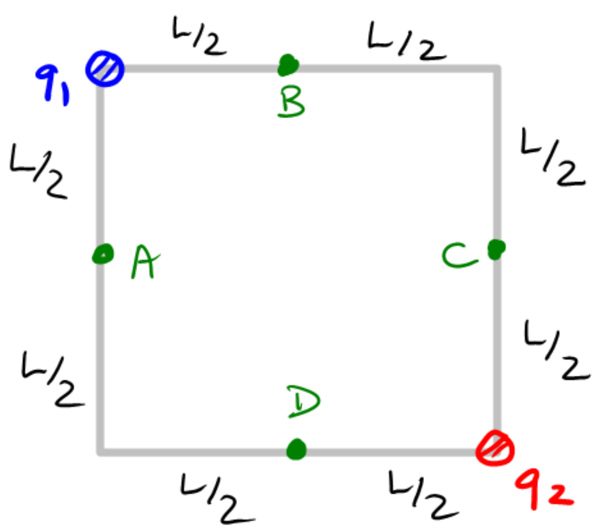
PhysicsTutor

Point charges: electric potential

Problem:

- Two point charges, q_1 = - 3.5 nC and q_2 = + 4.5 nC are located at diagonally opposite corners of a square with L = 2.5 cm. Calculate the electric potential at the points, *A*, *B*, *C*, *D*.



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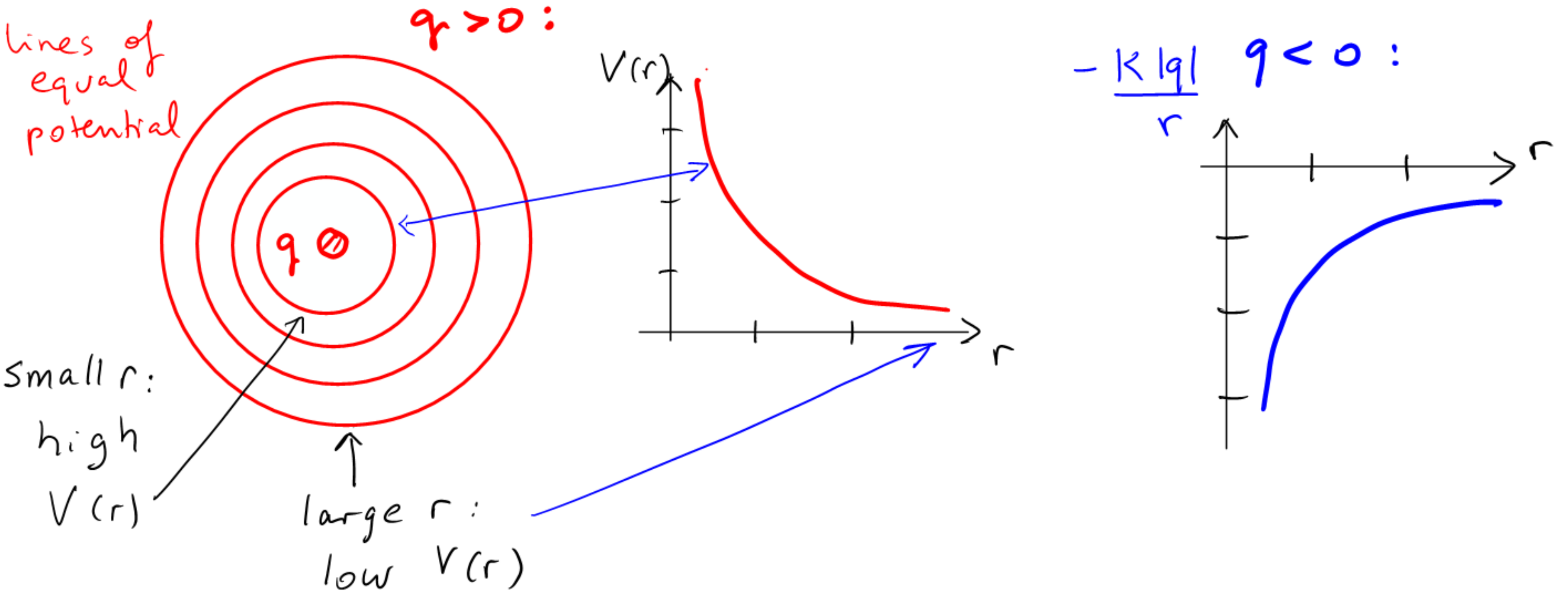
- Electric potential from a point charge q: falls with distance r according to $V(r) = Kq/r$



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- Electric potential from a point charge *q*: falls with distance *r* according to $V(r) = Kq/r$
- Electric potentials for multiple charges add (superposition principle). Total *V* represents net potential energy divided by probe charge.

