PhysicsTutor

Polarizer

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Problem:

- Construct a device that rotates the axis of linearly polarized light by 45 degrees from a sequence of ideal polarizers (no loss).
- Each polarizer has an axis that makes the same angle with the adjacent polarizer axis.
- How many polarizers do you need, and what is the angle between adjacent polarizer axes for an intensity reduction of at most 10% ?

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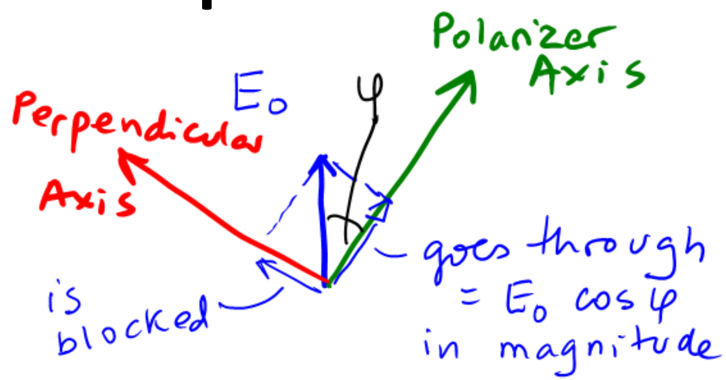
- A single polarizer at $\varphi = 45°$ would cut I_0 in half.
- A sequence of polarizers will work, since $\cos^2(\varphi) \approx 1$ for small φ.

Equations associated with ideas:



$$I_0 \cos^2\varphi = E_0^2 \cos^2\varphi$$

= transmitted intensity of an ideal polarizer
(@ $\varphi = 0$ perfect transmission)

n polarizers, $\varphi_{tot} = \frac{\pi}{4}$ desired $\therefore$ $\varphi = \frac{45^\circ}{n}$ for each

$$I_0 \underbrace{\left(\cos^2 \frac{45^\circ}{n}\right) \cdot \left(\cos^2 \frac{45^\circ}{n}\right) \cdots \left(\cos^2 \frac{45^\circ}{n}\right)}_{n \text{ times}} = I_0 \cos^{2n}\varphi$$

$$\cos^{2n}\left(\frac{\pi}{4n}\right) \geq \frac{9}{10}$$

← transcendental equation can't solve, but can tabulate LHS

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- We are looking for the smallest n for which the attenuation factor reaches 0.9, i.e., the loss is about 10% .
- The inequality is unlikely to be solvable in closed form, we need to generate a table of values for n=1,2,3...

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- $n = 6$: $\varphi = \frac{45^\circ}{6} = 7.5^\circ$

Q: can we figure out what happens for a real polarizer? $\varepsilon I_0 \cos^2\varphi$ where $\varepsilon = 0.8$?

→ add ε^n to the mix, and an ideal n-value may emerge! Of course, the attenuation will be big!