# PhysicsTutor

Doppler effect for EM waves

Giambattista 22.59

# Problem:

- A police car's radar gun emits microwaves with frequency $f_1$=36 GHz. The beam reflects from a car that speeds away from the cruiser with 43 m/s. The receiver in the police car detects the reflected waves at $f_2$.
- Which frequency is higher, $f_1$ or $f_2$?
- Calculate the difference $f_2 - f_1$.

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$$f_{obs} = f_{src}\sqrt{\frac{1 + v_{rel}/c}{1 - v_{rel}/c}} \approx f_{src}\left(1 + \frac{v_{rel}}{c}\right)$$

$\frac{v_{rel}}{c} << 1$

$v_{rel} > 0$ for approaching observer - source $\longrightarrow \quad \longleftarrow$

$v_{rel} < 0$ for receding $\longleftarrow \quad \longrightarrow$

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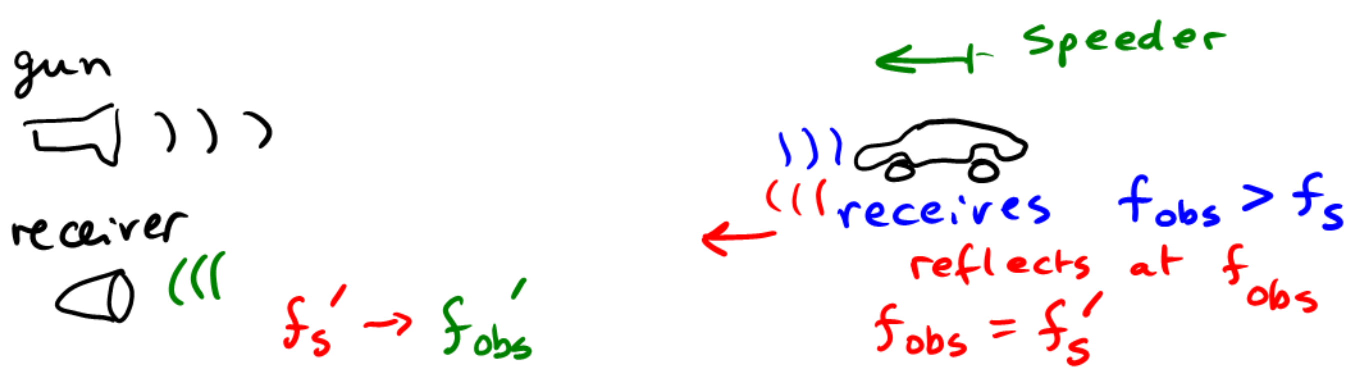
- In radar (emitting waves, observing frequency difference to waves reflected off a moving object) the Doppler effect appears twice.



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- The Doppler effect for EM waves is derived in special relativity. It only involves the relative velocity between source and observer.
- In radar (emitting waves, observing frequency difference to waves reflected off a moving object) the Doppler effect appears twice.
- EM waves travel in vacuum. No medium is involved. We treat air like vacuum here.

In the $v_s / v_{prop} \ll 1$ approximation all formulas become the same and depend on relative velocity only.

# Equations associated with ideas:

$$f_{obs} = f_{src}\sqrt{\frac{1 + v_{rel}/c}{1 - v_{rel}/c}} \underset{\frac{v_{rel}}{c} \ll 1}{\approx} f_{src}\left(1 + \frac{v_{rel}}{c}\right)$$

reflected waves are 'new' source :

$$f_{src}' = f_{obs} \approx f_{src}\left(1 + \frac{v_{rel}}{c}\right)$$

they are observed at:

$$f_{obs}' \approx f_{src}'\left(1 + \frac{v_{rel}}{c}\right)$$

$$= f_{src}\left(1 + \frac{v_{rel}}{c}\right)^2$$

$$= f_{src}\Big[1 + \underbrace{2\frac{v_{rel}}{c}}_{\text{1st order quantity}} + \underbrace{\cancel{\left(\frac{v_{rel}}{c}\right)^2}}_{\text{2nd order in small } v_{rel}/c}\Big]$$

$$= f_{src}\left[1 + \frac{2v_{rel}}{c}\right]$$

$v_{rel} > 0$ for getting closer

$v_{rel} < 0$ for getting away

# Strategy

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- Calculate the frequency of the radar waves as received by the speeding car.

$$f_{obs} \approx f_{src}\left(1 + \frac{v_{rel}}{c}\right) \qquad v_{rel} < 0 \text{ here!}$$

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- The reflected waves have this shifted frequency, and are emitted from a moving source.

$$f'_{src} = f_{obs} \approx f_{src}\left(1 + \frac{v_{rel}}{c}\right) \qquad (v_{rel} < 0)$$

$$f'_{src} < f_{src}$$

# Strategy

- Calculate the frequency of the radar waves as received by the speeding car.
- The reflected waves have this shifted frequency, and are emitted from a moving source.
- The receiver in the police car picks up the waves at a shifted frequency, since the source is moving away.

$$f'_{obs} \approx f'_{src}\left(1 + \frac{v_{rel}}{c}\right)$$

$v_{rel} < 0$

$f'_{obs} < f'_{src}$

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- $f_{obs} = f_{src}\left(1 + \frac{v_{rel}}{c}\right) = 36\times10^9\left(1 - \frac{43}{3.0\times10^8}\right) = 35.999995\,GHz$

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- $f'_{obs} = f_{obs}\left(1 + \frac{v_{rel}}{c}\right) = f_{src}\left(1 + \frac{v_{rel}}{c}\right)^2 \approx f_{src}\left(1 + 2\frac{v_{rel}}{c}\right)$

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- $f_{rec} = f'_{obs} = 36\ \text{GHz}\left(1 + \frac{86}{3.0\times10^8}\right) = 35.99999\ \text{GHz}$

$\Delta f = f_2 - f_1 = -10.3\ \text{kHz}$ ← This is needed to high accuracy to be able to issue a ticket!

Note: by measuring $\Delta f$, the frequency difference, which is +ve for an approaching speeder, the cruiser radar equipment calculates $v_{rel}$. When the cruiser moves, it has to have a separate measure of its speed