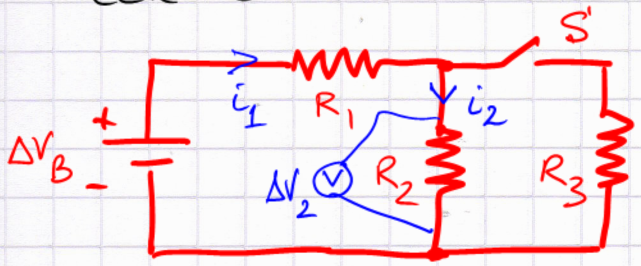
Week 16 19.23



$R_1 = R_2 = R_3 = R$

S closes $\rightarrow$

changes in i_1 (A), ΔV_2 (B), i_2 (C)

Solution

With S open : $i_1 = i_2$ controlled by $R_{12} = R_1 + R_2$

$\therefore R_{12} = 2R$

$$\therefore i_1 = i_2 = \frac{\Delta V_B}{2R}$$

$$\therefore i_1^{before} = i_2^{before} = \frac{\Delta V_B}{2R} \quad \left(= \frac{\mathcal{E}}{2R} \right)$$

S closes $\therefore$ R_1 in series with R_{23}^{eq}

R_{23}^{eq} : parallel $\therefore \frac{1}{R_{23}^{eq}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$

$\therefore R_{23}^{eq} = \frac{R}{2}$ $\quad \therefore R_{tot} = R + \frac{R}{2} = \frac{3}{2} R$

$$\therefore i_1 = i_{tot} = \frac{\Delta V_B}{\Delta R_{tot}} = \frac{\Delta V_B}{3/2\, R}$$

$$\frac{i_1^{after}}{i_1^{before}} = \frac{\frac{2}{3} \Delta V_B / R}{\frac{1}{2} \Delta V_B / R} = \frac{4}{3}$$

$\therefore$ current i_1 increases by a factor of $4/3$ (A)

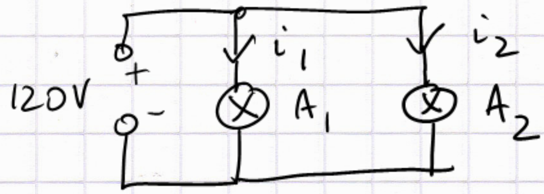
(B): $\Delta V_2^{before} = \frac{1}{2} \Delta V_B$ (voltage divider with $R_1 = R_2$)

ΔV_2^{after} : voltage divider R_1 & $R_{23}^{eq} = \frac{R}{2}$ $\quad \therefore \Delta V_2^{after} = \frac{1}{3} \Delta V_B$

ratio : $\frac{1}{3} : \frac{1}{2} = \frac{2}{3}$ $\quad \Delta V_2^{after} = \frac{2}{3} \Delta V_2^{before}$

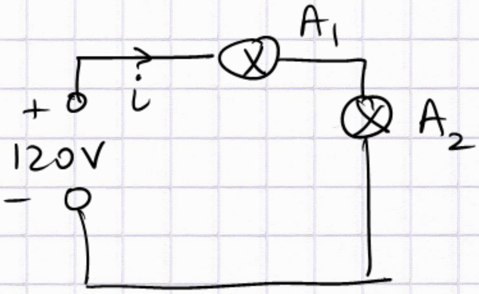
(C): $i_2^{after} = \frac{1}{2} i_1^{after} = \frac{1}{2} \cdot \frac{4}{3} i_1^{before} = \frac{2}{3} i_1^{before} = \frac{2}{3} i_2^{before}$

19.33



$i_1 = 2.0$ A
$i_2 = 3.5$ A

vs



Solution

Ohm's law : $R_1 = \dfrac{120\ \text{V}}{2.0\ \text{A}} = 60\ \Omega$, $R_2 = \dfrac{120\ \text{V}}{3.5\ \text{A}} = \dfrac{240}{7.0}\ \Omega$

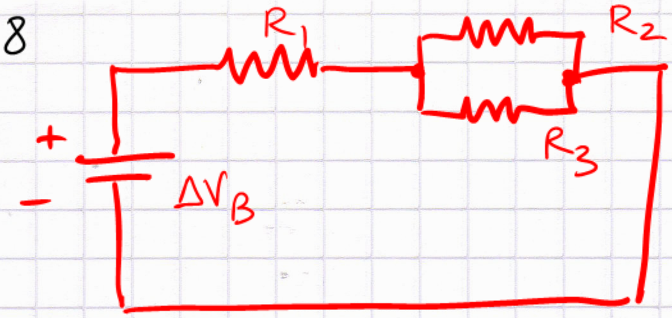
series connection : $R_{eq} = R_1 + R_2 = \left(60 + \dfrac{240}{7.0}\right)\Omega$

$$i = \Delta V / R_{eq} = \frac{120\ \text{V}}{\frac{420 + 240}{7.0}\ \Omega} = \frac{7.0 \cdot 6}{21 + 12}\ \text{A}$$

$$= \frac{42}{33}\ \text{A} = 1.3\ \text{A}$$

$\rightarrow$ less current than was passing through either appliance when connected properly (in parallel)

