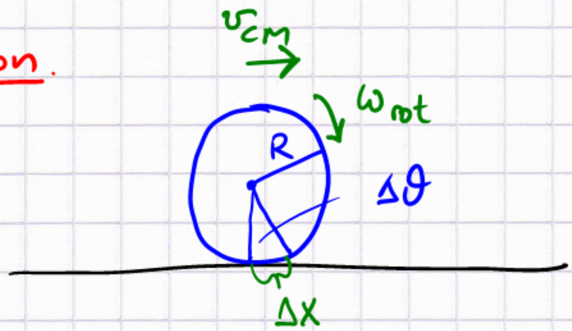
9.29 A cylinder ($m = 7.0$ kg, $R = 0.25$ m) rolls on a level floor. $v_{CM} = 1.5$ m/s. Find KE_{CM} and KE_{rot}

Solution.



$$R\,\Delta\vartheta = \Delta x$$

$$R\,\frac{\Delta\vartheta}{\Delta t} = \frac{\Delta x}{\Delta t}$$

$$\therefore \quad R\omega = v_{CM}$$

$$I_{CM} = \frac{1}{2} m R^2$$

$$KE_{rot} = \frac{1}{2} I \omega^2$$

$$KE_{CM} = \frac{m}{2} v_{CM}^2$$

total kinetic energy $TKE = KE_{CM} + KE_{rot}$

$$TKE = \frac{1}{2}\cdot\frac{1}{2} m R^2 \omega^2 + \frac{1}{2} m v_{CM}^2$$

know v_{CM}

$\therefore$ convert $\omega \to v_{CM}$

$$= \frac{1}{2} m \left(\frac{1}{2} v_{CM}^2 + v_{CM}^2\right) = \frac{3}{4} m v_{CM}^2$$

It does not depend on the radius

$$TKE = 0.75 \cdot 7.0 \cdot 2.25 \text{ J}$$

$$= 11.8 \text{ J} = 12 \text{ J}$$

2 sign.

a) 7.9 J KE_{tr} b) 3.9 J KE_{rot}

9.67 Large cylinder ($m_2 = 270$ kg, $R_2 = 0.22$ m) has a small cylinder attached concentrically ($m_1 = 67.5$ kg, $R_1 = 0.11$ m) from which a mass ($m = 2.3$ kg) is falling with a rope over R_1 causing the large cylinder to roll. ②

m falls by $h = 0.35$ m; the system was at rest before.

a) Calculate I_{CM} and M_{tot} b) $TKE^{fin} = ?$ c) $v_{CM}^{fin} = ?$

d) $x_{CM}^{fin} = ?$

Solution

a) $M_{tot} = 270 + 67.5 + 2.3 = 340$ kg

you can ignore m @ 2 sign. digits

$$I_{CM} = \frac{1}{2} m_2 R_2^2 + \frac{1}{2} m_1 R_1^2 = \frac{1}{2} m_2 R_2^2 \left(1 + \frac{1}{4} \cdot \frac{1}{4}\right) = \frac{17}{32} m_2 R_2^2$$

$$I_{CM} = 6.94 \text{ kg m}^2$$

b) By energy conservation: $TKE = -\Delta PE = +mgh$

$$TKE = 2.3 \cdot 9.8 \cdot 0.35 = 7.9 \text{ J}$$

c) ignore the motion of m (horizontal and vertical)

$$TKE = \frac{1}{2} I \omega^2 + \frac{1}{2} M v_{CM}^2$$

$v_{CM} = R_2 \omega$ rolling radius

$$= \frac{1}{2} \left(\frac{I}{R_2^2} + M \right) v_{CM}^2$$

$$= \frac{1}{2} (143 + 340) v_{CM}^2$$

$$\therefore \quad v_{CM} = \sqrt{\frac{2 \cdot TKE}{483}} = 0.18 \text{ m/s}$$

d) rolling relates: $\Delta x_{CM} = R_2 \, \Delta\vartheta$

unwinding rope relates: $\Delta y = R_1 \, \Delta\vartheta$

Thus, $$\frac{\Delta x_{CM}}{\Delta y} = \frac{R_2 \, \Delta\vartheta}{R_1 \, \Delta\vartheta} = \frac{R_2}{R_1} = 2$$

$$\therefore \quad x_{CM}^{fin} = 2 \cdot \Delta y = 0.7 \text{ m}$$

Extra:

Now set up the 2nd law (in the $m \ll m_1$ limit)

Main question: what makes the assembly move to the right ?? $\rightarrow f_s$!

1) m - vertical motion: $$m a_y = mg - T$$

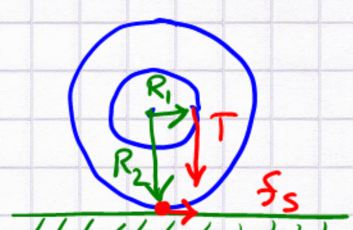
2) $m_1 + m_2$ - horizontal: $$(m_1 + m_2) \, a_{CM} = f_s$$

3) $$I\alpha = \tau_{net}$$ (it is clockwise, but we ignore the "−" sign)

$$\left(\alpha = \frac{a_{CM}}{R_2}\right)$$

τ_{net} has two contributions: T pulls CW and is causing rotation. f_s opposes it to ensure motion of the CM to the right!

$$\frac{17}{32} m_2 R_2^2 \left(\frac{a_{CM}}{R_2}\right) = R_1 T - \underbrace{R_2 f_s}_{\text{why?}}$$



3 eqs in 3 unknowns: $a_{CM} = 2a_y$, T, f_s