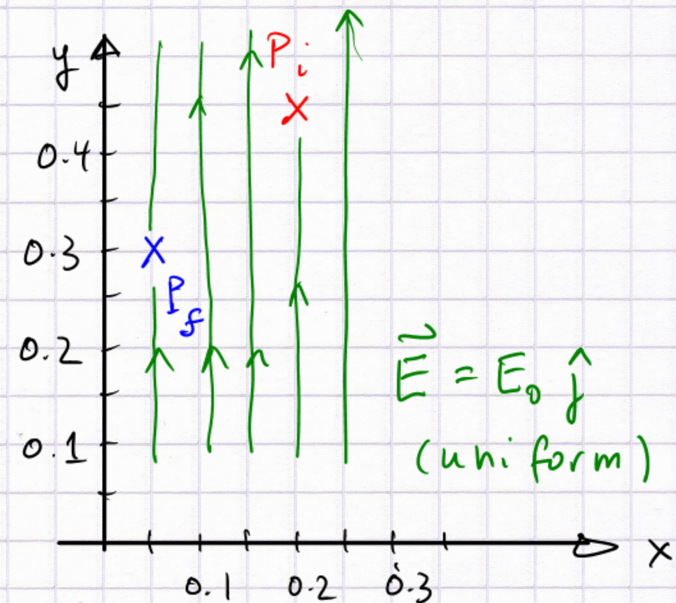
Week 14 18.8 Uniform $\vec{E}$ field in some region of space, $E_0 = 400$ N/C $\vec{E} = E_0 \hat{j}$. (a) Calculate change $\Delta(PE)$ for a charge $q = 3.5\ \mu C$ moved from $(0.2, 0.45)$ to $(0.05, 0.3)$
(b) $\Delta(PE)$ for moving q the same distance along x.



Solution.

$$\Delta(PE) = -qE_0 \Delta y$$

why?

$$\Delta(PE) = -W = -\vec{F} \cdot \Delta\vec{r}$$ ($\vec{F}$ is $\vec{F}_0$, constant magnitude + direction!) $\vec{F} = q\vec{E}$ in our case

$$\Delta y = 0.3 - 0.45 = -0.15 \text{ m}$$ ← order matters!

$$qE_0 = 3.5 \times 10^{-6} \times 400 = 14 \times 10^{-4} \text{ N}$$

$$\Delta(PE) = +14 \times 0.15 \times 10^{-4} \text{ J} = 0.21 \text{ mJ}$$

(b) moving the charge through the field perpendicular to the field lines:

NO CHANGE in P.E.!

($\vec{F} \cdot \Delta\vec{r} = 0$ in this case, since $\cos(90^\circ) = 0$)

18.12 An e^- and a p^+ are separated by $d = 7.5 \times 10^{-9}$ m. How much energy is required to double d?

Solution. Eq. 18.6 for a pair of point charges $PE = \frac{K q_1 q_2}{r}$

follows from $F = \frac{K q_1 q_2}{r^2}$

since $F = -\frac{d}{dr}(PE)$

Calculate $PE_{in} = \frac{K e(-e)}{d} = \frac{-9.0 \times 10^9 (1.6 \times 10^{-19})^2}{7.5 \times 10^{-9}}$ J

$= -3.1 \times 10^{-12}$ J $= -0.19$ eV

PE_{fin} is one half this value

$\therefore$ ΔPE is positive, and one half of the above magnitude

$\Delta(PE) = 0.096 \text{ eV} = 1.5 \times 10^{-20}$ J

NB: conversion J $\leftrightarrow$ eV ?

Volt is an SI unit

e = elementary charge

$= 1.60 \times 10^{-19}$ $\underbrace{C}_{SI}$

$\therefore$ 1 eV $= 1.60 \times 10^{-19}$ J

18.20 A proton changes location from A to B.
At A the voltage is 75 V, at B it is -20 V
(a) $\Delta(KE) = ?$ (b) for an e^- doing the same, $\Delta(KE) = ?$

Solution. Remember the force - field relation:

$$F = q\,E$$

Likewise: $PE = q\,V$ (E, V are space properties due to some other charges present)

