Week 10 9.8

Car wheel $\rightarrow$ $M = 18\ kg$, $d = 0.4\ m$

$\hookrightarrow$ travels @ $20\ m/s$ $KE_{tot}^{wheel} = ?$

Solution

$$KE_{tot} = KE_{transl.} + KE_{rot} = \frac{1}{2} M V_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$$

$$I_{CM} = \frac{1}{2} M R^2 = \frac{1}{2} M \left(\frac{d}{2}\right)^2 = \frac{1}{8} M d^2$$

assumption of disk ?!
(equal mass distribution)
assumed

car translation = wheel (CM) translation ; no slip

magnitudes: $V_{CM} = R\,\omega$

avoid $d = 2R$

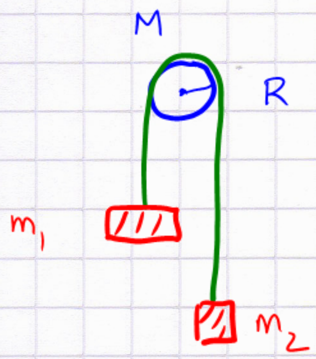
$$KE_{tot} = \frac{1}{2} M \left(V_{CM}^2 + \frac{1}{2} R^2 \frac{V_{CM}^2}{R^2} \right) = \underbrace{\frac{3}{4} M V_{CM}^2}_{\text{one wheel!}}$$

$$KE_{tot} = \frac{3}{4} (18\ kg) \left(20\ \frac{m}{s}\right)^2 = 5400\ J$$

$$= 5.4\ kJ$$

9.14 $M = 8.0\,kg$ $m_1 = 15\ kg$ $m_2 = 9.0\ kg$

$R = 0.20\ m$



a) Which forces can do work on masses ; on pulley system which conserves energy to be defined

b) crates released from rest @ equal height!? → $m_1 \to \Delta y = 2.0\ m$ reaches v_f

c, d) → energies? e, f) $v_f = ?$, $\omega = ?$

17) acceleration $a_f = ?$

Solution. → Exercise in setting up energy analysis.

a) Gravity $(F_{\text{on } m} = mg\,\Delta y)$ does work on the masses, but not on the pulley

Tension force : does work on masses and on pulley

No-slip assumption, no friction @ pivot :

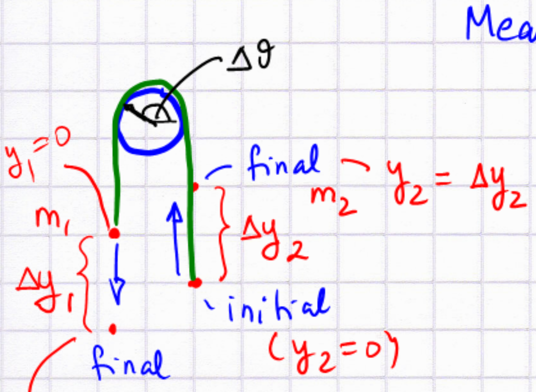
E_{tot} is conserved when considering total system: (m_1, m_2, M).

$$E_{tot} = \underbrace{(KE_1 + PE_1)}_{E^{tot}_{m_1}} + \underbrace{(KE_2 + PE_2)}_{E^{tot}_{m_2}} + \underbrace{\tfrac{1}{2} M R^2 \omega^2}_{KE_{pulley}} \quad \leftarrow \text{generic}$$

At $t = 0$: $E_{tot} = PE_1 + PE_2 + 0 = 0$ for choice $y_1 = y_2 = 0$

At $t = t_f$: $E_{tot} = \left(\tfrac{1}{2} m_1 v_f^2 + m_1 g\,\Delta y_1\right) + \left(\tfrac{1}{2} m_2 v_f^2 + m_2 g\,\Delta y_2\right) + \tfrac{M}{4} R^2 \omega_f^2$

Measure displacement Δy_i with consistent sign!



$\uparrow \hat{j}$ is the same for both

$\Delta y_2 > 0$, $\Delta y_1 = -\Delta y_2$, $PE_{in} = 0$, $PE_{fin} = mg\,\Delta y$

$y_1 = 0 \,/\, y_2 = 0$ for initial set-up ; doesn't need to be equal ?

(looks ugly mathematically if $y_i \neq y_2$)

@ $t = t_f$: $E_{tot} = \left(\tfrac{1}{2} m_1 v_f^2 - m_1 g\,\Delta y\right) + \left(\tfrac{1}{2} m_2 v_f^2 + m_2 g\,\Delta y\right) + \tfrac{M}{4} v_f^2$

9.14/17 cont'd   By choice of $y_1 = 0$, $y_2 = 0$: $E_{tot} = 0$

$$\therefore \quad \frac{1}{2}\left(m_1 + m_2 + \frac{M}{2}\right) v_f^2 = -(m_2 - m_1)\, g\, \Delta y$$

$$\therefore \quad v_f^2 = \frac{2(m_1 - m_2)\, g\, \Delta y}{m_1 + m_2 + M/2}$$

put in values in SI :

$$v_f = \sqrt{\frac{2(15-9)\cdot 9.8 \cdot 2.0}{15 + 9 + 4.0}} = 2.90 \ \frac{m}{s}$$

$$v_f = 2.9 \ \frac{m}{s}$$ ← indep. of pulley radius, but does depend on its mass

$$\omega = \frac{v_f}{R} = 14.49 \ \frac{rad}{s} = 14.5 \ \frac{rad}{s}$$

m_1 moves down
m_2 moves up
with this velocity
at the moment of $\Delta y = 2.0$ m.

