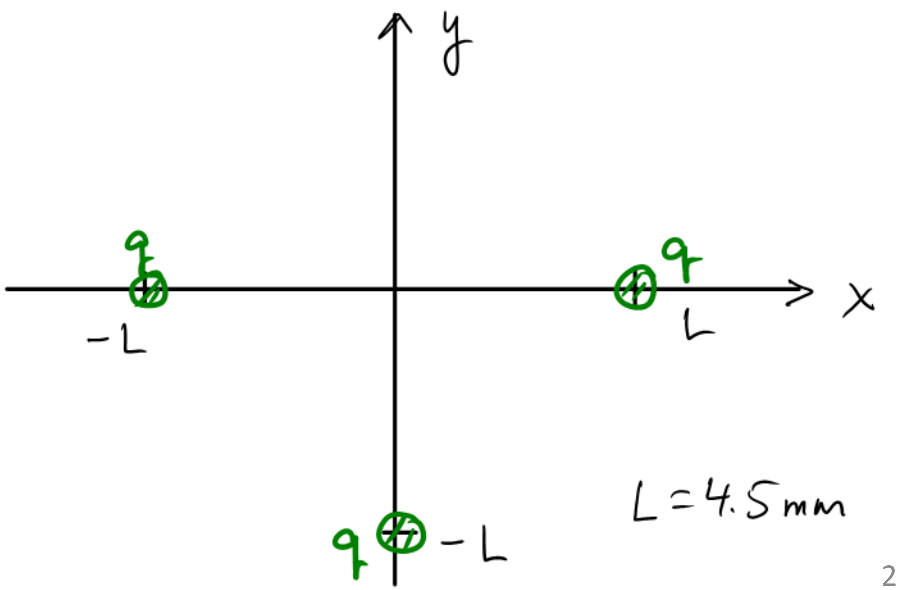
PhysicsTutor

Point charges: electric field

Giordano 17.47

Problem:

- Three point charges with $q = -\ 8.2\ \mu C$ each are located as shown with $L = 4.5$ mm.
- What are the magnitude and direction of the electric field at (a) the origin, (b) at $y = 6.8$ mm on the y-axis?



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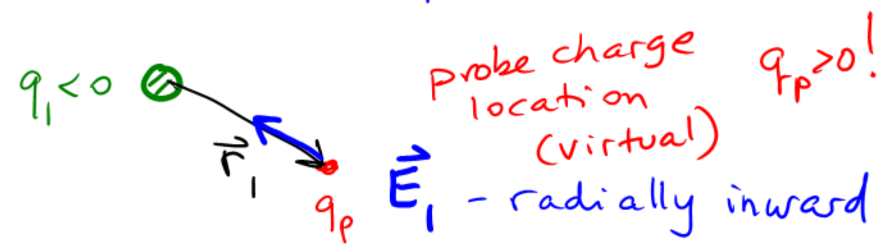
- Use symmetry to simplify calculations: work in Cartesian (x,y) coordinates when specifying the components of **E**.

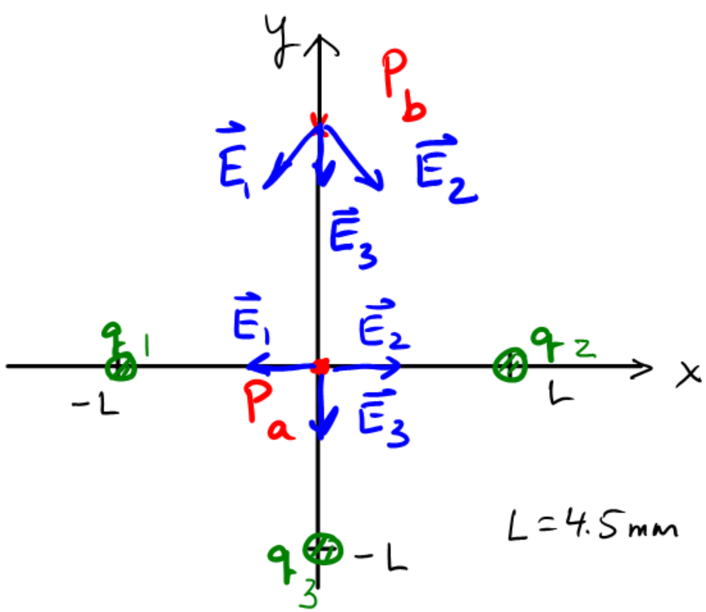
Equations associated with ideas:

$$\vec{F}_{Coulomb} = \underbrace{q_p}_{\text{probe charge}} \vec{E}$$

→ Coulomb force due to q_1 on q_p (virtual)

$$\vec{E}_1 = \frac{-1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2}\hat{r}_1$$





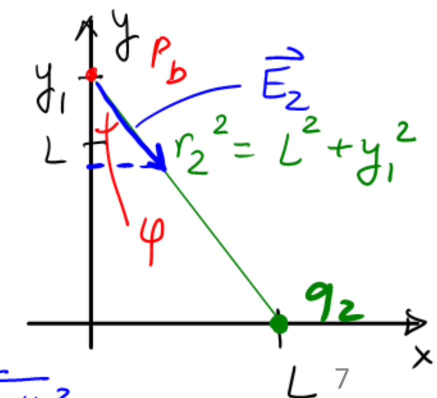
By inspection:

$$\vec{E}_{net} = \sum_{i=1}^{3} \vec{E}_i \sim -\hat{j}$$ both @ P_a and P_b

$$E_{1,x} + E_{2,x} = 0$$

$$E_{1,y} = E_{2,y}$$

$$\frac{E_{2,y}}{E_2} = \cos\varphi = \frac{y_1}{\sqrt{L^2 + y_1^2}}$$



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- At $(0,y_1)$: charges at $x = \pm L$ contribute to $E_{net,y}$.

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- $E_{net}^{(a)} = \frac{9.0\times10^9 \; 8.2\times10^{-6}}{4.5^2\times10^{-6}} \frac{Nm^2 \; C}{C^2 \; m^2} = 3.6\times10^9 \frac{N}{C}$ along $-y$ dir'n

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- $E_{net}^{(b)} = E_3 + 2E_{2,y}$; $E_3 = \frac{K|q_3|}{(L+y_1)^2} = \frac{9.0\times10^{9} \cdot 8.2\times10^{-6}}{(4.5+6.8)^2\times10^{-6}} \frac{N}{C} = 5.78\times10^{8}\ \frac{N}{C}$

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- $E_{2,y} = E_2 \cdot \frac{y_1}{\sqrt{y_1^2+L^2}} = \frac{K|q_2|y_1}{(y_1^2+L^2)^{3/2}} = \frac{9.0\times10^{9} \cdot 8.2\times10^{-6} \cdot 6.8\times10^{-3}}{(6.8^2+4.5^2)^{1.5} \cdot (10^{-3})^3} \frac{N}{C}$

$E_{2,y} = \frac{9.0 \cdot 8.2 \cdot 6.8}{(6.8^2+4.5^2)^{1.5}} \times 10^{9} \frac{N}{C} = 0.926\times10^{9} \frac{N}{C}$

$E_3 + 2E_{2,y} = 2.43\times10^{9} \frac{N}{C}$ $\qquad E_{net}^{(b)} = 2.4\times10^{9} \frac{N}{C}$ along $-y$ dir'n