PhysicsTutor

Standing wave mode on a string.

Young & Freedman, 15.47

Problem:

- A guitar string of length 63.5 cm is tuned to play the B3 note (245 Hz) as a fundamental.
- What is the speed of transverse waves on the string?
- The tension is increased by 1%. Which frequency is obtained?

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- Standing wave on a string of length L.
- First harmonic: n=1.
- Propagation speed depends on tension and on linear mass density.
- Tension force change implies propagation speed change. Wavelength remains the same, but frequency changes.

Equations associated with ideas:

$$v_w = \lambda_s \cdot f \qquad \lambda_n = \frac{2L}{n} \quad \text{with } n = 1$$

$$v_w = \sqrt{\frac{F_t}{\mu}} \qquad \mu = \frac{M}{L}$$

$$\frac{\Delta F_t}{F_t} = 1\% \quad \therefore \quad \frac{\Delta v_w}{v_w} = 0.5\%^{(*)}$$

solution ideas for part (b)

$$\therefore \quad \frac{\Delta f}{f} \sim \frac{\Delta v_w}{v_w} \sim 0.5\%$$

$$(*) \quad \sqrt{x + \Delta x} = \sqrt{x}\sqrt{1 + \frac{\Delta x}{x}} = \sqrt{x}\left(1 + \frac{1}{2}\frac{\Delta x}{x} + \dots\right) = \sqrt{x} + \frac{1}{2}\frac{\Delta x}{\sqrt{x}} + \dots$$

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- From wavelength and frequency we find the propagation speed $v_w = \lambda_1 \cdot f_1$

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- From wavelength and frequency we find the propagation speed _________________
- The tension force increase results in an increase of the propagation speed ___________, not linear, but ________________
- The frequency change is proportional to the propagation speed change, since $f_1 = v_w / \lambda_1$

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- $\frac{\Delta f}{f} = \frac{1.2}{245} = 5 \times 10^{-3} = 0.5\ \%$

This confirms a result known from error analysis: errors in formulae involving powers of the measured quantity: $y = (x + \Delta x)^n$