9.17 : What is the acceleration when $\Delta y = 2.0$ m ? does it change in time ?

To answer it is easiest to return to the 2nd law treatment (class notes). A constant net force $(m_1 - m_2) g$ acting on the effective system mass $(m_1 + m_2 + M/2)$

$$\therefore \quad a = \frac{(m_1 - m_2) g}{m_1 + m_2 + M/2}$$

$\therefore$ a = const

$a = 2.1 \ m/s^2$

Another way to proceed: (after figuring out a = const):

$$v_f^2 = v_i^2 + 2 a \Delta y \ ; \ v_i = 0$$

$$a = \frac{v_f^2}{2 \Delta y} = \frac{2.9^2}{4.0} = 2.1 \ m/s^2$$

Textbook answer: M = 5.0 kg was chosen for pulley → $2.3 \ m/s^2$

9.15 Marble ($I_{CM} = \frac{2}{5} MR^2$) $R = 0.01\,m$ $m = 8\times10^{-3}\,kg$

rolls (no-slip) down a ramp ($\Delta y = 0.2\,m$)

$v_f = ?$

Solution. (by energy conservation)

$E_{tot} = Mg\,\Delta y$ (initially $KE = 0$)

$E_{tot} = KE_{tr.} + KE_{roll}$ (finally $PE = 0$)

Mechanical energy is conserved even though a friction force (static) is required and 'slows' the CM. $\vec{F}_s$ at the point of contact does no work since there is no displacement while it acts. (not easy to accept!)

$$\frac{1}{2} \cancel{M} v_{CM}^2 + \frac{1}{2} \cdot \frac{2}{5} \cancel{M} \cancel{R^2} \left(\frac{v_{CM}}{\cancel{R}} \right)^2 = \cancel{M} g\,\Delta y$$

$$v_{CM}^2 \left(\frac{5+2}{5} \right) = 2g\,\Delta y$$

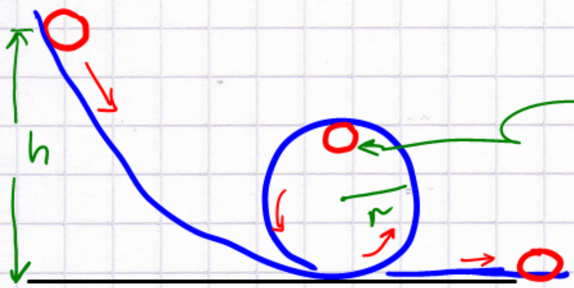
$$v_{CM}^2 = \frac{10}{7} g\,\Delta y \quad \therefore \quad v_{CM} = \sqrt{\frac{10}{7} g\,\Delta y}$$

$$v_{CM} = 1.67\,\frac{m}{s} \rightarrow 1.7\,\frac{m}{s}$$

9.21

$r = 5.0$ m

critical @ top of loop
($v_{CM} = \sqrt{g\,r}$ as derived previously)



ball is upside down?

Solution

By mech. energy conservation: $E_{tot} = mgh$ (= PE)

At the top of loop (inside) $E_{tot} = mg(2r) + \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2$

Not knowing the radius of the ball, R, we assume $R \ll r$

$\therefore$ height $\approx 2r$

$$2\cancel{m}gr + \frac{1}{2}\cancel{m}v_{CM}^2 + \cancel{\frac{1}{2}}\frac{2}{5}\cancel{m}\cancel{R^2}\left(\frac{v_{CM}}{\cancel{R}}\right)^2 = \cancel{m}gh$$

$$\left(\frac{1}{2} + \frac{1}{5}\right)v_{cm}^2 = g(h-2r)$$

$$v_{CM}^2 = \frac{10}{7}g(h-2r)$$

critical height: $v_{CM}^2 = gr \quad \therefore \quad gr = \frac{10}{7}g(h_{cr} - 2r)$

$$\therefore \quad \cancel{g}r\left(1 + \frac{20}{7}\right) = \frac{10}{7}\cancel{g}h_{cr} \quad \therefore \quad \underline{\underline{h_{cr} = \frac{27}{10}r}} = 2.7r$$

$$\therefore \quad h_{cr} = 2.7 \times 5.0 \text{ m} = 13.5 \text{ m}$$

Message: rolling doesn't help when it comes to the motion of the CM carrying out centripetal motion. For $v < v_{cr}$ we get slip + falling faster. Interesting question: actual trajectory?