19.38



$$R_{23}^{eq} = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{tot}^{eq} = R_1 + R_{23}^{eq}$$

$$= \frac{R_1(R_2+R_3) + R_2 R_3}{R_2 + R_3}$$

$$R_{tot}^{eq} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

$$i = \frac{\Delta V_B}{R_{tot}^{eq}} = \frac{\Delta V_B (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

values: $R_1 = 2.4\ k\Omega$; $R_2 = 1.4\ k\Omega$, $R_3 = 4.5\ k\Omega$

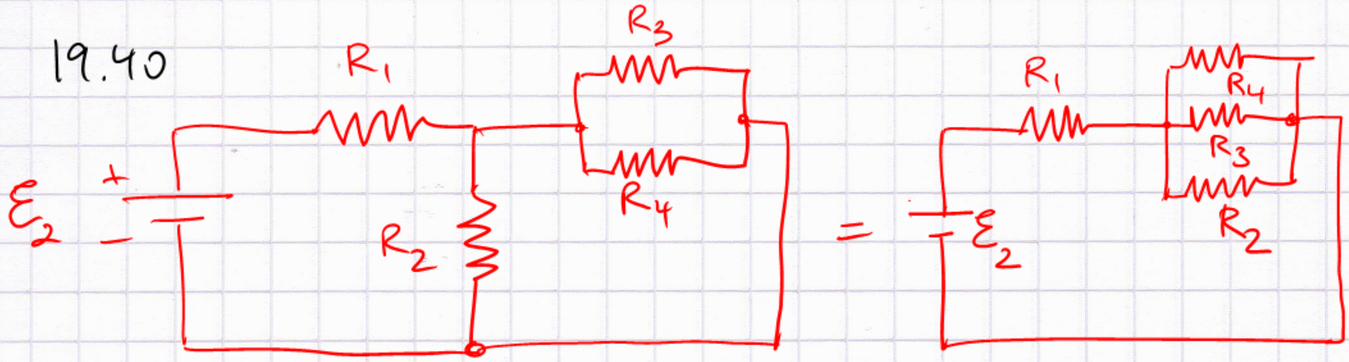
$\Delta V_B = 1.5\ V$

$R_{23}^{eq} = 1.1\ k\Omega$ $\quad R_{tot}^{eq} = (1.1 + 2.4)\ k\Omega = 3.5\ k\Omega$

$$i_1 = \frac{1.5 V}{3.5\ k\Omega} = 0.43\ mA$$

This is also the current through the battery!

19.40



$$\therefore \quad R_{tot}^{eq} = R_1 + R_{234}^{eq}$$

$$\frac{1}{R_{234}^{eq}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \qquad i = \frac{\Delta V_B}{R_{tot}^{eq}}$$

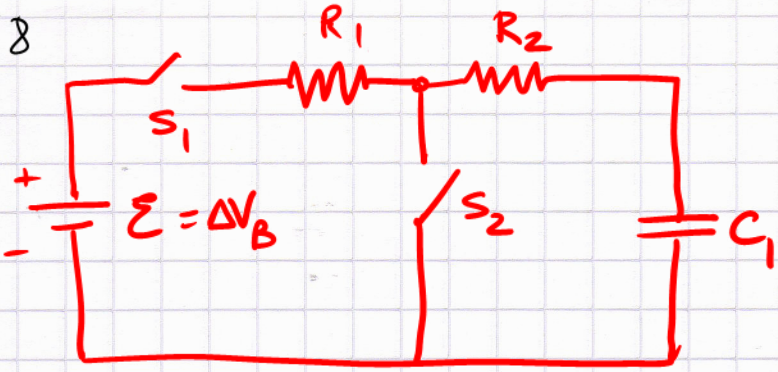
$$\frac{1}{R_{234}^{eq}} = \left(\frac{1}{1.4} + \frac{1}{4.5} + \frac{1}{6.0}\right)\frac{1}{k\Omega}$$

$$R_{234}^{eq} = 0.906\ k\Omega \rightarrow 900\ \Omega$$

$$R_{tot}^{eq} = 2.4\ k\Omega + 0.9\ k\Omega = 3.3\ k\Omega$$

$$i_1 = i_{batt} = \frac{3.0V}{3.3k\Omega} = 0.91\ mA$$

19.58



close S_1

→ what is i_{batt} ?
just after S_1 closes

Solution

with S_2 open we have $R_{12}^{eq} = R_1 + R_2$ in series with C_1, i.e, an RC circuit.