$V_i(r) = \frac{Kq_i}{r}$ r is the distance from the point charge (radial distance)

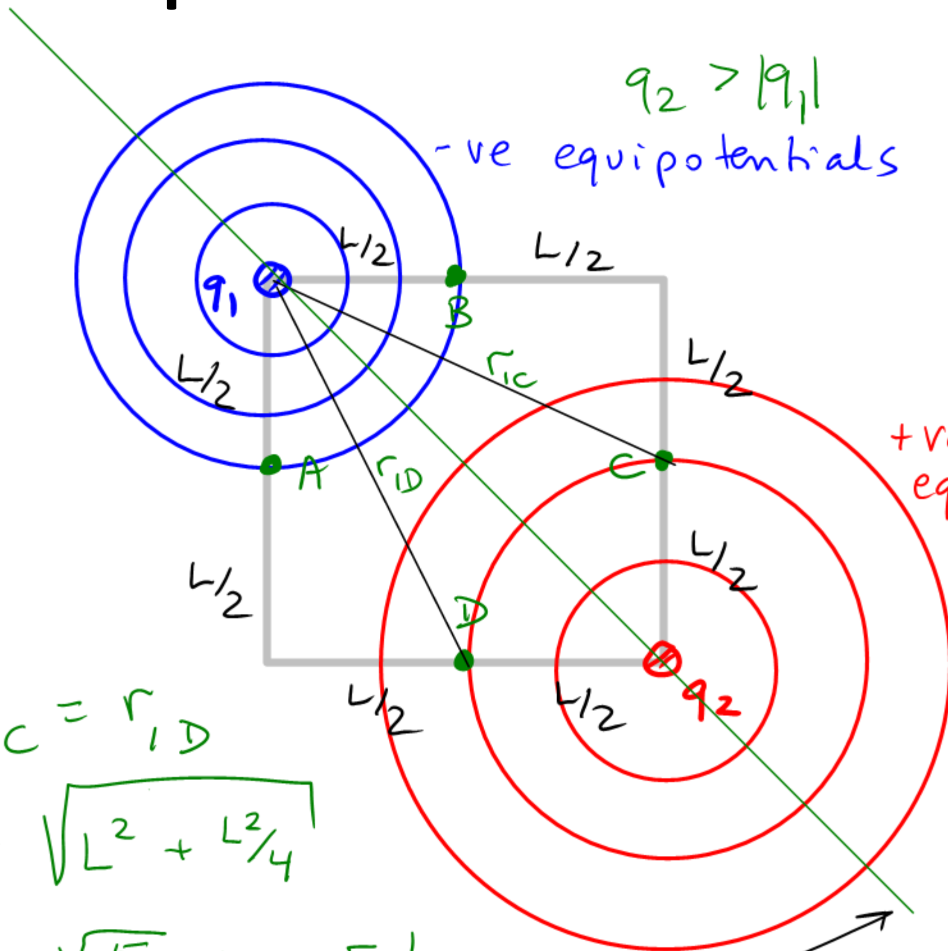
We need to add : $i = 1, 2$ here

Relevant ideas:

- Electric potential from a point charge *q*: falls with distance *r* according to $V(r) = Kq/r$
- Electric potentials for multiple charges add (superposition principle). Total *V* represents net potential energy divided by probe charge.
- Use geometry and symmetry: Is the potential the same at some of the points?

Look for points with equal distances to each charge
respectively → 2 pairs
confirm finding by understanding reflection symmetry

Equations associated with ideas:



A, B @ equal distance to q_1 and to q_2

$\therefore$ same potential there

Likewise C, D.

$\therefore V_A = V_B$,

$V_C = V_D$

@ A, B:

$$V_1 = \frac{Kq_1}{(L/2)} \; ; \; V_2 = \frac{Kq_2}{\sqrt{L^2 + (L/2)^2}}$$

@ C, D:

$$V_1 = \frac{Kq_1}{\sqrt{L^2 + (L/2)^2}} \; , \; V_2 = \frac{Kq_2}{(L/2)}$$

$$r_{1C} = r_{1D} = \sqrt{L^2 + L^2/4} = \sqrt{5/4}\, L = \sqrt{5}\,\frac{L}{2}$$

Note:

(reflection) symmetry axis

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- Draw schematic equipotential lines in the vicinity of the point charges, use different color for positive vs negative charges q.
- Realize how symmetry allows for shortcuts.
- Evaluate the potentials from q_1 and q_2 at the required locations and add them.
- It is easy to calculate the potential anywhere: just scalar addition required.

Solution

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- a A, B : $V_1 = \frac{Kq_1}{L/2} = \frac{2Kq_1}{L} = \frac{2 \cdot 9 \cdot 10^9 \cdot (-3.5) \cdot 10^{-9}}{2.5 \cdot 10^{-2}} \frac{Nm}{C} = -2{,}520\,V$

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- $V_2 = \frac{Kq_2}{\sqrt{5/4\ L^2}} = \sqrt{\frac{4}{5}} \frac{Kq_2}{L} = .894 \frac{9 \cdot 10^9 \cdot 4.5 \cdot 10^{-9}}{2.5 \cdot 10^{-2}} \frac{Nm}{C} = 1{,}450\,V$

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- $V_{tot}^{A,B} = (-2520 + 1450)\,V = -1{,}070\,V = -1{,}100\,V$
- @ C,D : $V_2 = \frac{2Kq_2}{L} = 3{,}240\,V$; $V_1 = \frac{2}{\sqrt{5}} \frac{Kq_1}{L} = -1{,}130\,V$

$V_{tot}^{C,D} = 2{,}110\,V = 2{,}100\,V$

Note: expressing the potentials as $\frac{2Kq_i}{L}$ vs $\frac{2}{\sqrt{5}} \frac{Kq_i}{L} \approx \frac{2Kq_i}{2.2..L}$

allows one to see: $V_1^D \lessapprox 0.5\, V_1^A$, etc. → compare w. plot: $r_D \approx 2 r_A$