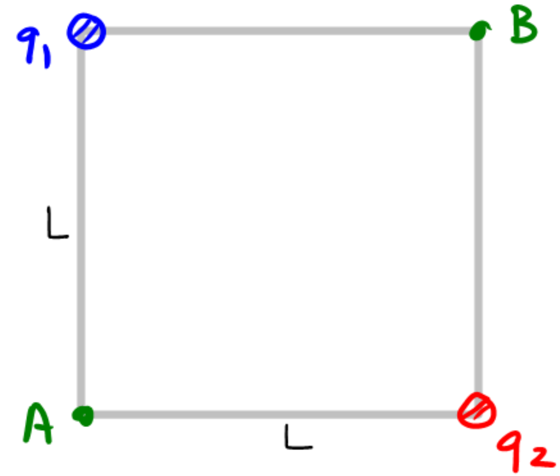
PhysicsTutor

Point charges: electric potential

Problem:

- Two point charges, q_1 = - 3.5 nC and q_2 = + 2.5 nC are located at diagonally opposite corners of a square with L = 2.5 cm. What is the electric potential at the other two corners of the square, *A* and *B*?



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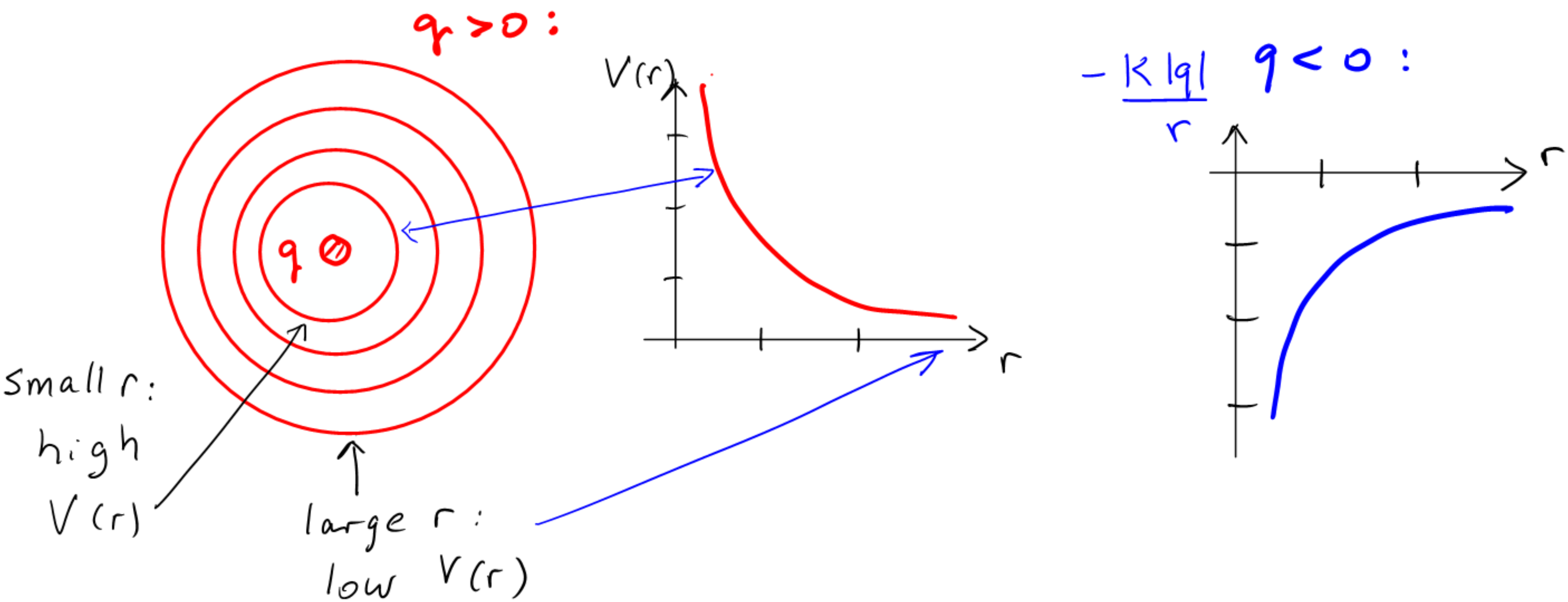
- Electric potential from a point charge q: falls with distance r according to $V(r) = Kq/r$



Relevant ideas:

- Electric potential from a point charge *q*: falls with distance *r* according to $V(r) = Kq/r$
- Electric potentials for multiple charges add (superposition principle). Total *V* represents net potential energy divided by probe charge.