With C_1 uncharged at this instant, $\Delta V_c = 0$ ($= Q/C_1$, with $Q = 0$).

Thus R_{12}^{eq} limits the current: $i_{batt} = \dfrac{\Delta V_B}{R_{12}^{eq}}$

$$R_{12}^{eq} = (1.5 + 2.4)\ \text{k}\Omega = 3.9\ \text{k}\Omega$$

$\Delta V_B = 5.5\ \text{V}$ $\quad \therefore \quad i_{batt} = \dfrac{5.5}{3.9}\ \text{mA} = 1.4\ \text{mA}$.

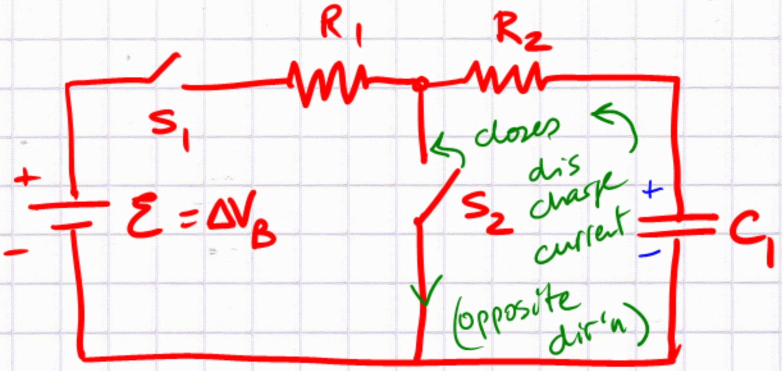
NB: This current changes rapidly:

$$i(t) = \frac{\Delta V_B}{R_{eq}} e^{-t/\tau}$$

where $\tau = R_{eq} C$ is the time constant

after τ seconds $i(\tau) \approx \frac{1}{3} i(0)$ $\quad (1/2.7\ldots)$

19.59



S_1 was closed for a long time

Now simultaneously S_1 is opened and S_2 is closed

- what is the current through S_2 just when it closed?
- show a graph of $i_2(t)$
- $i_2(t \to \infty) = ?$

Solution.

S_1 was closed for long $\therefore$ $\Delta V_{C_1} = \Delta V_B$; $i_1 = 0$

all charge separation has taken place, C_1 is charged to $Q = C_1 \Delta V_B$

since there is no current, by Ohm's law there is no voltage drop across R_1 or R_2 $\therefore \Delta V_{C_1} = \Delta V_B$!

- simultaneously opening S_1 & closing S_2:

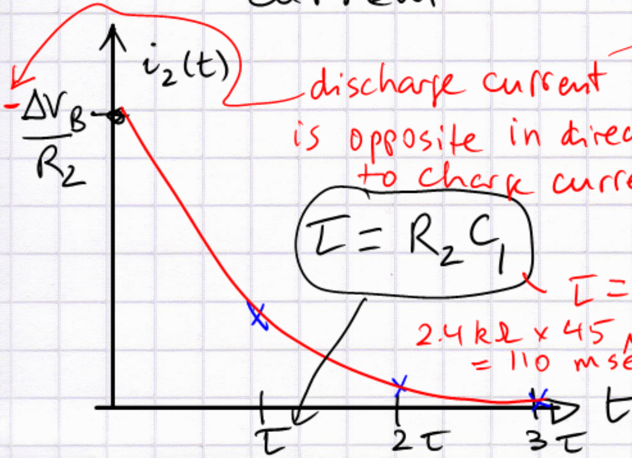
a) battery is disconnected

b) C_1 starts discharging through R_2: $i_2(0) = \dfrac{-\Delta V_B}{R_2}$

$-2.3\,mA$

R_2 limits the discharge current

$|\Delta V_{R_2}| = |\Delta V_{C_1}|$ (Kirchhoff loop) $\left| \dfrac{5.5V}{3.9k\Omega} \right.$



discharge current is opposite in direction to charge current

$\boxed{\tau = R_2 C_1}$

$\tau = 0.11s$

$2.4k\Omega \times 45\mu F = 110\ msec$

Long time (multiples of τ)

$\to$ current is zero

(C_1 plates are discharged)