PhysicsTutor

Standing wave mode on a string.

Knight, 21.33

Problem:

- A 2.0 m long string vibrates at its second-harmonic frequency with maximum amplitude of 2.0 cm.
- One end of the string is at x=0 cm.
- Find the oscillation amplitude at x=10, 20, 30, 40, and 50 cm.

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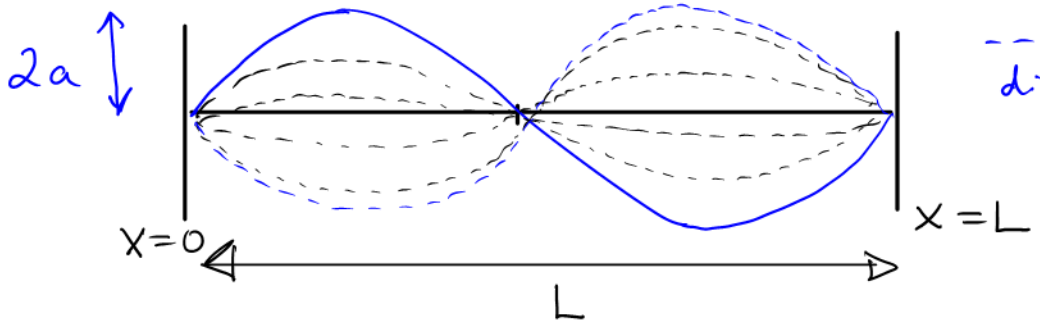
- Standing wave: a product of time-dependent and spatially dependent factors.
- Spatial part will yield the answers.

Equations associated with ideas:

$\lambda_n = \frac{2L}{n}$; $n=2 \rightarrow \lambda_2 = L$ (given)

$D(x,t) = 2a \cos(\omega_2 t) \sin\left(\frac{2\pi}{\lambda_2} x\right)$; $a = 1.0$ cm

satisfies $D(0,t) = 0$ (for all times)



different $\omega_2 t$ between 0 and π

$= 2a \sin\left(\frac{2\pi x}{L}\right)$ since $\lambda_2 = L$

Strategy

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- Given L and n=2 we find the wavelength

$\lambda_2 = \frac{2L}{2} = L$

Strategy

- Given L and n=2 we find the wavelength _____
- The spatial form is given by $2a \sin kx = 2a \sin\left(\frac{2\pi x}{\lambda}\right)$ which is multiplied by $\cos(\omega_2 t) = \cos(2\pi f_2 t)$

Strategy

- Given L and n=2 we find the wavelength _____
- The spatial form is given by_____ which is multiplied by _____
- We can evaluate the amplitude at the x-values given, keeping in mind that the $\cos(\omega_2 t)$ factor provides the time modulation, and is irrelevant for the amplitude

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- $A(0.2) = 11.8\ mm \quad ; \quad A(0.3) = 16.2\ mm \quad ; \quad A(0.4) = 19.0\ mm \quad ; \quad A(0.5) = 20\ mm$

Compare these with the graph: going from $x = 0$ to $x = \frac{L}{4}$

Note: the stretching of the string is ignored which is reasonable for $2a/L \sim 10^{-2}$