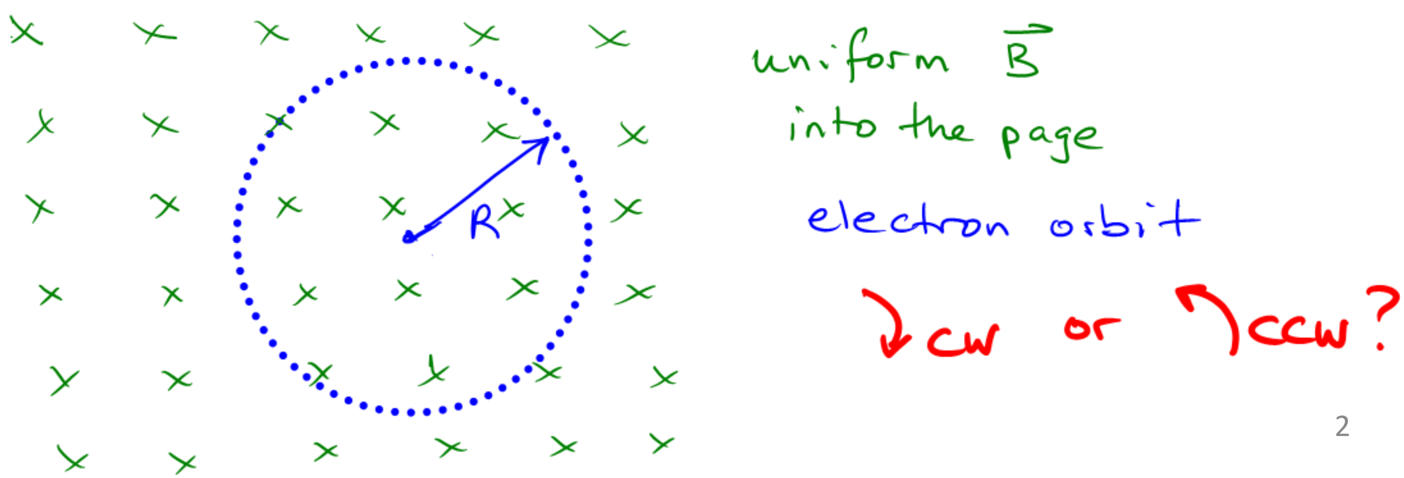
PhysicsTutor

Magnetic field and electron motion

Giambattista 19.112

Problem:

- An electron moves in a circle of radius *R* in a uniform field **B**, which is into the page.
- Does the electron move clockwise or counterclockwise?
- Derive an expression for the orbit time.



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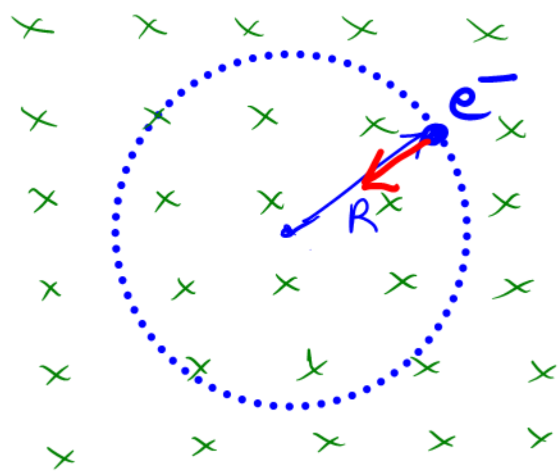
$$\frac{mv^2}{R} = F_M$$



$\vec{a}_{cp}$

$$a_{cp} = \frac{v^2}{R}$$

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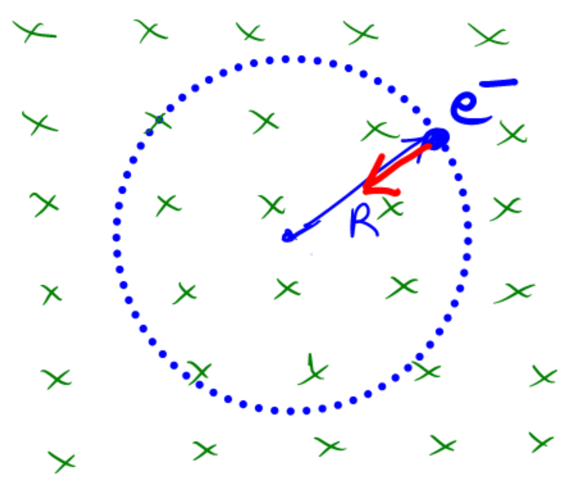
- The magnetic force provides the centripetal acceleration.
- The orientation of the orbit follows from the evaluation of the cross-product rule: **v** cross **B**, multiplied by the negative electron charge.



$\vec{a}_{cp}$

assume: e^- $\vec{v}$ clockwise

$\vec{F}_M = q\,\underbrace{\vec{v}\times\vec{B}}$

$q=-e$ reverses direction

$\vec{F}_M$ e^- $\vec{v}$

$\vec{v}\times\vec{B}$ orientation

∴ yes, clockwise works!

Relevant ideas:

- The magnetic force provides the centripetal acceleration.
- The orientation of the orbit follows from the evaluation of the cross-product rule: **v** cross **B**, multiplied by the negative electron charge.
- Linear speed *v* and circumference determine the orbit time.

$$v = \frac{2\pi R}{T}$$

← one revolution

← period = orbit time

Equations associated with ideas:

$$\vec{F}_M = q\,\vec{v} \times \vec{B} \quad , \quad |\vec{F}_M| = |q v B \sin(\measuredangle \vec{v}, \vec{B})|$$

$$a_{cp} = \frac{v^2}{R} \qquad v = \frac{2\pi R}{T} \quad \therefore \quad T = \frac{2\pi R}{v}$$

$$\sin(\measuredangle \vec{v}, \vec{B}) = 1 \quad \therefore \quad |F_M| = |q| v B = e v B$$

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- The time for one orbit: $T = 2\pi R/v$. The speed follows from the condition derived above.
- The orientation for traversing the orbit: use the right-hand rule for the direction of $\mathbf{v}\times\mathbf{B}$, then reverse the sign ($q=-e$) for an electron.

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- Orientation of the orbit: "trial-and-error" method.
- Assume an orientation (e.g., CW), with the direction of $\vec{v}$ evaluate $q\,\vec{v}\times\vec{B}$ orientation ($q=-e<0$ for e^-), check whether $\vec{F}_M = m\vec{a}_{cp}$ is consistent with assumption. → accept → reject and confirm.