$$V_i(r) = \frac{Kq_i}{r} \rightarrow V_i(\vec{r}) = \frac{Kq_i}{|\vec{r} - \vec{r}_i|}$$

q_i is located @ (0,0) → q_i is located at $\vec{r}_i = (x_i, y_i)$

N charges: $$V_{net}(\vec{r}) = \sum_{i=1}^{N} \frac{Kq_i}{|\vec{r} - \vec{r}_i|}$$

add scalars, but take position vectors $\vec{r}_i$ and location $\vec{r}$ into account

potential energy for probe q_p: $PE = q_p V_{net}(\vec{r})$

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- Electric potential from a point charge q: falls with distance r according to $V(r) = Kq/r$
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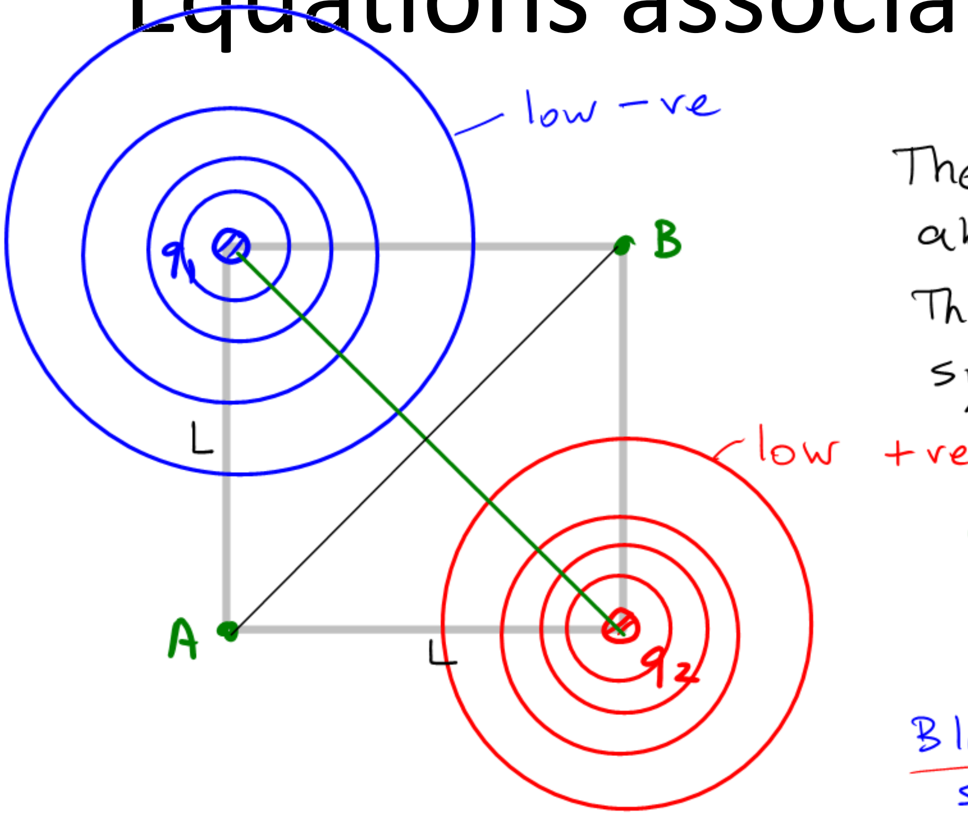
- Use geometry and symmetry: Is the potential the same at points A and B?

Equations associated with ideas:



There is reflection symmetry about ╲ diagonal.

There would be an additional symmetry about ╱ for $q_1 = q_2$.

Q: where is the potential zero?

It would be on ╱ for $q_1 = -q_2$

Blue circles have larger radii, since $|q_1| > |q_2|$

Choose $\vec{r}_A = (0,0)$, then $\vec{r}_1 = L\hat{j}$, $\vec{r}_2 = L\hat{i}$

$$V_1(\vec{r}_A) = \frac{kq_1}{|\vec{r}_A - \vec{r}_1|} = \frac{kq_1}{|-\vec{r}_1|} = \frac{kq_1}{r_1}$$

Simple, after all: just add $V_1(L) + V_2(L)$!

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- Draw schematic equipotential lines in the vicinity of the point charges, use different color for positive vs negative charges q.
- Realize now the symmetry for points A and B.
- Evaluate the potentials from q_1 and q_2 at location A and add them.
- Realize how easy it is to calculate the potential anywhere: just scalar addition required.

Solution

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- @ A: $V_1 = \frac{Kq_1}{L} = \frac{9.0 \cdot 10^{9} \cdot (-3.5) \cdot 10^{-9}}{2.5 \cdot 10^{-2}} \frac{Nm}{C} = -1260\,V$

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- $V_2 = \frac{Kq_2}{L} = \frac{9.0 \cdot 10^9 \cdot 2.5 \cdot 10^{-9}}{2.5 \cdot 10^{-2}} \frac{Nm}{C} = +900 V$

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- $V_{net} = -360\ V$. same value @ B
- Note: the contribution from each point charge to the total potential $V_{net}(\vec{r}_A)$ depends only on the distance $d_i = |\vec{r}_A - \vec{r}_i|$: $V_{net} = \sum_{i=1}^{N} \frac{Kq_i}{d_i}$.

The electric field components (E_x, E_y) at $\vec{r}_A$ can be obtained from knowledge of V_{net} in the vicinity of $\vec{r}_A$:

$$E_x^{net} = -\frac{d}{dx} V_{net}\Big|_{\vec{r}_A}, \quad E_y^{net} = -\frac{d}{dy} V_{net}\Big|_{\vec{r}_A}$$