2\pi d}$$

strength of B falls off with distance d



Spacing of field lines grows with d.

Relevant ideas:

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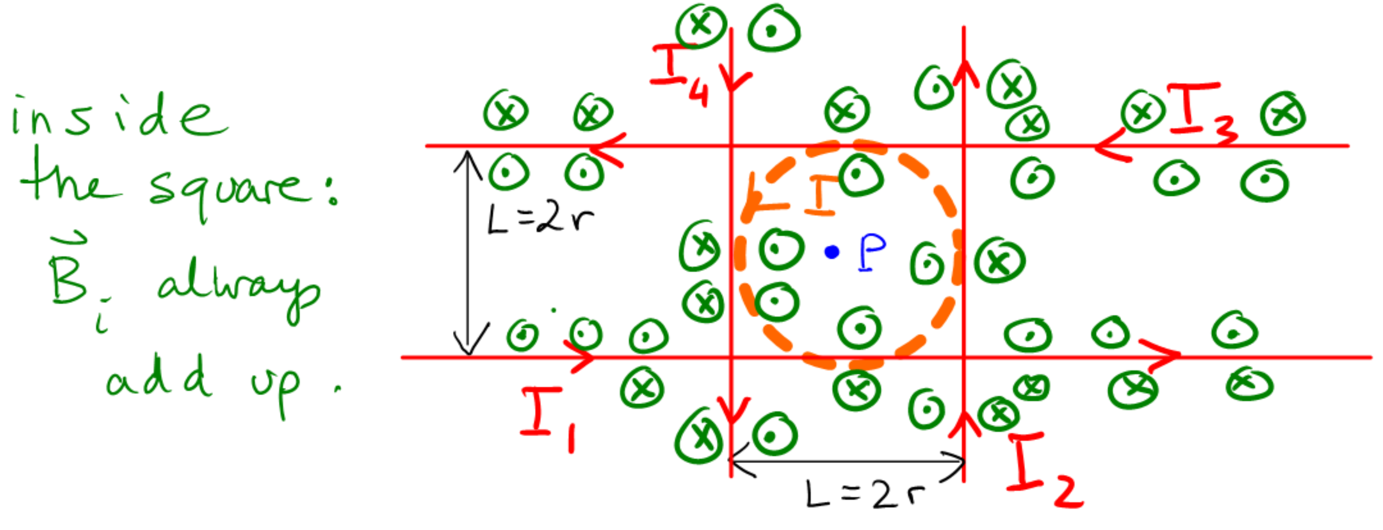
- The field lines are circles about the wire. Use the Right-Hand rule to find the direction of $\mathbf{B}_i$ at *P* for wires i = 1,..,4.



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- A long current-carrying wire is surrounded by a magnetic field whose strength drops as 1/d.
- The field lines are circles about the wire. Use the Right-Hand rule to find the direction of $\mathbf{B}_i$ at P for wires $i = 1,..,4$.
- The net magnetic field adds all contributions.

Equations associated with ideas:

$$B = \frac{\mu_0 I}{2\pi d} \; ; \qquad B_i = \frac{\mu_0 I_i}{2\pi d} \qquad \left(d = \frac{L}{2} = r\right)$$

$$B_{net} = \sum_{i=1}^{4} B_i$$

← inside the square fields allways add (do not cancel)

$$B_{circ.\,loop} = \frac{\mu_0 \cdot I}{2r}$$

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- In the comparison with the circular loop result look for a change of the type "$2\pi r$ becomes $8r$", as is the case for the circumference of the circle versus the perimeter of a square.

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- $B_i = \dfrac{\mu_0 I_i}{2\pi d}$ $\qquad B_{net} = \sum_{i=1}^{4} B_i = \dfrac{\mu_0 I}{2\pi} \dfrac{1}{(L/2)} \underbrace{\sum_{i=1}^{4} 1}_{=4}$

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- $B_{net} = \dfrac{2\mu_0 I}{\pi r}$; $B_{loop} = \dfrac{\mu_0 I}{2r}$
- $\dfrac{B_{net}}{B_{loop}} = \dfrac{4}{\pi} \approx 1.3$
- Field is stronger by the same ratio: $\dfrac{4 \cdot 2r}{2\pi r}$

as in the perimeter/circumference problem.

At first, this is not obvious that $B_{net} > B_{loop}$, in the loop case the current is closer to P on average!
Straight-wire case: the wires are assumed to be very (∞-ly) long!