PhysicsTutor mh

Capacitor and Energy

Giambattista 17.75

Problem:

- A parallel-plate capacitor holds a charge of 5.5×10^{-7} C on its plates (positive on one, negative on the other). The distance between the plates is increased by 50%, while the charge on the plates stays the same.
- What happens to the energy stored in the capacitor?

Relevant ideas:

Relevant ideas:

- Energy stored in a capacitor: PE = $Q^2/(2C)$

Note: using $\Delta V_c = \frac{Q}{C}$ this can also be written as $PE = \frac{(C\Delta V_c)^2}{2C} = \frac{C}{2}\Delta V_c^2$, but we are given Q, which is held constant.

Relevant ideas:

- Energy stored in a capacitor: PE = $Q^2/(2C)$
- Capacitance C changes with the plate separation. Smaller distance = larger capacitance.

$C \sim A$ (A=area of plates)

$C \sim \frac{1}{d}$ (d = plate separation)

$C = \varepsilon_0 \frac{A}{d}$

material property of capacitor

- change ε_0 : insert dielectric instead of air
- change area of overlap → typical variable capacitor (old fashioned)

change d → not very practical, has to be small

Relevant ideas:

- Energy stored in a capacitor: PE = $Q^2/(2C)$
- Capacitance C changes with the plate separation. Smaller distance = larger capacitance.
- Plate distance d is increased, i.e., capacitance is reduced. PE increases in proportion with d, since the charge Q remains the same.

$C \sim \frac{1}{d}$ $\quad$ $PE \sim \frac{1}{C}$ $\quad \therefore \quad$ $PE \sim d$

increases with d

Equations associated with ideas:

$PE = \frac{1}{2C} Q^2$ $\qquad$ $C = \varepsilon_0 \frac{A}{d}$

$\Delta V_C = \frac{Q}{C}$ ← not needed here, but we may want to know

$PE = \frac{C}{2} (\Delta V_C)^2$

ΔV_C will change like PE, as $Q = \text{const}$, C decreases

quadratic increase in $(\Delta V_C)^2$ is compensated by decrease in C.

Strategy

Strategy

- Relate the change in capacitance C_2/C_1 after the distance change $d_2/d_1=3/2$.

Strategy

- Relate the change in capacitance C_2/C_1 after the distance change $d_2/d_1=3/2$.
- Use the ratio of the capacitances C_2/C_1 to express the potential energy change $\mathrm{PE}_2/\mathrm{PE}_1$.

Strategy

- Relate the change in capacitance C_2/C_1 after the distance change d_2/d_1=3/2.
- Use the ratio of the capacitances C_2/C_1 to express the potential energy change $\mathrm{PE}_2/\mathrm{PE}_1$.
- Additional question: what happens to the voltage drop across the plates, ΔV_C?
- It also increases linearly, since $\Delta V_C = Q/C$.

Solution

Solution

- $d_2/d_1 = 3/2$ (50% increase). $\frac{C_2}{C_1} = \frac{\varepsilon_0 \frac{A}{d_2}}{\varepsilon_0 \frac{A}{d_1}} = \frac{d_1}{d_2}$

Solution

- $d_2/d_1 = 3/2$ (50% increase). $\dfrac{C_2}{C_1} = \dfrac{\varepsilon_0 \frac{A}{d_2}}{\varepsilon_0 \frac{A}{d_1}} = \dfrac{d_1}{d_2}$
- $\therefore \dfrac{C_2}{C_1} = \dfrac{2}{3}$ $\therefore \dfrac{PE_2}{PE_1} = \dfrac{Q^2/2C_2}{Q^2/2C_1} = \dfrac{C_1}{C_2} = \dfrac{3}{2}$

Solution

- $d_2/d_1 = 3/2$ (50% increase). $\frac{C_2}{C_1} = \frac{\varepsilon_0 \frac{A}{d_2}}{\varepsilon_0 \frac{A}{d_1}} = \frac{d_1}{d_2}$
- $\therefore \frac{C_2}{C_1} = \frac{2}{3}$ $\therefore \frac{PE_2}{PE_1} = \frac{Q^2/2C_2}{Q^2/2C_1} = \frac{C_1}{C_2} = \frac{3}{2}$
- PE increases by 50% (prop. to increase in d)

Solution

- $d_2/d_1 = 3/2$ (50% increase). $\dfrac{C_2}{C_1} = \dfrac{\varepsilon_0 \frac{A}{d_2}}{\varepsilon_0 \frac{A}{d_1}} = \dfrac{d_1}{d_2}$
- $\therefore \dfrac{C_2}{C_1} = \dfrac{2}{3}$ $\therefore \dfrac{PE_2}{PE_1} = \dfrac{Q^2/2C_2}{Q^2/2C_1} = \dfrac{C_1}{C_2} = \dfrac{3}{2}$
- PE increases by 50% (prop. to increase in d)
- Q: Who provides the additional energy?

Mechanical work is required to pull the plates apart $\longrightarrow$ the $+Q$ / $-Q$ charged plates attract each other.