PhysicsTutor

Thin-film Interference

Giordano 25.19

Problem:

- An very thin sheet of glass (n=1.55) floats on the surface of water (n=1.33). When illuminated with white light at normal incidence, the reflected light consists predominantly of the wavelengths 560 nm and 400 nm.
- How thick is the glass?

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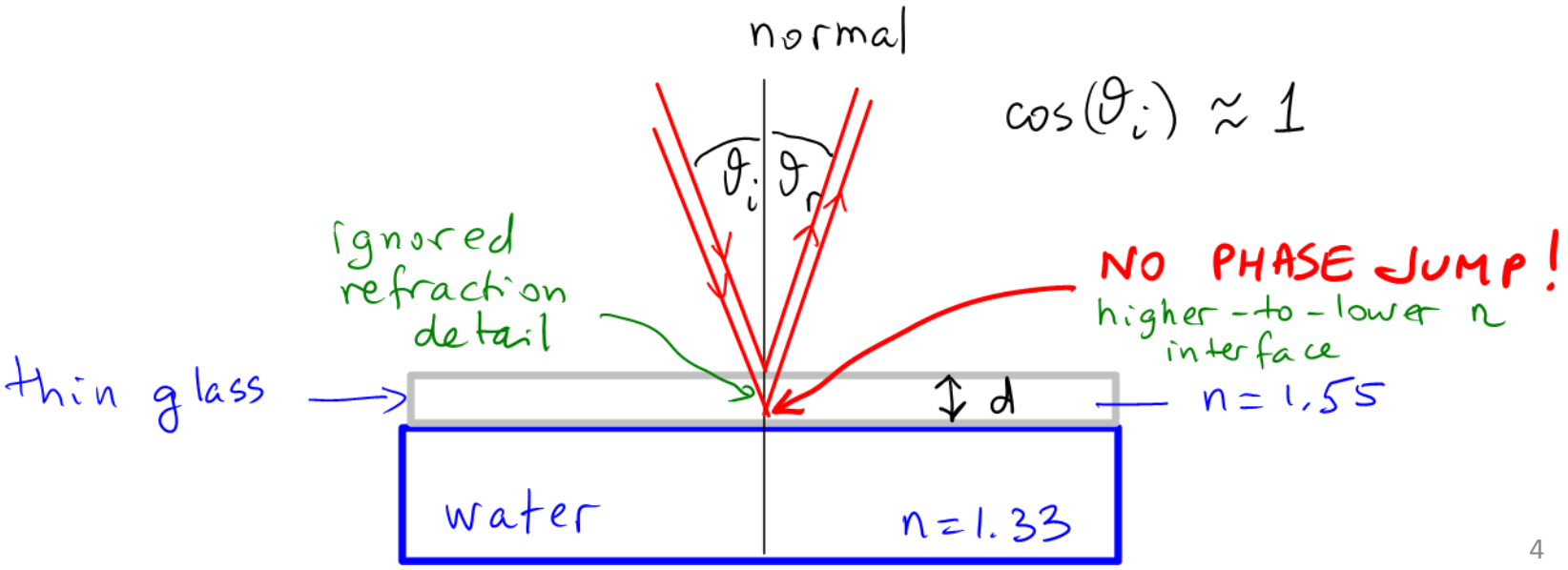

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- Constructive interference of light reflected from the top surface of the glass with light reflected from the glass-water interface.
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- Constructive interference of light reflected from the top surface of the glass with light reflected from the glass-water interface.
- The number of phase jumps is NOT the same for recombining beams.
- Find the optical path length difference between the two beams, phase shift of a multiple of 2π for both λ values.

Equations associated with ideas:

$$E_{1/2}(x,t) = E_0 \sin\left(\omega t - \frac{2\pi}{\lambda}x + \phi_{1/2}\right)$$

same x, t : $\phi_1 = \pi$, $\phi_2 = \frac{2\pi}{\lambda_{med}}\Delta x = \frac{2\pi}{\lambda_{vac}} n \Delta x$

$\phi_2 - \phi_1 = \Delta\phi$ is the accumulated difference phase

$$\Delta x = \frac{2d}{\cos\vartheta} \simeq 2d$$

$\Delta\phi = 2\pi m$ for two wavelengths, must be with different m-values, e.g., 1 and 2

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- Same arrival time for both paths: find the accumulated phase difference (PD) in space from the optical path length difference.
- Equate the PD to 2π for low-order constructive interference (the glass is thin).
- Repeat for the other wavelength.

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- $1 - 2m = \dfrac{4n\,d}{\lambda_{vac}}$ $\qquad$ use $m = 0, -1, -2, \ldots$
- $d = \dfrac{(1-2m)}{4n}\,\lambda_{vac}$ $\qquad$ try $m=0$ for $\lambda_i = 560$ nm

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- $d = \dfrac{(1-2m)}{4n}\lambda_{vac}$ $\qquad$ try $m=0$ for $\lambda_1 = 560$ nm

- $d = \dfrac{560\text{ nm}}{4 \cdot 1.55} = 90$ nm ; $d_2 = \dfrac{400 \cdot 3}{3.1 \cdot 2} = 193$ nm

$$\frac{1-2m_1}{2 \cdot 3.1} \cdot 560 = \frac{1-2m_2}{2 \cdot 3.1} 400 \quad \therefore (1-2m_1)\,56 = (1-2m_2)\,40$$

$$\therefore \frac{1-2m_1}{1-2m_2} = \frac{5}{7} \quad \therefore m_1 = -2,\ m_2 = -3 \quad \therefore d = \frac{5 \cdot 560\text{ nm}}{3.1 \cdot 2} = 450\text{ nm}$$