Energy is conserved: $E_{tot} = KE + PE = \text{const}$

$$\therefore \quad \Delta E_{tot} = 0 = \Delta(KE) + \Delta(PE)$$

$$\therefore \quad \Delta(KE) = -\Delta(PE)$$

proton: $q = +e$; $PE_A = +e \cdot 75 = +75\ eV$

$PE_B = +e\,(-20) = -20\ eV$

$\Delta(PE) = PE_B - PE_A = -20 - 75$
$= -95\ eV$

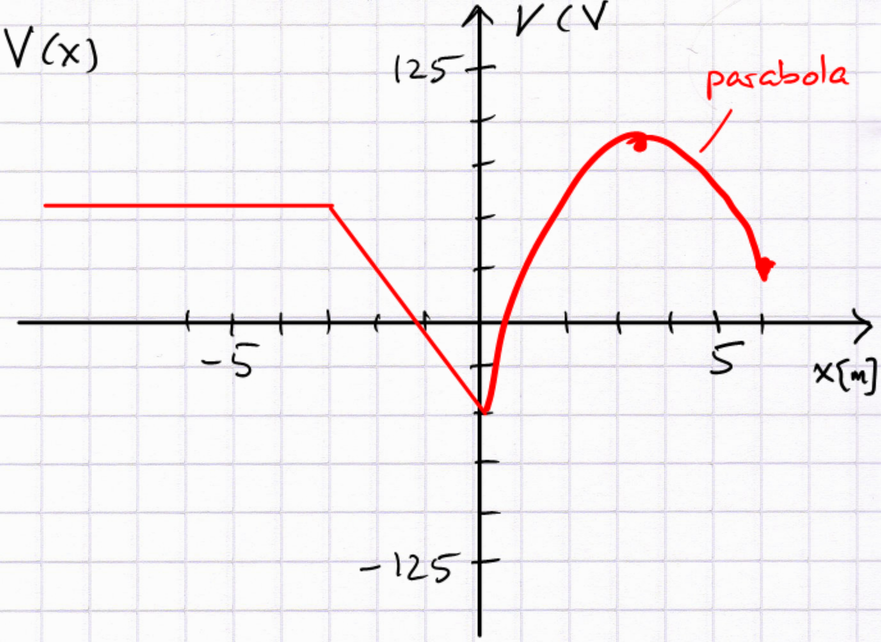
$\Delta(KE) = -\Delta(PE) = +95\,eV$

The proton gains 95 eV KE (it was accelerated by the field)

$\Delta(KE) = 1.5 \times 10^{-17}\ J$

(b) An electron has the opposite charge, it has to work against the field, it loses 95 eV ($\Delta KE = -95\,eV$). It must have had more than 95eV KE @A!

18.26 Electric potential $V(x)$ is shown.



Show $E_x(x)$

Solution

$$E_x = -\frac{d}{dx}V(x) \qquad \left(= -\frac{\Delta E}{\Delta x}\right)$$

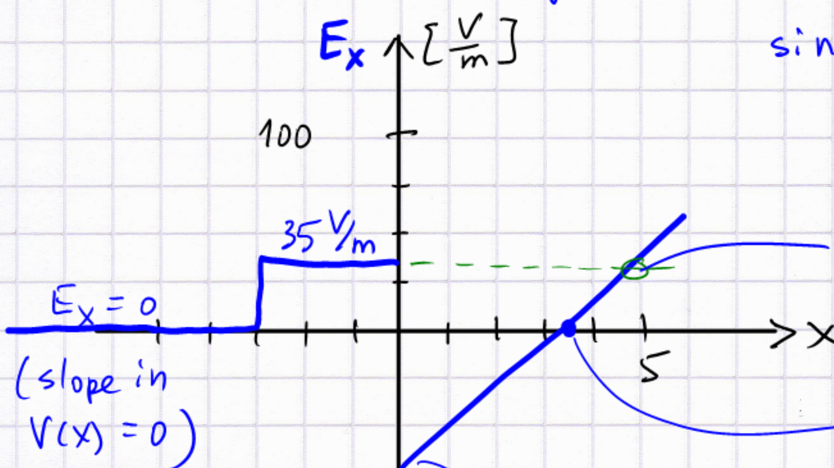
For $x < -3$: $E_x = 0$

$-3 < x < 0$: $E_x =$ pos. const. $\sim \dfrac{(50+60)V}{3\,m}$

neg. slope, but want $-\frac{d}{dx}V$! $\quad E_x \approx 35\,\frac{V}{m} \quad \left(= \frac{N}{C}\right)$

$0 < x < 6$: $E_x = a\,x + b$ derivative of parabola = linear f'n

inverted parabola, but slope a should be positive since $E_x = -\dfrac{dV}{dx}$ (- sign!)



(slope in $V(x) = 0$)

about the same slope at $x \approx 5$ as between -3 and 0

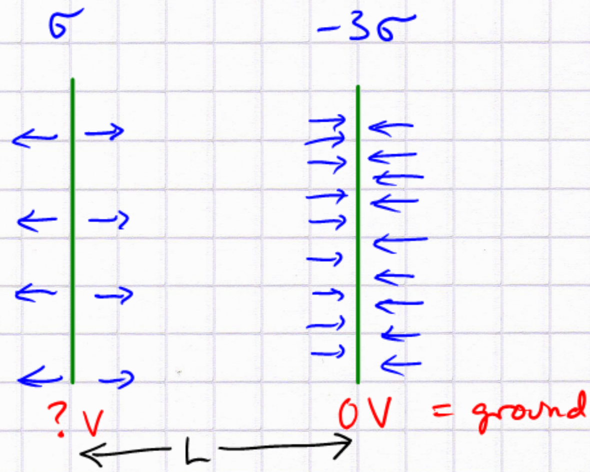
crossing 0 at $x = 3.5$ (apex of parabola)

slope magnitude ~ two times the value from the left

Note: a discontinuous E_x is not easy to make in the lab.

18.32 Two infinite parallel plates, uniformly charged to $+\sigma$ and $-\sigma$ respectively, are separated by L. RightPlate is at 0V (ground). What is V_L?

(use σ, L)



Solution

arrows indicate the $\vec{E}$ fields due to L and R plates.

These fields add, and they are constant, everywhere!

Between the plates the fields are additive

(arrows point in the same direction)

Figure out the potential by moving a probe charge L → R

$$E_{between} = \frac{\sigma}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0} = 2\frac{\sigma}{\varepsilon_0}$$

From $E = -\frac{\Delta V}{\Delta x}$ find $\Delta V = -E\Delta x$

$$|\Delta V| = |E|\,|\Delta x|$$

We know $V_L > 0$ since $\vec{E}$ points from high to low potential

$$V_L = 2\frac{\sigma}{\varepsilon_0}L$$