PhysicsTutor mh

Guitar string.

Problem:

- A guitar B string of vibrating length 63.5 cm is made of steel wire (diameter 0.406 mm, mass density: 7800 kg/m^3) to play the B_3 note (247 Hz) as a fundamental.
- What is the tension of the string?
- The string is now tuned down from B_3 to A_3 (220 Hz). What is the percentage decrease in the tension?

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- Propagation speed depends on tension force and on linear mass density.
- Volume of the string is known, as is material mass density; linear mass density should follow.

Equations associated with ideas:

string modes: $\lambda_n = \frac{2L}{n} \quad \therefore \quad \lambda_1 = 2L$

$$v_w = \lambda_1 \cdot f_1$$

$$v_w = \sqrt{\frac{F_t}{\mu}} \quad \therefore \quad F_t = \mu v_w^2 \quad ; \mu = \frac{M}{L}$$

$$M = \rho V \quad , \quad V = \pi r^2 L = \frac{\pi d^2}{4} L$$

$f_1 \rightarrow \tilde{f}_1 = f_1 - \Delta f$ implies $v_w \rightarrow \tilde{v}_w$

$$\tilde{v}_w = v_w - \Delta v_w \quad \therefore \quad F_t \rightarrow \tilde{F}_t = F_t - \Delta F_t$$

$\Delta f \sim \Delta v_w$, but $\Delta F_t \sim 2 \Delta v_w$

why? $F_t = \mu v_w^2$ implies $\Delta F_t = 2\mu v_w \Delta v_w$

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- From wavelength and frequency we find the propagation speed $v_w = \lambda_1 f_1$

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- The tension force will follow from $v_w = \sqrt{F_t/\mu}$, if we find the linear mass density μ=M/L.

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- From wavelength and frequency we find the propagation speed _________________
- The tension force will follow from ___________, if we find the linear mass density μ=M/L.
- The mass M follows from ___ $M = \rho V$ ___, where the volume is that of a cylinder ___ $V = \pi \frac{d^2}{4} L$ ___.

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- $F_t = \mu v_w^2 = 99.4\ N; \quad 247 \rightarrow 220\ Hz: \frac{\Delta f}{f} = -0.109$
- $\Delta F_t = 2 \mu v_w \Delta v_w \quad \therefore \quad \frac{\Delta F_t}{F_t} = 2 \frac{\Delta v_w}{v_w} = 2 \frac{\Delta f}{f} = -0.218$

An 11% decrease in frequency is obtained from a 22% decrease in string tension.